Simulation of Relativistic Jet Driven by a Spinning Black Hole

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Abstract

Relativistic jet phenomena are most often observed in the vicinity of black holes where the surrounding plasma accretion plays an important role in the formation of such jets. The presence of a magnetic field is crucial since it has a significant effect on the accretive behavior of the plasma. Primarily, the magnetic field links the central source with the ambient plasma and can be imagined as a set of wires which can transport energy towards the black hole and away by means of magnetohydrodynamical (MHD) waves. Moreover, the magnetic field is able to collimate the plasma flow and to form relativistic jets. The behavior of the magnetized plasma accretion around a spinning black hole is studied by the string approach, i.e., the magnetized plasma is modelled as a set of magnetic flux tubes/strings. It is shown that by the interaction of the magnetic flux tube with the spinning black hole energy can be extracted and, at the same time, a relativistic jet is created. In addition, magnetic reconnection within the jet leads to the formation of plasmoids which move away from the central source and carries energy and angular momentum. This process can be repeated many times and finally the jet is structured as a chain of plasmoids which propagate along the spin axis of the black hole.

Introduction

Starting with the original Penrose process (Penrose, 1969), several processes have been suggested for extracting spin energy from a black hole.

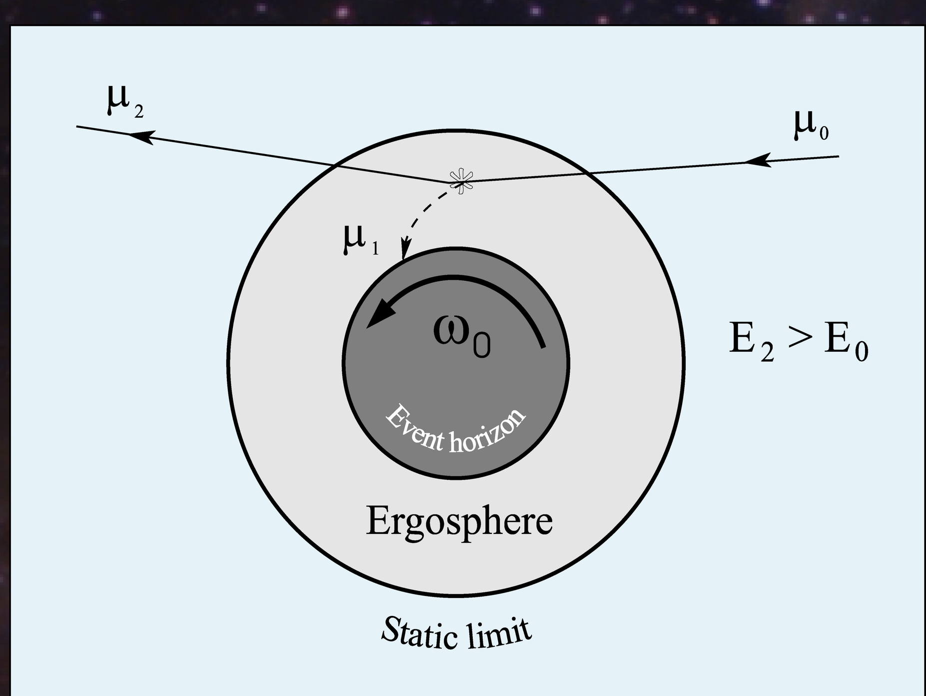

Fig. 1 A body falling from a certain distance enters the ergosphere of rotating black hole and explodes at any point close to the black hole surface in two fragments. One fragment is absorbed by black hole. The other one is ejected from the ergosphere having energy greater than the energy of the falling body.

Here we present a new mechanism based on the interaction of a magnetic flux tube, which can be modeled as a string, with a Kerr black hole. This suggestion builds on the notion that a plasma embedded in a magnetic field can be represented as a gas of non-linear strings (flux tubes), rather than a gas of particles (Semenov and Erkaev, 1992). Such an approach, based on examining the behaviour of a test flux tube, can provide clear physical meaning to the otherwise difficult problems of analyzing the dynamics of space plasmas.

