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USING RELATIVISTIC FREQUENCY SHIFT IN MULTI-SATELLITE GRAVITY FIELD MISSIONS

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INTRODUCTION

Einstein‘s general theory of relativity states that space-time and gravity are fundamentally connected. A clock is not only attracted by a mass‘ gravitation, its time is also running ‚slower‘. By simplifying Einstein‘s field equations with a post-newtonian approximation a connection between the potential difference and the time dilation of two atomic clocks can be derived. A satellite sends an electromagnetic wave to a geostationary receiver station which compares its frequency with a reference wave to compute the time dilation and thus, the gravitational potential.

Geophysical signals and resulting gravity field models of currently operating satellite gravity field missions like GRACE and GOCE can be attributed to earth‘s static or time variable gravity field. Moreover, the quality of the derived gravity field depends on the type of observable and its error structure, various combined geophysical signals and the orbit configuration of the mission. Thus, ocurring aliasing effects have to be taken into account in future satellite gravity field missions.

Current developments suggest that the short-term time measurement stability of future atomic clocks will make it possible to derive earth‘s gravity field with sufficient quality for low degrees. Previous simulations have shown that the use of relativistic time dilation for gravitational reconstruction is mathematically feasible. In terms of future satellite gravity field missions a combination of classical measurement methods with atomic clock observations would lead to more isotropic error strucures and lower deviations for low degrees and orders. The scope of this paper is to show how the derived gravity fields change when gravity satellite missions are addionally supported by atomic clock observations.

SIMULATION ENVIRONMENT

MISSION DESIGN

A GRACE-like mission setup has been chosen. A leader-follower configuration with 200 km separation on polar orbits at 450 km heigth was created. Range rate observations and additional atomic clock observations were created. For the time dilation observable two transmitter clocks situated in both satellites were simulated. A corresponding receiver clock was put in a geostationary orbit. It was assumet that the gravitational potential, the velocity of the geostationary satellite and its position are known error-free.

OBSERVABLES AND FUNCTIONAL MODEL

Two months time series for following gravity field functionals have been created:
- range rates from inter-satellite tracking
- time dilation from atomic clock observations

The correspoding orbits were generated by orbit integration up to degree/order 90. Based on orbit positions range rates have been computed. The time dilation observations have been computed by utilizing orbit positions and velocity and potential differences between the transmitter and receiver clocks.

For deriving the influence of measurement uncertainties white noise has been applied to the observations. The standard deviation of the noise types are:
- orbit position and velocity: 2 cm and $1\cdot10^{-7}$ m/s
- range rate: $2\cdot10^{-7}$ m/s²
- time dilation: $1\cdot10^{-18}$ s

Based on these standard deviations a complete variance-covariance propagation has been done. The obervables have been derived up to maximal degree/order 90.

