Plasma response to resonant magnetic field perturbations when modelling Tokamak plasmas

M. Mulec[1], M. F. Heyn[1], S. V. Kasilov[1,2]

[1]Institute of Theoretical and Computational Physics, Graz University of Technology, Austria

[2]Institute of Physics, National Science Center, 'Kharkov Institute of Physics and Technology', Ukraine

Motivation

- Application of 1D models to describe plasma response to magnetic field perturbations in DIII-D, JET, ITER.
- transport modelling using 3D Monte Carlo transport code E3D

Method

Application of 3D-divergence-free magnetic field for computations in cylindrical coordinates and flux coordinates

Divergence free field representation

To achieve $\nabla \cdot \mathrm{B} = 0$ up to a satisfying interpolation accuracy, a divergence-free field representation is used.

Cylindrical coordinates Perturbation magnetic field is given on an equidistant 3D grid in the cylindrical coordinates (R, φ, Z). For divergence-free representation we separate into axissymmetric and non-axissymmetric part

$$B^i(R,\varphi,Z) = \tilde{B}^i(R,\varphi,Z) + \bar{B}^i(R,Z) \qquad (1)$$

$\bar{B}^i$ is computed as average over the azimuthal angle φ.
Because of the symmetry components of the averaged part contain only $\bar{A}_\varphi$ and $\bar{A}_Z$

$$\bar{B}^R(R,Z) = -\frac{1}{R}\frac{\partial}{\partial Z}\bar{A}_\varphi(R,Z) \qquad (2)$$

$$\bar{B}^Z(R,Z) = \frac{1}{R}\frac{\partial}{\partial R}\bar{A}_\varphi(R,Z) \qquad (3)$$

$$\bar{B}^\varphi(R,Z) = -\frac{1}{R}\frac{\partial}{\partial Z}\bar{A}_Z(R,Z) \qquad (4)$$

and the non-symmetric part is calculated using the gauge $\tilde{A}_\varphi = 0$ and Fourier analysis over the toroidal angle φ

$$\tilde{B}^R(R,\varphi,Z) = \sum_{n=-\infty}^{\infty}\frac{in}{R}A_{Z,n}(R,Z)\,\mathrm{e}^{in\varphi} \qquad (5)$$

$$\tilde{B}^Z(R,\varphi,Z) = -\sum_{n=-\infty}^{\infty}\frac{in}{R}A_{R,n}(R,Z)\,\mathrm{e}^{in\varphi} \qquad (6)$$

$$\tilde{B}^\varphi(R,\varphi,Z) = \frac{1}{R}\sum_{n=-\infty}^{\infty}\left(\frac{\partial}{\partial Z}A_{R,n}(R,Z) - \frac{\partial}{\partial R}A_{Z,n}(R,Z)\right)\mathrm{e}^{in\varphi} \qquad (7)$$