Basic equations

A convenient mathematical formulation for studying the behaviour of flux tubes can be obtained through the introduction of lagrangian coordinates into the relativistic magnetohydrodynamic equations (RMHD) of general relativity. The RMHD equations are (Lichnerowicz, 1967):

$$\nabla_i \rho u^i = 0 \qquad (1)$$

$$\nabla_i (h^i u^k - h^k u^i) = 0 \qquad (2)$$

$$\nabla_i T^{ik} = 0 \qquad (3)$$

Here (1) – continuity equation, (2) – Maxwell's equation, (3) energy-momentum equation. Here u^i is the time-like vector of the 4-velocity, $u^i u_i = 1$, $h^i = {}^*F^{ik} u_k$ is the space-like 4-vector of the magnetic field, $h^i h_i < 0$, ${}^*F^{ik}$ is the dual tensor of the electromagnetic field, and T^{ik} is the energy-momentum tensor.

$$T^{ik} = Qu^i u^k - Pg^{ik} - h^i h^k / 4\pi \qquad (4)$$

$$P \equiv p - h^i h_k / 8\pi \qquad (5)$$

$$Q \equiv p + \varepsilon - h^i h_k / 4\pi \qquad (6)$$

Here p is the plasma pressure, P is the total (plasma + magnetic) pressure, e is the internal energy including ρc^2, g^{ik} is the metric tensor with signature (1,-1,-1,-1). Generally speaking, $\nabla_i h^i = 0$, but we can find a function q such that $\nabla_i q h^i \neq 0$. Then using (1) the Maxwell equation (2) can be rewritten in the form of a Lie derivative:

$$\frac{h^i}{\rho} \nabla_i \frac{u^k}{q} = \frac{u^i}{q} \nabla_i \frac{h^k}{\rho} \qquad (7)$$

and we can therefore introduce "frozen-in" coordinates, such that:

$$\frac{\partial x^i}{\partial \tau} = \frac{u^i}{q}, \quad \frac{\partial x^i}{\partial \alpha} = \frac{h^i}{\rho} \qquad (8)$$

with new coordinate vectors u^i/q, h^i/ρ tracing the trajectory of a fluid element and the magnetic field in a flux tube. Using (8) the energy-momentum equation (3) can be rearranged to form a set of string equations in terms of the frozen-in coordinates:

Using (8) the energy-momentum equation (3) can be rearranged to form a set of string equations in terms of the frozen-in coordinates:

$$-\frac{\partial}{\partial \tau}\left(\frac{Qq}{\rho}\frac{\partial x^i}{\partial \tau}\right) - \frac{Qq}{\rho}\Gamma^l_{ik}\frac{\partial x^i}{\partial \tau}\frac{\partial x^k}{\partial \tau} + \qquad (9)$$
$$+\frac{\partial}{\partial \alpha}\left(\frac{\rho}{4\pi q}\frac{\partial x^i}{\partial \alpha}\right) + \frac{\rho}{4\pi q}\Gamma^l_{ik}\frac{\partial x^i}{\partial \alpha}\frac{\partial x^k}{\partial \alpha} = -\frac{g^{il}}{\rho q}\frac{\partial P}{\partial x^i}$$

here $q = 1/(g_{ik} x_\tau^i x_\tau^k)^{1/2}$, and Γ_{ik}^l is the Christoffel symbol.

