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THE SIGNIFICANCE OF GPS/LEVELING POINTS FOR THE HIGH PRECISION GEOID COMPUTATION

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ABSTRACT

The computation of a geoid can be done by several ways. Least Squares Collocation (LSC) [1] as one of the possible tools has the big advantage that observations representing different functionals of the disturbing potential can be combined. Therefore GPS/leveling observations can be used as observation values. For the high precision geoid computation the importance of this kind of data is undisputed. Beside the assumption that these observations have a high level of accuracy, the question about the significance of highly weighted GPS/leveling observations for the prediction of gravity field quantities arises. Apart from proper weighting the different kind of observations, the fitting of the geoidal surface to the GPS/leveling benchmarks by a transformation surface can cause problems. For modeling the transformation surface an alternative approach using Radial Basis Functions (RBF) [2] in association with the Greedy algorithm [3] is used. The algorithm detects the minimum amount of GPS/leveling observations for geoid fitting by the transformation surface.

DATA

For all investigations which have been done, different gravity field quantities in Austria and the neighboring countries are available. These quantities and their spatial distribution can be seen in Figure 1.

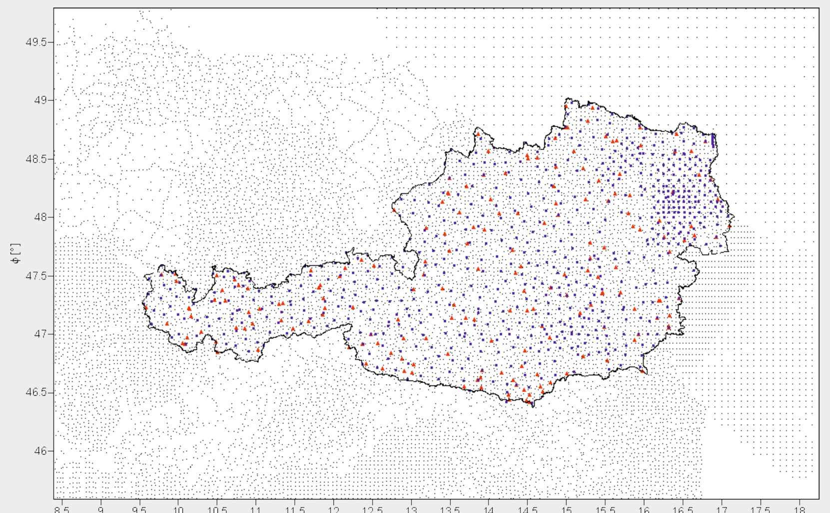

Fig.1: All available terrestrial gravity field quantities

The data set consists of 13689 gravity anomalies with an approximate distance of 4km (displayed in black), 672 deflections of the vertical (blue) and 192 GPS/leveling observations (red). For LSC computation a constant a-priori σ of Δg = 1mGal and ξ,η = 0.3" was assumed. For the reduction of the topographic masses in the frame of the Remove-LSC-Restore technique a Digital Terrain Model with a resolution of 44 x 49m has been used. A standard crustal density of 2670kg/m^3 was chosen. The isostatic part of the reduction is computed by an Airy-Heiskanen model. The computation of the local reductions has been performed with an adapted version of the program Terrain Correction (TC) [4]. The global part of the reduction is represented by the GOCO02S satellite-only gravity field model which is available up to Degree/Order (D/O) 250.

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GPS/LEVELING OBSERVATIONS

In case of fitting the geoid to GPS/leveling observations LSC needs information about the accuracy of all input data. For the GPS/leveling points different a-priori σ were chosen to find a proper weight using an empirical approach. In Figure 2 differences between the GPS/leveling points and the computed geoid heights can be seen. According to different weights the fitting residuals are increasing. For the prediction of geoid heights using LSC the optimal a-priori weight is achieved with σ = 1mm.