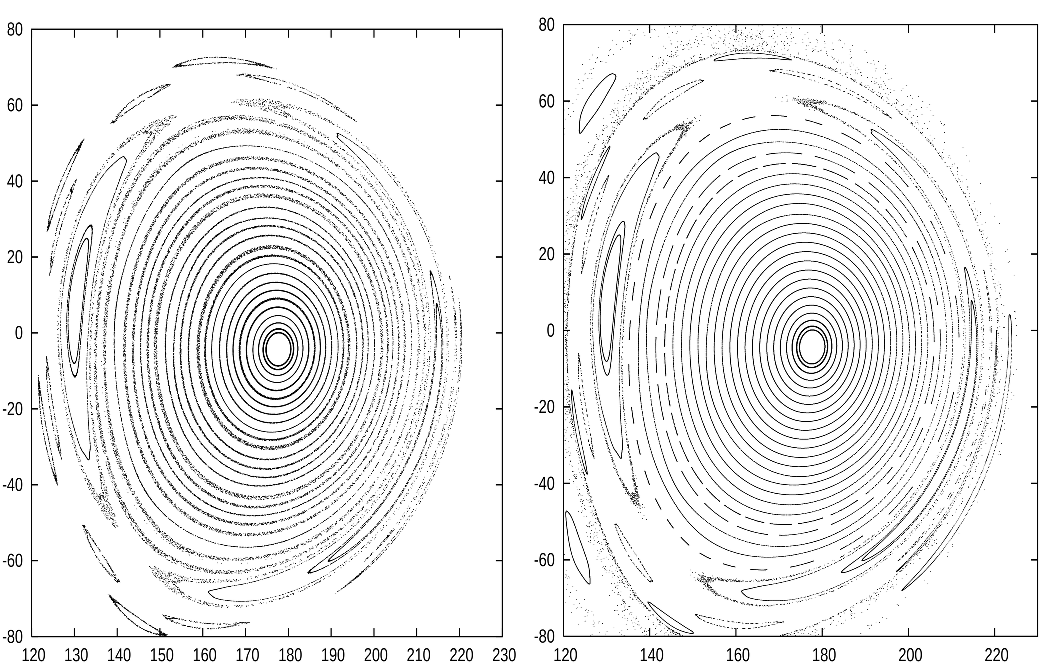

Figure 1: Poincare plot for a C-coil perturbation field at DIII-D for original field (left) and for the "divergence-free" field (right). A diffusive core on the left figure and ergodic field region on the right figure are clearly visible.

Fig.1 shows results for the DIII-D C-coil perturbation field:

- diffusive behaviour for field lines in the core region for the original field
- no diffusive core for divergence-free field
- present open ergodic region for the divergence-free field, absent for the original field

Flux coordinates Flux coordinates are useful to apply the model to Tokamak symmetry.
The intention is to represent the perturbation magnetic field in form of Fourier expansion in flux coordinates $(\psi, \vartheta, \varphi)$, ψ as poloidal flux normalised by 2π, ϑ as poloidal angle-like variable such that the field lines look as straight lines.
Using the safety factor q to define a rotational transform angle $\iota = \frac{1}{q}$, makes the straight field lines in coordinates (ϑ, φ) look

$$\mathrm{d}\vartheta = \iota(\psi)\,\mathrm{d}\varphi \qquad (8)$$

We compute the field line equations as ratios of the magnetic field components

$$\frac{\mathrm{d}\psi}{\mathrm{d}\varphi} = \frac{B^\psi}{B^\varphi} \approx -\frac{1}{\sqrt{g}B_0^\varphi}\frac{\partial A_\vartheta}{\partial \varphi} \qquad (9)$$

$$\frac{\mathrm{d}\vartheta}{\mathrm{d}\varphi} = \frac{B^\vartheta}{B^\varphi} \approx \frac{B_0^\vartheta}{B_0^\varphi} = \iota(\psi) \qquad (10)$$

We see, that $\tilde{A}_\vartheta$ determines formation of magnetic islands and is also responsible for generation of parallel plasma response currents which screen the perturbations on resonant flux surfaces.
Poloidal and toroidal harmonics are included using Fourier expansion

$$\tilde{A}^\psi = \sum_{m,n} A^\psi_{m,n}\mathrm{e}^{im\vartheta+in\varphi}\ ,\quad \tilde{A}_\vartheta = \sum_{m,n} A_{\vartheta;m,n}\mathrm{e}^{im\vartheta+in\varphi} \qquad (11)$$