NUMERICAL CASE STUDIES

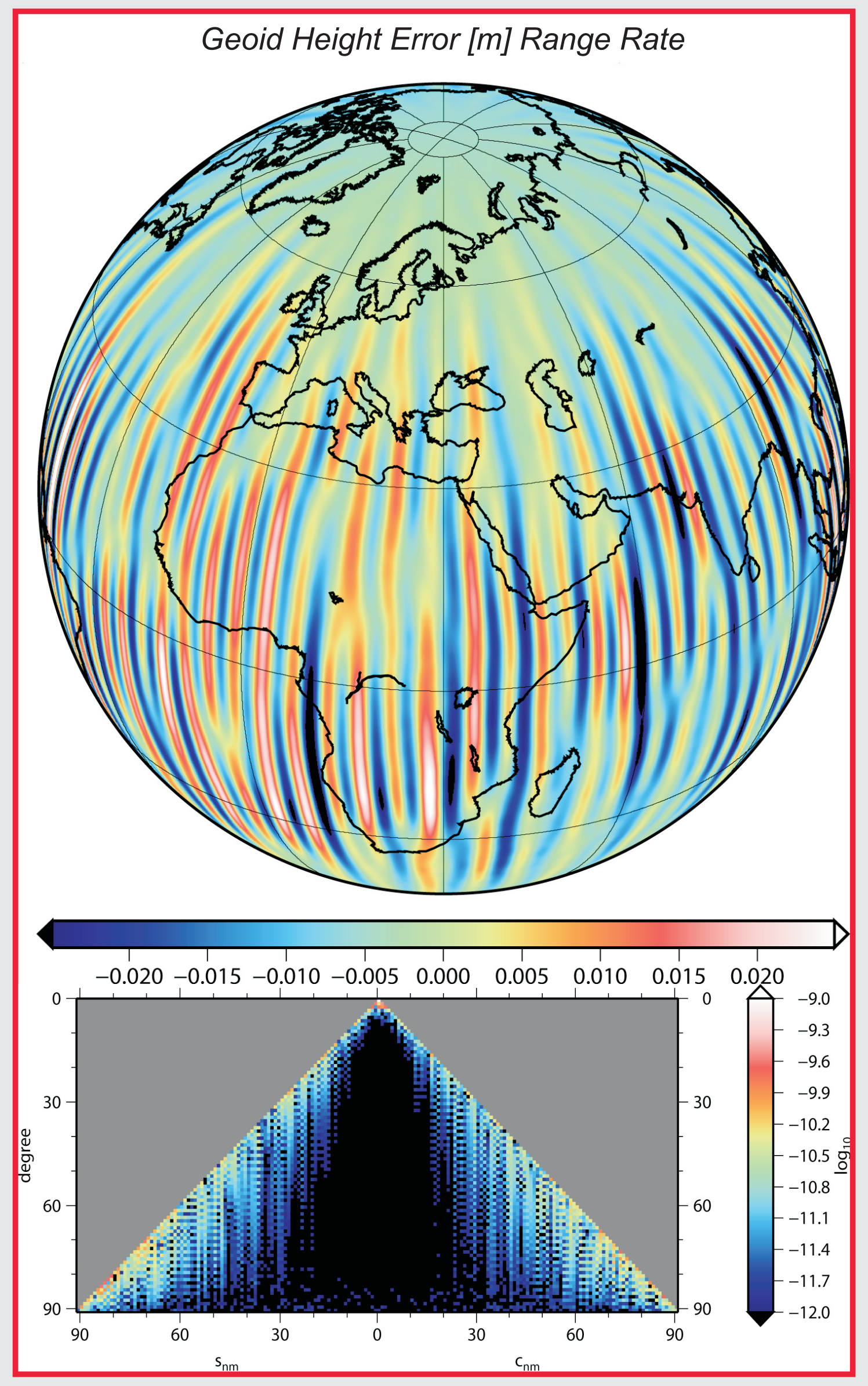

Fig. 1: Simulated error of static gravity field for range rate observations up to d/o 90 in terms of geoid heights [m] (upper) and coefficient deviations (lower).

STATIC GRAVITY FIELD

A static gravity field model has been derived from range rates observations (fig. 1) and one from atomic clock time dilations (fig. 2). GOCO-02S has been used as reference gravity field .

The geoid height error plots show that SST-ll observations lead to north-south directed pattern, so-called striping (fig. 1, upper). This is caused by the non-isotropic attribute of the range rate observation. In contrast to this the geoid height error pattern from the mission utilizing time dilation is perfectly isotropic (fig. 2, upper). Consider the different scaling of both geoid height error plots. While the overall deviation is for the SST-ll mission in the centimetre range, the geoid height error from the atomic clock mission is in the decimetre range. This can also be seen in the coefficient error plots (fig. 1 and 2, lower). The degree error deviation plot shows that the time dilation mission scales with the degree and is better than the range rate mission for very low degrees (fig. 3).