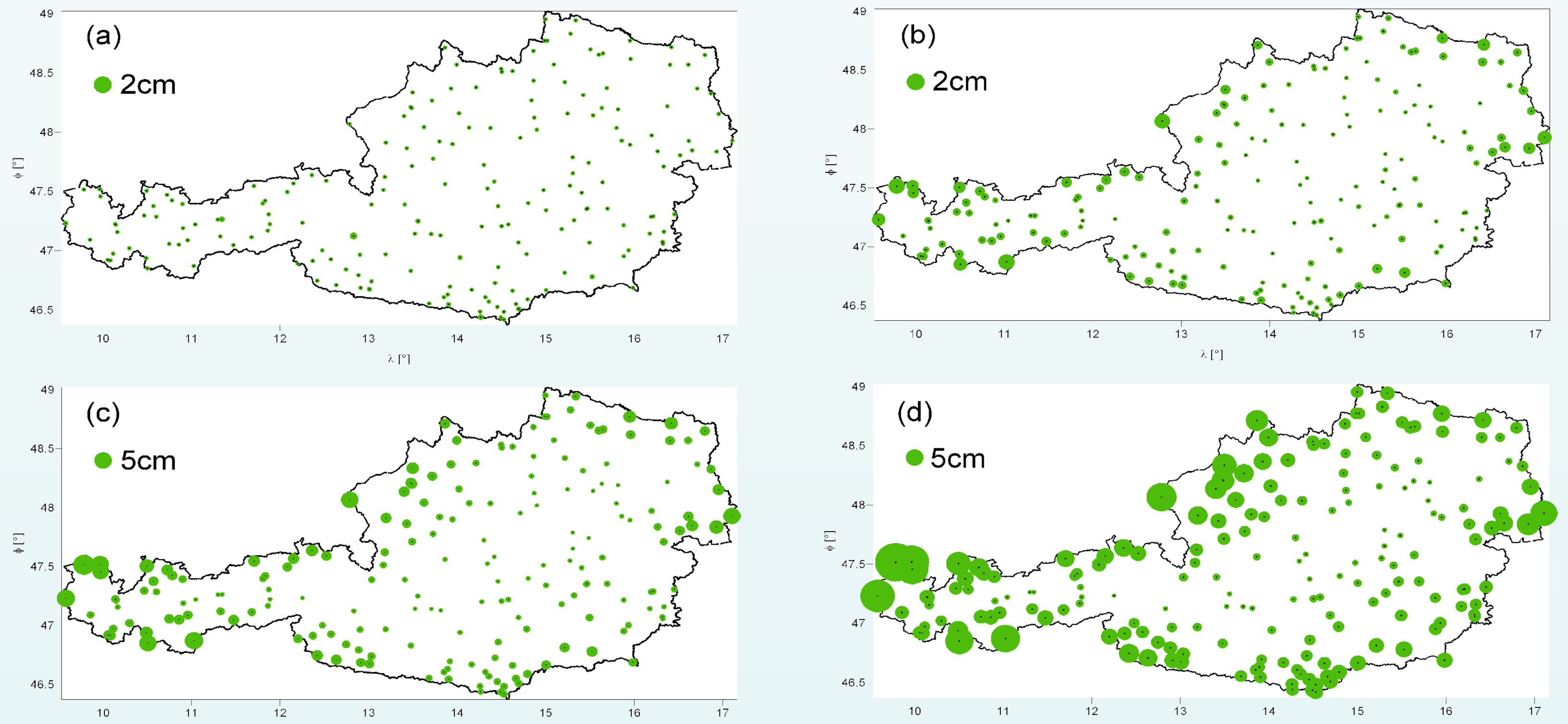

Fig.2: Differences in 192 observations between GPS/leveling geoid and the geoid solutions based on 1mm (a), 5mm (b), 10mm (c) and 20mm (d) a-priori weights (displayed as absolute values)

GRAVITY ANOMALIES

This investigation will reveal the impact on predicted gravity anomalies if GPS/leveling points with varying a-priori σ were taken into account. In a first step the optimal weight for geoid heights is selected (σ = 1mm). In Figure 3 differences between predicted gravity anomalies and the used input data can be seen. On the one hand the computation is done with and on the other hand without GPS/leveling observations. The differences are depending on the chosen D/O of the global reduction. Also the area near the GPS/leveling points is affected. This means that the predicted gravity anomalies are strongly influenced by the highly weighted GPS/leveling observations. In a next step the observations are continuously downweighted to σ = 20mm. Henceforth no significant impact on gravity anomalies can be observed anymore but the fitting residuals are increasing (see Figure 2). For the prediction of gravity field quantities with LSC the chosen a-priori weights are always depending on the desired output.