Nonlinear string in Kerr metric

The Kerr metric in Boyer-Lindquist coordinates is given by the following line element (Misner et al. 1973):

$$ds^2 = \left(1 - \frac{2Mr}{\Sigma}\right)dt^2 - \frac{\Sigma}{\Delta}dr^2 - \Sigma d\theta^2 - \qquad (10)$$
$$-\left(r^2 + a^2 + \frac{2Mra^2}{\Sigma}\sin^2\theta\right)\sin^2\theta d\phi^2 + \frac{4Mra}{\Sigma}\sin^2\theta d\phi\, dt$$
$$\Delta = r^2 - 2Mr + a^2, \quad \Sigma = r^2 + a^2\cos^2\theta$$

Here M and a are the mass and angular momentum of the hole, respectively, and we have used a system of units in which c = 1, G = 1. For cyclic variables t and φ, the energy and angular momentum conservation laws for the flux tube can be written as:

$$\int_{\alpha_1}^{\alpha_2} \frac{Qq}{\rho}(g_{tt} t_\tau + g_{t\phi}\phi_\tau)\, d\alpha = E \qquad (11)$$

$$\int_{\alpha_1}^{\alpha_2} \frac{Qq}{\rho}(g_{t\phi} t_\tau + g_{\phi\phi}\phi_\tau)\, d\alpha = -L \qquad (12)$$

By means of the inhomogeneous rotation of the space around the black hole, the flux tube becomes stretched and twisted and gravitational energy is partially converted into magnetic energy of the swirling tube. The strong magnetic field evidently slows down rotation of the flux tube as it falls (fig. 2d), hence this part of the tube must have negative momentum with respect to the rotation of the hole (12). On the other hand, angular momentum of the tube as a whole needs to be conserved (12), and to compensate positive angular momentum has to be radiated to infinity.

For sufficiently strong rotation of the black hole and sufficiently strong initial magnetic field the leading part of the falling flux tube should have negative energy inside the ergosphere where $g_{tt} < 0$ (Misner at al. 1973). Since the energy of the tube as a whole has to be conserved (11), the part of the tube with positive energy ends up with an energy greater than the initial tube energy, E (Semenov at al., 2002). To some extent this is similar to the Penrose (1969) process (fig. 1), but now we don't need to invoke the interaction or decay of particles or tubes, since just a single tube can extract energy from the hole.

Results of the numerical simulation

In the spinning Kerr geometry the leading part of the falling flux tube loses angular momentum and energy as the string/tube brakes, which leads to creation of negative energy inside the ergosphere (fig. 3b) (Semenov, 2000). To conserve energy and angular momentum for the tube as a whole, the positive energy and angular momentum has to be generated for the trailing part of the tube (fig. 3b). This is a variant of the Penrose process (fig. 2) (Penrose, 1969), but now the energy is extracted via the interaction of a flux tube with the rotating black hole (Semenov et al. 2002).