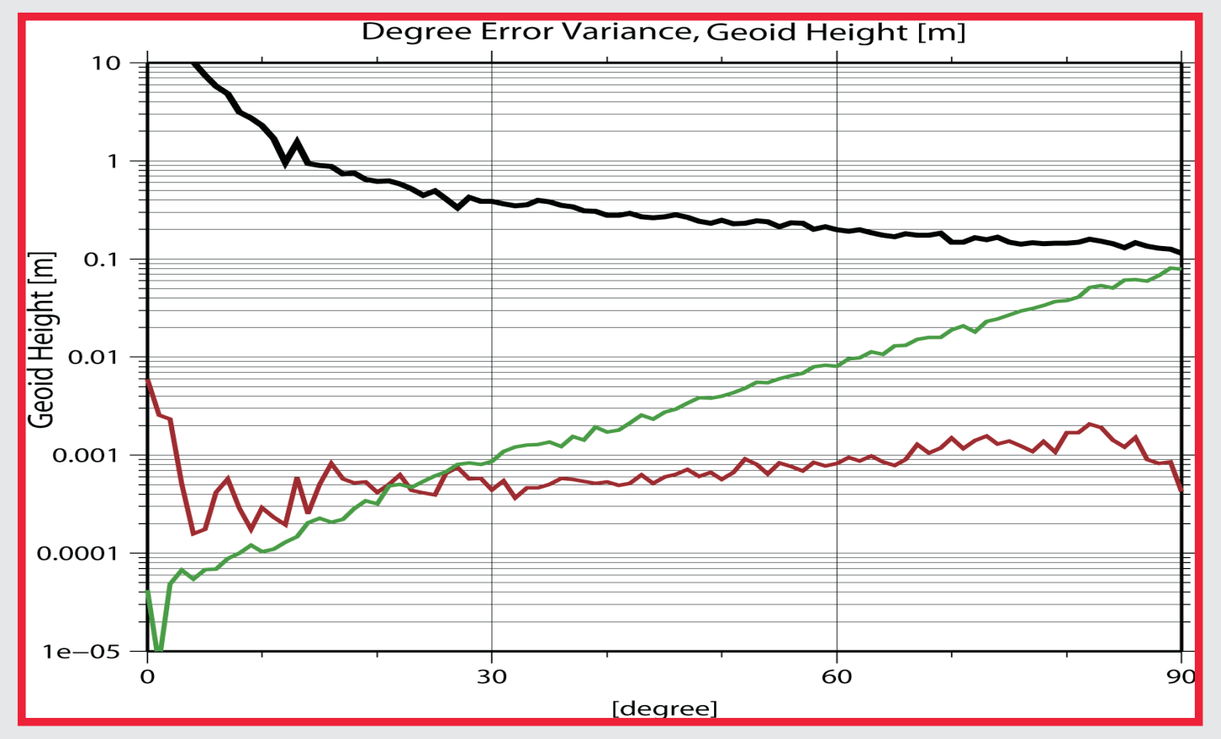

Fig. 3: Degree error variances. Black: reference GOCO-02S, Red: SST-ll, Green: time dilation.