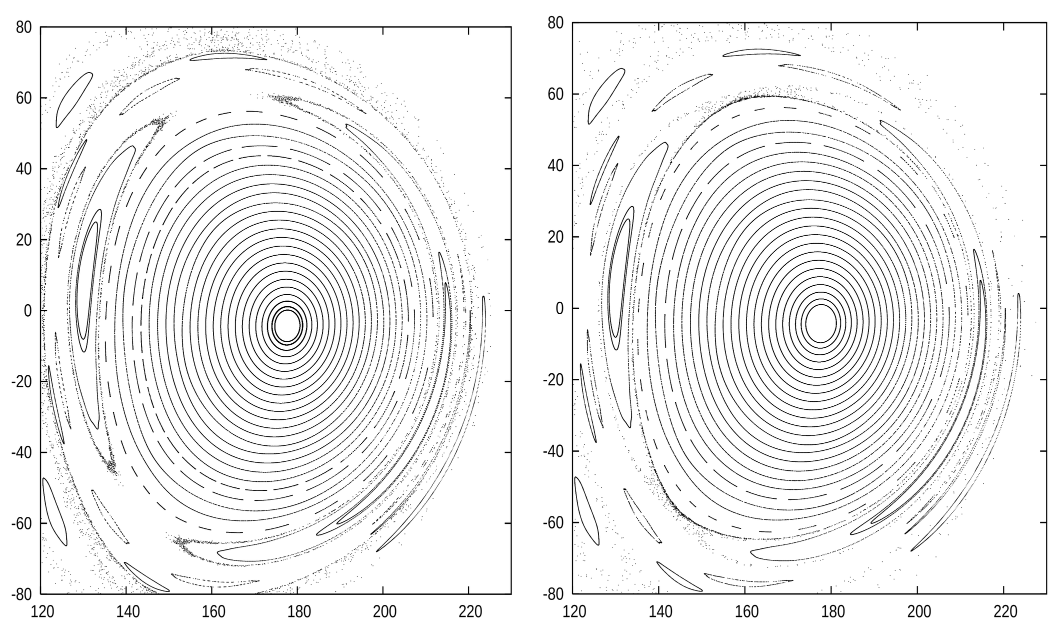

Figure 2: Poincare plot for "divergence-free" field in cylindrical coordinates (left) and via Fourier expansion in magnetic coordinates (right) for C-coil perturbation field at DIII-D .

Cylindrical coordinates and Fourier expansion in magnetic coordinates show a satisfying accordance in

- island position
- ergodic region

Integration along field lines

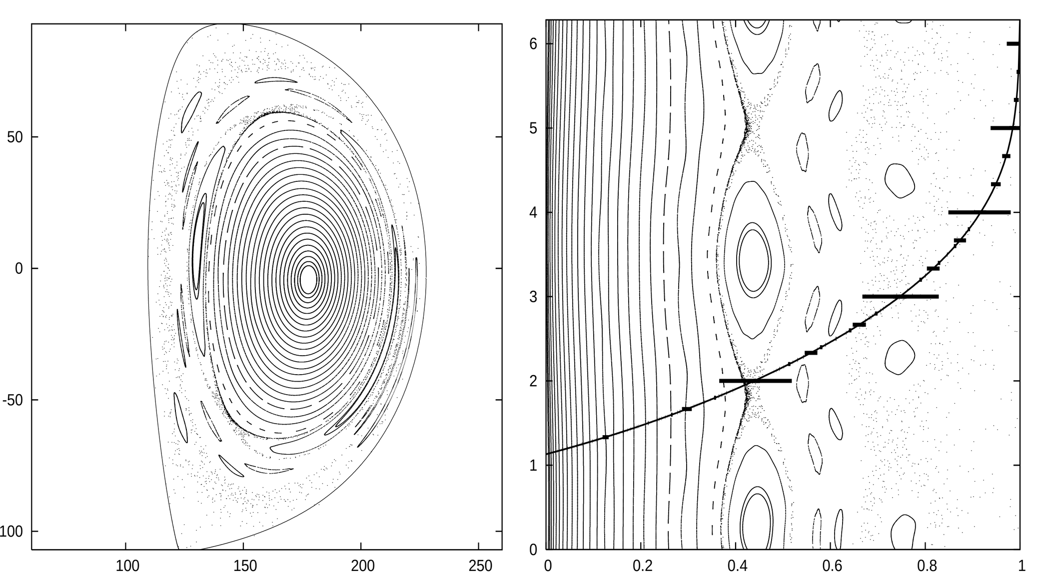

Figure 3: Same Poincare plot as the right in Fig.2 shown for Fourier expansion in flux coordinates with unperturbated separatrix (left) and in magnetic coordinates (s, ϑ) with q-profile and island sizes (right)

- a clear q-profile and island positions can be shown in magnetic coordinates
- only resonances for $n = 1$ are responsible for the formation of the ergodic layer
- islands for $n = 3$ and higher harmonics are significantly smaller