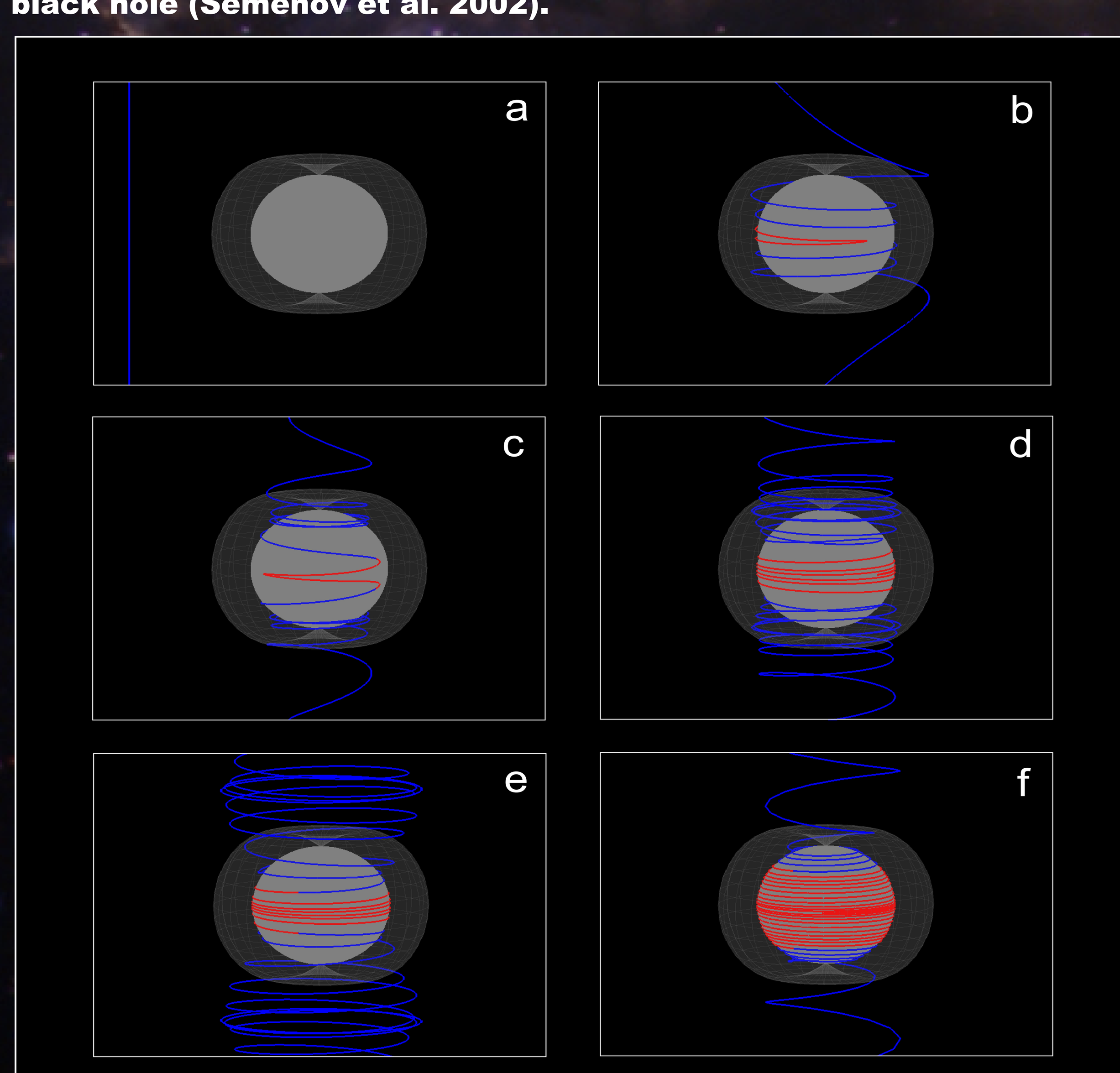

Fig. 3 This figure shows the different moments of simulations. (a) – corresponds to the initial configuration of the flux tube; (b) - the negative energy creation onset (part of the string with negative energy labeled by red color), (c) – beginning of the spiral structure creation, (d) and (e) – development of bipolar spiral structure creation along the axis of rotation; (f) - the last moment of simulation.

The deeper the tube falls in the hole, the more the tube is stretched (fig.3c,d,e,f), the stronger the magnetic field gets, the slower the central part of the tube rotates, the more negative energy is generated (fig.3c,d,e,f), and the more positive energy is created. Eventually, plasma ejected from the ergosphere producing relativistic jet (fig.4) (Semenov et al. 2004).