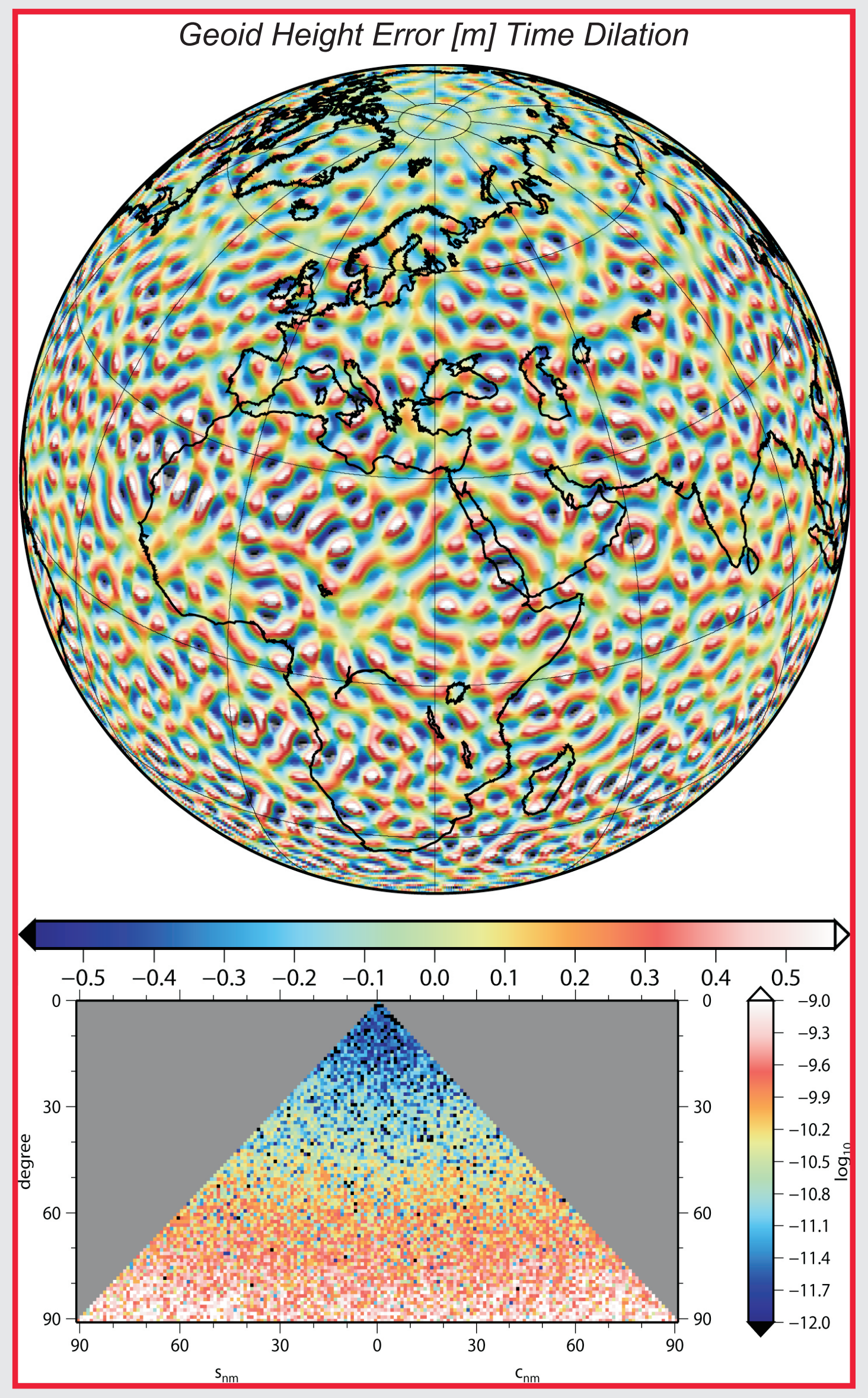

Fig. 2: Simulated error of static gravity field for time dilation observations up to d/o 90 in terms of geoid heights [m] (upper) and coefficient deviations (lower).

TIME VARIABLE GRAVITY FIELD

For future satellite gravity field missions the ability to map earth‘s time variable gravity field will be crucial. Beside the static gravity field model GOCO-02S the global ocean tide model EOT08a has been used to create the observations.

The time variable gravity field observations have been directly used for deriving spherical harmonic coefficients, no reference tides have been substracted. Thus, the ocean tide causes aliasing in the SST-ll solution, the overall quality decreases to 0.5 metres geoid height error (fig. 4, upper left) with strong striping. This can also be seen in the sectorial coeffcients (fig. 4, upper right). In an attempt to reduce ocean tide aliasing effects, additional atomic clocks have been simulated. It can be seen that the striping pattern has been reduced (fig. 4, lower left) and low frequency and sectorial coefficients are much better estimated (fig. 4, lower right). Thus, the isotropic error structure of satellite missions using additional atomic clocks leads to gravity field models with less sensitivity to temporal aliasing.