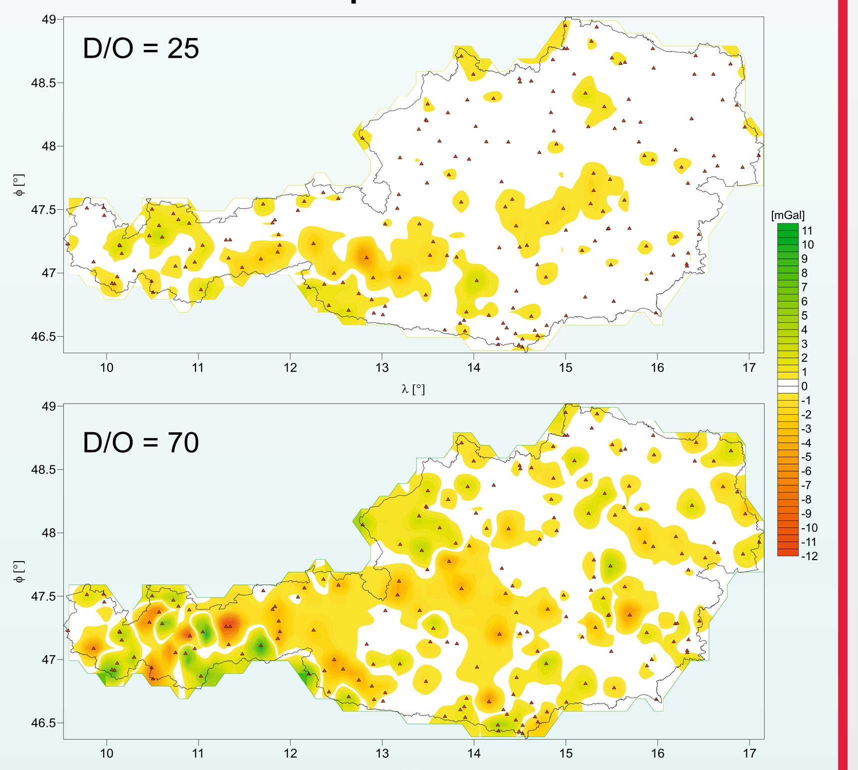

Fig.3: Differences of input and output gravity anomalies predicted with LSC

TRANSFORMATION SURFACE BY GREEDY ALGORITHM AND RBF

The occurring differences between gravimetric and GPS/leveling geoid heights are attributed to datum inconsistencies, GPS errors or different tidal systems. All biases may be represented by a correction surface and can be modeled using low degree polynomials p(x) or Radial Basis Functions Φ with the following interpolation function:

$$s(x) = p(x) + \sum_{i=1}^{N} \lambda_i \phi(|x - x_i|)$$

The λ_i are denoted as RBF coefficients, the x stands for the interpolation points and the x_i are the RBF centers. For the RBF modeling the Greedy algorithm was adapted to find the minimum number and location of the GPS/leveling observations that are necessary for the reconstruction of the correction surface [5]. The aim is to find redundant information in the data (see Figure 4). The Greedy

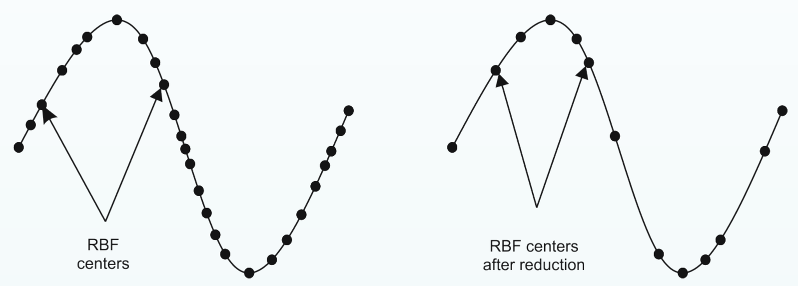

Fig.4: Reduction of RBF centers using Greedy algorithm

algorithm can provide the desired information. First starting points and a geometrical error margin has to be chosen. The error margin as an important input parameter is representing the maximum error occurring due to the reduction of the points. The amount of Greedy points is directly related to this error. The set of 192 GPS/leveling points can be split into Greedy points (needed for the reconstruction) and validation points. In case of an error of ± 1cm the algorithm detected 130 Greedy and 62 validation points (Figure 5). If the error margin is ± 2cm (Figure 6) 86 Greedy points are needed for the surface reconstruction. The remaining points can be used to check the solution. Geoid heights are predicted for the validation points with LSC. The differences between predicted and measured geoid heights are shown in Figure 7.