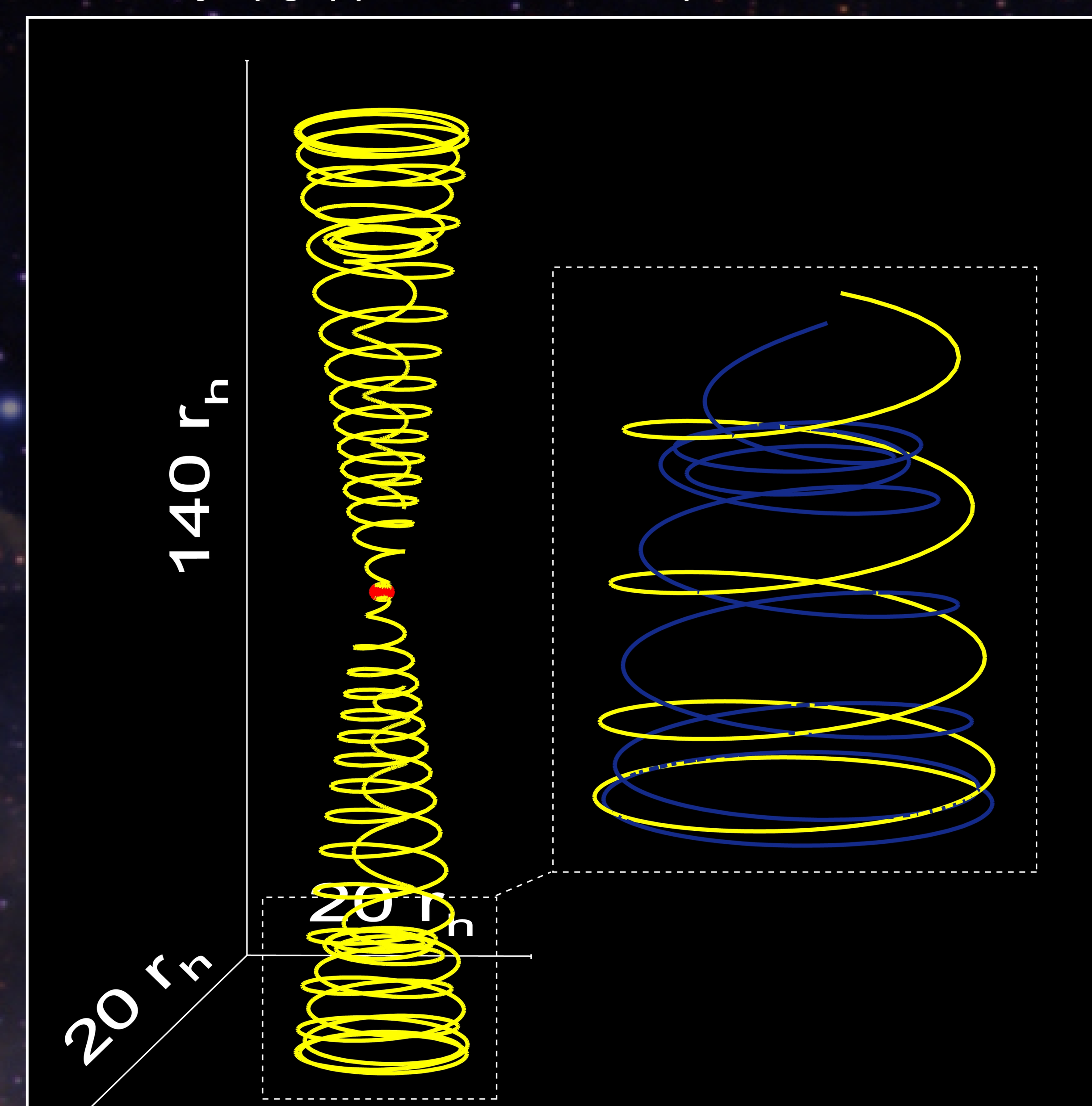

Fig. 4 The reconnection process may lead to the formation of plasmoids since the initial jet has double spiral structure (blue internal and yellow – external spirals) with nearly antiparallel magnetic field.

Continuously development of the jet leads to the monotonic string/tube stretching (fig. 4) which stops by reconnection. Topology of the flux tube is changed and it gives rise to the plasmoid formation. The chain of plasmoids propagate along the spin axis of the hole with relativistic speed carrying off the energy and angular momentum away from the black hole (fig. 5).

