# Supplementary Material - Plane-based Surface Regularization for Urban 3D Reconstruction 

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## 1 Additional Experiments

We evaluated on the additional dataset Building Complex. This dataset consists of four similarly shaped buildings mainly composed of planar surfaces. The input point cloud was created by PMVS2 [ $[$ ] and consists of approx. 3.4M points. In Fig. 1, an input image and results of our method are illustrated in comparison to others. As one can see, the planar surfaces on the building surface and on the ground floor are reconstructed well with the proposed method. Previously detected planar structures are used to denoise and simplify the scene. The effects are clearly visible in comparison to Labatut et al. [ $]$ ] which does not aim to simplify surface structures in urban environments. In contrast, Li et al. [ $\left[\begin{array}{l}\text { ] makes }\end{array}\right.$ strong (Manhattan) assumptions about the scene and can only deal with orthogonal planar structures. Fig. 2 shows very similar results on the Block Building dataset.

In sum, our method can be seen as a hybrid approach where the user can define the smoothness and level-of-detail continuosly between no regularization such as Labatut et al. [ $\mathbb{\square}]$ and very strong regularization and simplification such as Li et al. [ $[\boldsymbol{B}]$.

## 2 Tetrahedra Subdivision

Each tetrahedron (also named cell) with at least one intersected edge with the plane gets subdivided into multiple ones. Adjacent cells to intersected ones also need to get subdivided adequately to enforce consistency within the whole triangulation.

To enforce the consistency with the neighbors and internally between the newly created cells, each intersected cell gets divided into multiple cells using a fixed subdivision scheme.


Figure 1: Comparison of results on the Building Complex dataset. While the result without shape priors (Labatut et al. [D]) contains a lot of noise, the result of the proposed method produces planar surfaces and sharp edges. Due to a lower point density at surfaces in between the buildings, no planes were detected in these parts and they hence remain noisy. The results of Li et al. [ B ] over-simplify the geometry and several empty parts between the buildings are not correctly identified, because the method is very sensitive to noise.


Figure 2: Comparison of results on the Block Building dataset. In comparison to Labatut et al. [ $\square]$ which does not use any shape priors, our approach simplifies the geometry of surfaces along planar primitives, while keeping any desired level of detail. The strict Manhattan assumption of Li et al. [ B$]$ heavily simplifies the scene and leaves little or no control to adjust the level of detail.

### 2.1 Tetrahedra Intersected by Plane

Depending on how the plane intersects the tetrahedron, different subdivision cases have to be defined. The possible subdivision cases are illustrated in Fig. 3. If a neighboring cell from an intersected cell is already subdivided, the current subdivision is oriented accordingly to the already subdivided neighboring cell so that the adjacent cells are consistent (i.e., the cell is rotated until it fits to the neighboring cell). However, when two or more already divided neighbor cells exist, the triangulation might not stay consistent. Therefore, changes in the subdivision schemes are necessary.

### 2.2 Consistency Adoptions

When an intersected cell has two or more already divided cells as neighbor, the default subdivision scheme from Fig. 3 may not lead to a consistent triangulation. Therefore, the subdivision scheme needs to be adopted depending on the adjacent cells. In some cases, this


Figure 3: Subdivision schemes of tetrahedra intersected by a plane. Depending on the amount of edges intersected by the plane and their locations, the cell needs to be divided differently. The cut by the plane is illustrated with green edges, additional edges which need to be inserted are illustrated in red. In (a) and (b) there is an edge-plane intersection which results in a new vertex for each intersection. Four new cells are created in (a) and six new cells are created in (b). In (c), the cell-plane intersection follows exactly an edge. Therefore, just one new vertex and two new cells are created. In (d), the cell-plane intersection comprises one cell vertex. The cell gets divided into three new cells by adding two new vertices.
may lead just to a differently oriented subdivision with the same amount of resulting cells. However, there exist cases where the subdivision results in more cells (up to 12). These more sophisticated consistency adoptions only happen at 3-point and 4-point intersections (as illustrated in Fig. 3). A subset of these adoptions can be seen in Fig. 4 and Fig. 5. At a vertex intersection, an adoption might also be necessary. However, just a simple one: Only the newly inserted edge needs to be flipped.

### 2.3 Margin Tetrahedra

Cells not directly intersected by a plane but adjacent to a subdivided cell must also be subdivided in order to keep the whole triangulation consistent.

Depending on the adjacent face of the subdivided neighbor cell (i.e., if the neighboring face is divided or not), the margin cells need to get divided differently:

- If a cell has more than two divided neighboring faces, this cell also needs to be subdivided as if it would be intersected by the plane.
- If a cell has only one intersected neighbor with a not divided adjacent face, this cell does not need to be subdivided. Only the neighbor relationship needs to be updated.
- If a cell has one divided neighboring face, this cell has to be divided into three new cells, which are consistent to the neighbor. Fig. 6 (left) illustrates this subdivision.
- If a cell has no intersected neighbor, but contains a newly created vertex in an edge (i.e., an incident cell of an edge was intersected by the plane), the cell has to be divided into two cells. Fig. 6 (right) illustrates this subdivision scheme.


## References

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Figure 4: Consistency adoptions of cells with 3-point intersection. Edges to be changed in order to stay consistent with neighbors are depicted in blue. In the first two subdivision schemes from left, only one edge and its corresponding cells need to be changed. No additional change in the internal structure is necessary. In the third subdivision scheme from left, also only one edge needs to be changed to stay consistent to its neighbors. However, to stay consistent within the new cell structure, a new vertex needs to be inserted in the middle of the cell and finally the cell gets divided into 9 cells (instead of four). In the figure to the right, two edges need to be changed to stay consistent with the neighbors. Also in this case an additional vertex needs to be inserted in the middle of the cell and the cell gets divided into 9 cells. For the other subdivision schemes with two changed edges, the internal structure does not need to be changed.


Figure 5: Consistency adoptions of cells with 4-point intersection. Edges to be changed in order to stay consistent with neighbors are depicted in blue. Top row: Subdivision schemes, where only one edge and its corresponding cells need to be changed to stay consistent with the neighbors. In the first two schemes from left, no additional change in the internal structure is necessary. In the first two schemes from right, also the internal (newly created) edge lying on the plane needs to be changed. Bottom row: In the first two schemes from left, two outer edges need to be changed. To stay consistent internally, an additional vertex and edge needs to be inserted on the intersection plane. Hence, the cell gets divided into 12 cells (instead of 6). In the first two schemes from right, three outer edges need to be changed. To stay internally consistent, also the internal edge lying on the plane needs to be changed. For the other subdivision schemes with two and three changed edges, the internal structure needs not be changed.


Figure 6: Subdivision schemes of margin tetrahedra. Left: Margin tetrahedron adjacent to an intersected cell. The cell gets divided into three cells. Right: Margin tetrahedron incident to an intersected edge. The cell gets divided into two cells.
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