Quadrics of Revolution On Given Points

Anton Gfrerrer
Institute of Geometry
gfrerrer@tugraz.at

Paul Zsombor-Murray
Department of Mechanical Engineering, McGill University
paul@cim.mcgill.ca

There are very early contributions of E. Laguerre [4] to the task of finding all quadrics of revolution on 5 given points P_1, P_2, P_3, P_4, P_5: Laguerre interlinked this problem to the circumspheres determined by the 5 quadruples P_i, P_k, P_j, P_l that can be established from the five given points. More recently, the problem of determining all right cylinders on 4 given points P_i has been solved by H. Schaal [10], [11]: There is a 1-parametric set of cylinders of revolution and in case of non-coplanar points P_i their axes establish an algebraic surface of degree 3. The task of finding all right cylinders on 5 points is an algebraic problem of order 6: In the generic case there exist 6 solution cylinders at most [14]. The cones of revolution on 4 given points establish a 2-parametric set, the locus of vertices is an algebraic surface of order 14, see [12], [13]. O. Röschel [9] determined all quadrics of revolution containing a given conic section. There are also early contributions to this problem by A. Narasinga Rao and M. S. Srinivasachari [7], [8].

In my presentation I discuss the approach [3] which allows to identify different kinds of quadrics of revolution on a given number n of points. By means of this method one can easily check that the axes of the 1-parametric set of quadrics of revolution on 6 prescribed points intersect the plane of infinity along a conic section. Moreover I will show that there are at most 4 quadrics of revolution on 7 given points and at most 12 right cones on 6 given points in the generic case. The forementioned task of determining the right cylinders on 5 points can also be treated by the introduced method. In each case I can give examples where the maximal solution numbers 4, 12 and 6 are obtained by *real* quadrics of revolution, right cones and right cylinders, respectively.

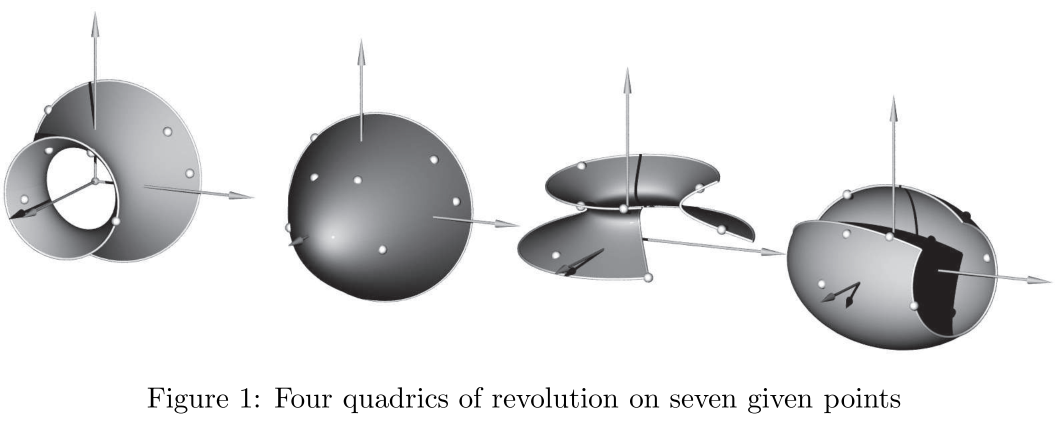

Figure 1: Four quadrics of revolution on seven given points

References

[1] C. Beder, W. Förstner, Direct Solutions for Computing Cylinders from Minimal Sets of 3D Points, *Proc. of the European Conference on Computer Vision, Graz, Austria* (2006), 135–146.

[2] O. Devillers, B. Mourrain, F. P. Preparata, P. Trebuchet, Circular Cylinders by Four or Five Points in Space, *Discrete and Computational Geometry* **29** (2003), 83–104.

[3] A. Gfrerrer, P. J. Zsombor-Murray, Quadrics of Revolution on Given Points, *Journal for Geometry and Graphics*, **13**, 2 (2009), 131–144.

[4] E. Laguerre, Sur la courbe enveloppée par les axes des coniques qui passent par quatre points donnés et sur les axes des surfaces de révolution du second ordre qui passent par cinq points donnés. Sur les lignes spiriques, *Nouv. Ann. Math. (2)* **18** (1879), 206–218.

[5] D. Lichtblau, Cylinders Through Five Points, Complex and Real Enumerative Geometry, *F. Botana, E. Roanes-Lozano (eds): Automated Deduction in Geometry ADG* (2006).

[6] E. Müller, Durch vier Punkte einen Drehkegel von vorgegebener Achsenrichtung zu legen, *Arch. d. Math. u. Phys. (3)* **22** (1914), 225–226.

[7] A. Narasinga Rao, Quadrics of Revolution through a Given Conic, *J. Ind. Math. Soc.* **17** (1928), 213–222.

[8] A. Narasinga Rao, M. S. Srinivasachari, Quadrics of Revolution through a Pair of Skew Lines, *J. Annamali Univ.* **2** (1933), 71–77.

[9] O. Röschel, Drehflächen zweiter Ordnung durch einen Kegelschnitt, *Stud. Sci. Math. Hung.* **29**, n. 3-4 (1994), 379–386.

[10] H. Schaal, Ein geometrisches Problem der metrischen Getriebesynthese. *Sb. d. Österr. Akad. d. Wiss., math.-nat. Klasse, Abt II* **194**, Heft 1–3 (1985), 39–53.

[11] H. Schaal, Konstruktion der Drehzylinder durch vier Punkte einer Ebene. *Sb. d. Österr. Akad. d. Wiss., math.-nat. Klasse, Abt II* **195**, Heft 4–7 (1986), 406–418.

[12] U. Strobel, Über die Drehkegel durch vier Punkte. *Sb. d. Österr. Akad. d. Wiss., math.-nat. Klasse* **198** (1989), 281–293.

[13] U. Strobel, Über die Drehkegel durch vier Punkte. Teil II. *Sb. d. Österr. Akad. d. Wiss., math.-nat. Klasse* **200** (1991), 91–109.

[14] P.J. Zsombor-Murray, S. El Fashny, A cylinder of revolution on five points, *Journal for Geometry and Graphics* **10**, 2 (2007), 125–131.