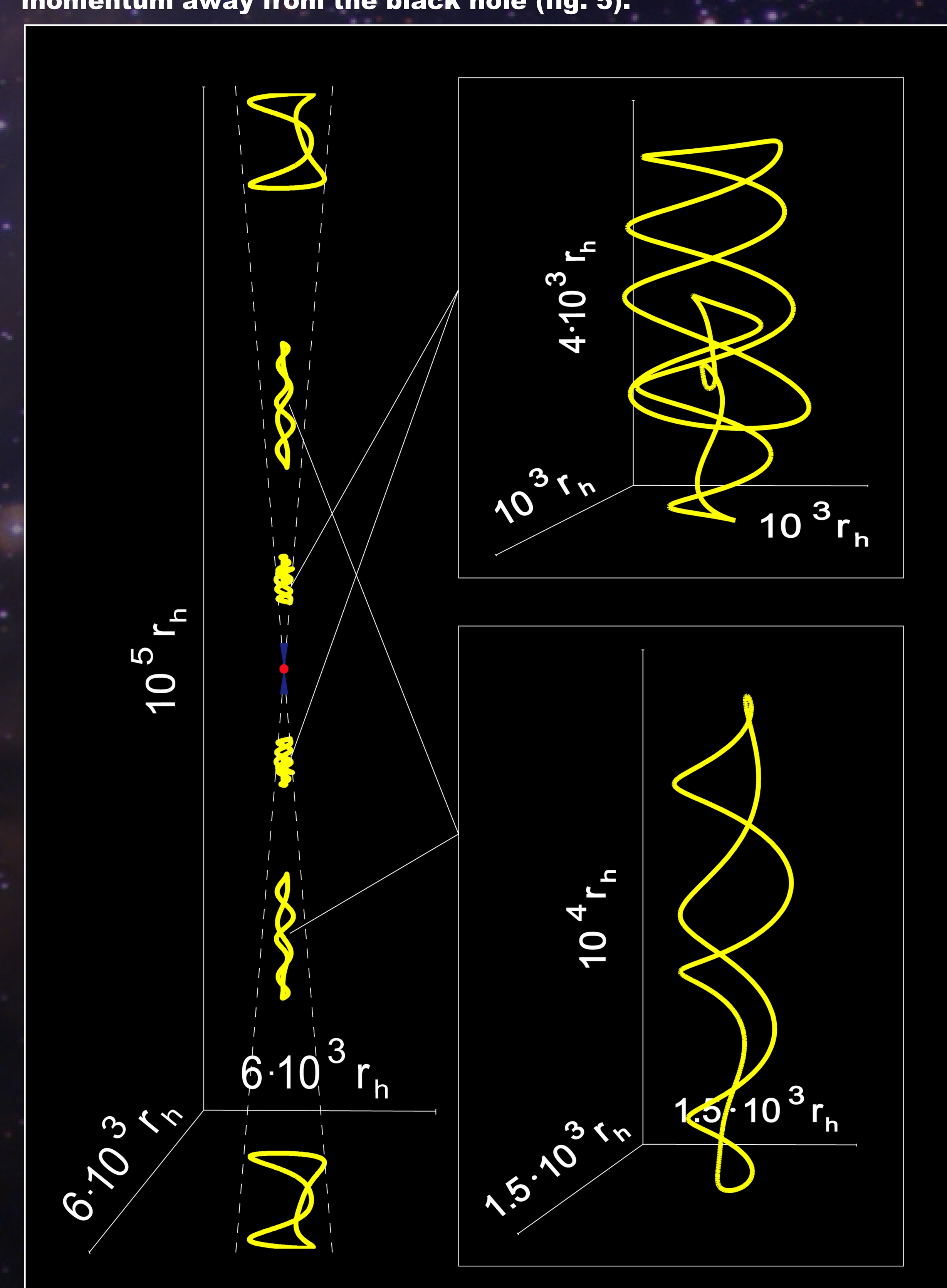

Fig. 5 Plasmoid evolution. Propagating as a solitary object, the plasmoid reveals sufficiently strong collimation, which is a result of the equilibrium between the centrifugal forces of plasma rotation and magnetic tension. Gradual simplification of the helical structure in the course of time leads to conversion of magnetic energy into the plasma energy.

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