Integration along the unperturbated field lines gives Fourier amplitudes for $A^\psi_{m,n}$ and $A_{\vartheta;m,n}$ as well as coordinate dependencies $\psi(R,Z)$, $\vartheta(R,Z)$ by formulas (9), (10) and

$$F_m(\psi) = \frac{\iota}{2\pi}\int_0^{\frac{2\pi}{\iota}} \mathrm{d}\varphi\, F\left(R\left(R_b, Z_b; \varphi\right), Z\left(R_b, Z_b; \varphi\right)\right)\mathrm{e}^{-im\iota\varphi} \qquad (12)$$

for a field line starting point $(R_b, Z_b, 0)$.

Presence of linear plasma response

For estimation of the screening effect in plasma the Fourier amplitudes of the vector potential are modified by form factors

$$A^\psi_{m,n} \rightarrow A^\psi_{m,n} f^{(\psi)}_{m,n} \qquad A_{\vartheta;m,n} \rightarrow A^\psi_{\vartheta;m,n} f^{(\vartheta)}_{m,n} \qquad (13)$$

which are determined as ratios of magnetic field in presence of plasma and in vacuum, mostly estimated in cylindrical coordinates (r, θ, z)

$$f^{(\psi)}_{m,n} \approx \frac{B^{(plasma)}_{\theta;m,n}}{B^{(vacuum)}_{\theta;m,n}} \qquad f^{(\vartheta)}_{m,n} \approx \frac{B^{(plasma)}_{r;m,n}}{B^{(vacuum)}_{r;m,n}} \qquad (14)$$

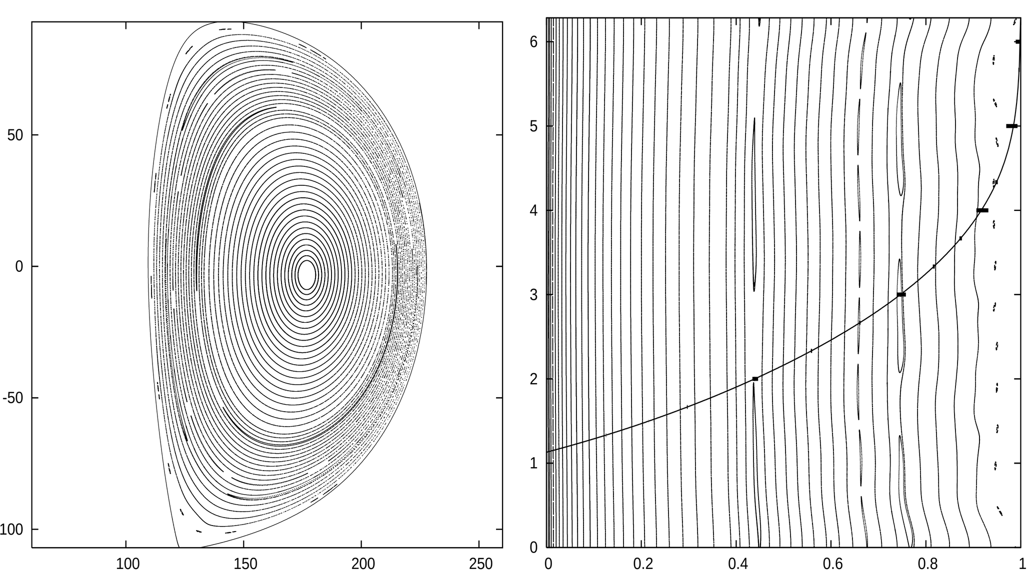

Figure 4: Poincare plots for C-coil perturbation field at DIII-D like in Fig.3 in presence of plasma response currents.

- strong reduction of island size by factor 30
- reduction in size of the ergodic layer
- negligibly small effect of perturbation on the core plasma

Conclusion and outlook

Screening effects significantly reduce ergodisation of the magnetic field configuration:

- strong effect of plasma response current
- reduction of the size of the ergodic layer
- plots showing an almost unperturbated core

The purpose of all activities is to reimplement models and study of ELM mitigation by RMPs in DIII-D, JET, ITER.

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References

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