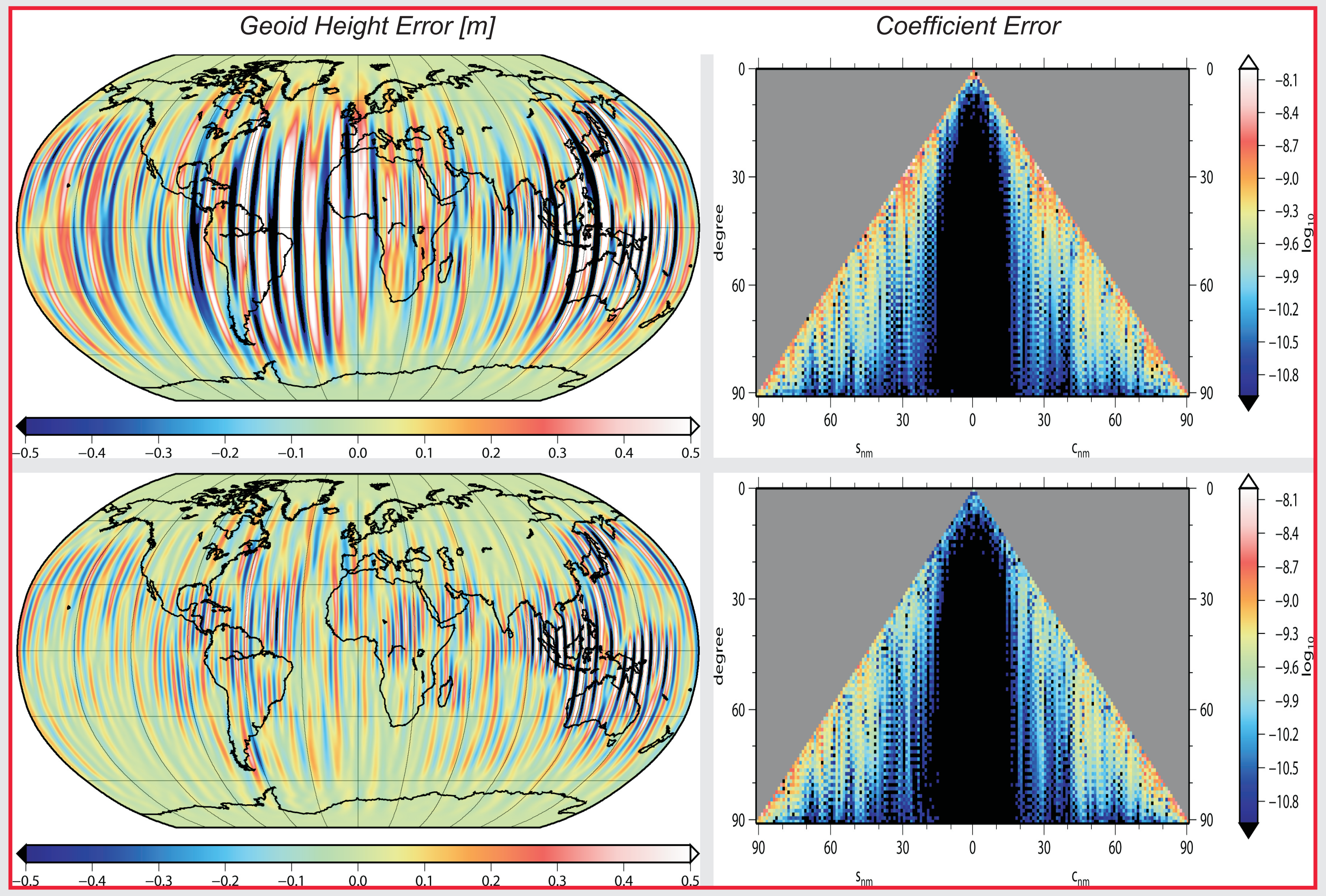

Fig. 4: Geoid height error [m] (left) and coefficient error (right) of SST-ll only mission (upper) and combination of SST-ll and time dilation mission (lower) including tides.