Wave Propagation in Viscoelastic and Poroelastic Continua: A Boundary Element Approach

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Summary

A novel boundary element formulation in time domain has been presented based on the convolution quadrature method. This numerical quadrature formula determines its integration weights from the Laplace transformed fundamental solution and a linear multistep method. Hence, boundary element time-stepping techniques for elastodynamic, viscoelastodynamic, and poroelastodynamic continua have been developed although in case of viscoelasticity and poroelasticity only Laplace domain fundamental solutions are known. So, this method combines the advantage of the Laplace domain with the advantage of a time domain calculation. Finally, wave propagation in a 1-d poroelastic column and in visco- or poroelastic half spaces has been considered.

In turn, time-dependent integral equations contain fundamental solutions which are convoluted with time-dependent boundary data. In the presented formulation, this convolution integral is approximated by the convolution quadrature method. The integration weights of this quadrature rule are determined by the Laplace transformed fundamental solution and a linear multistep method. Beside some numerical aspects, this way to establish a time-stepping BE formulation has two main advantages:

1. Only Laplace transformed fundamental solutions are used enabling a time-dependent BE formulation without the knowledge of the time-dependent fundamental solution.
2. The stability of the time-stepping procedure is improved, whereas with different underlying multistep methods different ``optimal'' time step sizes can be achieved.

Focused on the first advantage, this boundary element method makes it possible to establish time domain boundary element formulations in cases which traditionally are solved in Laplace domain with a subsequent inverse transformation. This traditional procedure is not possible for transient boundary conditions, e.g., contact problems or moving surfaces, and is dependent on a proper choice of some method dependent parameters.

Here, a viscoelastic and a poroelastic BE formulation has been developed without the knowledge of the time-dependent fundamental solutions. Further engineering problem solutions achievable by following the new approach can be found. For every problem which has at least a fundamental solution in Laplace domain the convolution quadrature based boundary element method leads to a solution. Further, as the complexity of the Laplace domain fundamental solution is mostly less than the corresponding time domain solution, a BE formulation using the convolution quadrature is always advantageous.

The second advantage, listed above, concerning stability was shown in chapter 4 at the example of an elastodynamic boundary element formulation. As in all BE time-stepping procedures a lower critical time step size exists below which the algorithm become unstable. This critical value is approximately ten times smaller as in the classical formulation. More important, for a fine enough spatial discretization this value tends to zero, i.e., in the limit no critical lower stability bound exists. Additionally, the underlying multistep method has a strong influence. It was found that a *A*-stable method with stability in infinity has to be used, e.g., BDF 2.

The convolution quadrature method based BE formulation is introduced and evaluated to model viscoelastic as well as poroelastic continua in the chapters 5 and 6. Concerning the spatial and temporal discretization the viscoelastic formulations behave like the elastodynamic one, whereas the poroelastic formulation needs a much finer spatial discretization as a qualitatively comparable elasto- or viscoelastodynamic calculation.

The influence of the viscoelastic and poroelastic material modeling is studied in chapter 7 at the example of wave propagation in a half space. Modeling the half space viscoelastic, i.e., taking higher order time derivatives in the stress-strain relation into account, results in a more stiff behavior of the half space compared to an elastic modeling. The poroelastic material model, i.e., dissipation caused by the friction between the elastic solid skeleton and the interstitial viscous pore fluid is introduced, results in the following effects:

- The displacement amplitudes caused by the Rayleigh wave are increased compared to the viscoelastic and the elastic case.
- The Rayleigh wave causes a compressional wave in the pore fluid.
- As expected from theory, the effect of the Rayleigh wave vanishes with depth.

- The second slow compressional wave in the poroelastic medium is found in the limit of an inviscid pore fluid, but for the analyzed materials, Berea sandstone and a soil, for realistic values of the permeability its effect vanishes after a short traveling distance.

These results show that modeling a half space viscoelastic or poroelastic is quite different, i.e., a fluid saturated material should not be modeled viscoelastic.

Summarizing, the proposed boundary element formulation based on the convolution quadrature method combines the advantage of the Laplace domain, i.e., the derivation of a fundamental solution is mostly simpler as in time domain, with the advantage of a time domain calculation, i.e., transient boundary conditions can only be modeled in time domain.

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