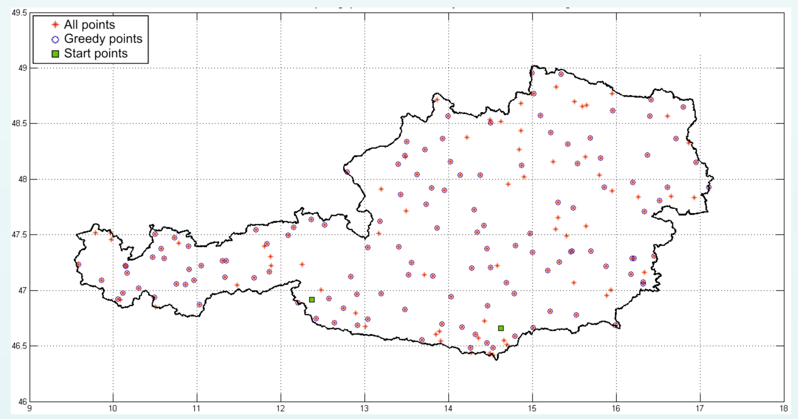

Fig.5: Greedy vs. validation points; used error margin: ± 1cm

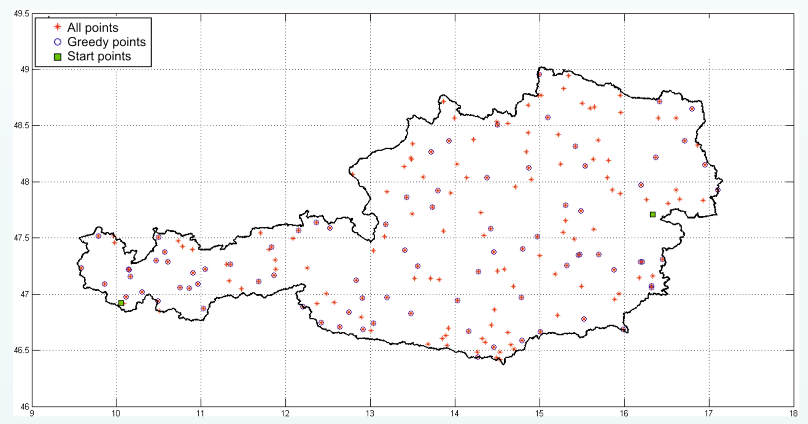

Fig.6: Greedy vs. validation points; used error margin: ± 2cm

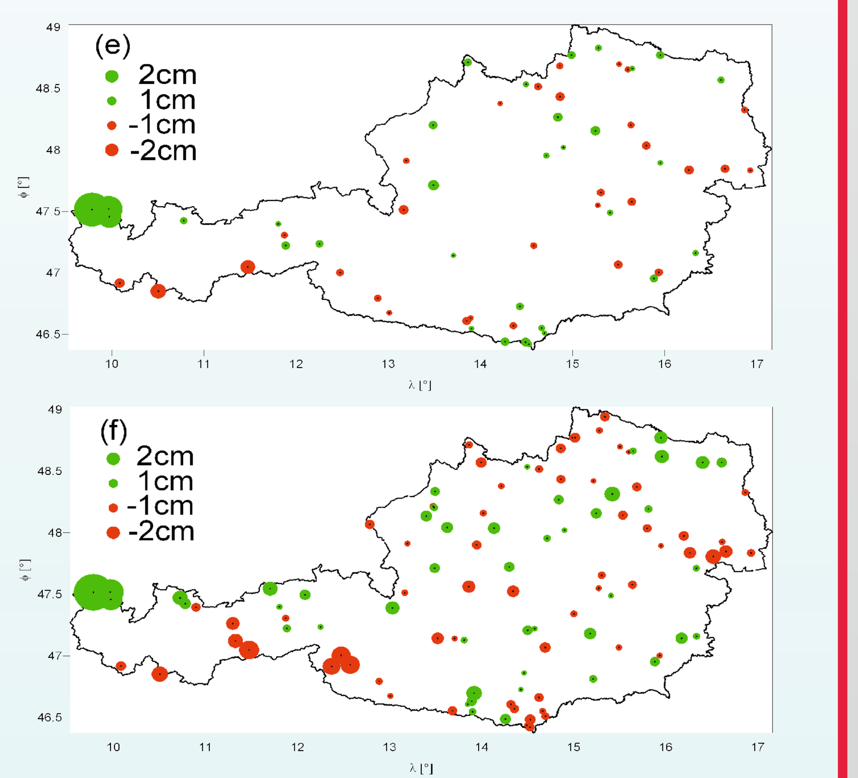

Fig.7: Achieved differences in validation points error margin: ± 1cm (e) and ± 2cm (f)