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Application of NEWEUL in Robot Dynamics

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We consider the simulation of the dynamics of a complex robot system modeled as multibody system with rigid links and flexible drives. The symbolic equations, linearized with respect to a nominal motion, are generated by the program system NEWEUL. From these equations the dynamic behaviour of the system can be determined, using numeric standard procedures.

The features of the program system as well as some problems occuring with the use of a symbolic formalism for the determination of equations of motion are discussed. The application of NEWEUL during the simulation of a robot system with complex dynamics leads to efficient program codes in the simulation program and guarantees high accuracy and stability during the numeric evaluation.

1. Introduction

Industrial robots are free programmable manipulation systems, with various degrees of freedom. They are equipped with tools or grippers and are already available for many applications in tool and object manipulation. Increasing demands on the accuracy and speed have motivated progress in the analysis and simulation of complex robot systems, including general force-laws and control-laws. Theoretical simulations provide a deep insight into the dynamic behaviour at most various requirements. They are helpful for studies during planning, development and application.

The large motions of a robot's links are the reason for its highly nonlinear dynamic characteristic, which in turn leads to complex system equations. Typically, one also has to include at least the main characteristics of its actuating system. Therefore, simulation as well as the design of control-laws is a difficult task.

However, for a wide class of robots, the links can be considered as more or less rigid, whereas the elastic properties of the actuating drives need to be taken into account. Thus multibody systems are well qualified to model the mechanics of a complex robot system.

This paper is concerned with the modeling of the mechanics and with the generation of the symbolic–numeric equations of motion for robot types that consist of nearly rigid links and flexible actuating drives. The paper looks at the case of a three axes robot with six degrees of freedom of deflection from the nominal motion, whose position is defined in generalized relative coordinates.

2. Computer-aided Generation of Equations of Motion for Multibody Systems

The equations of motion describe the interaction between the acting applied forces and the motion of the mechanical system. Program systems for the automatic generation of

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the numeric equations of motion and their solution are available, including convenient pre- and postprocessors, e.g. ADAMS (Chace, 1984), IMP (Shet & Uicker, 1972). However, recently symbolic formalisms for the generation of the dynamic equations have become increasingly important, e.g. NEWEUL (Kreuzer, 1979), MESA VERDE (Wittenburg & Wolz, 1985).

Symbolic formalisms allow in particular:

compatibility with free programmable modules describing force-laws and control-laws;

parameterized presentation of dynamic relations;

generation of a computationally efficient simulation code and high accuracy of results during the numerical evaluation;

possibility of error-detection in the equations of motion prior to the numerical simulation phase.

In the following case, the program system NEWEUL was used for the generation of the symbolic equations of motion.

The scientific foundations of the program NEWEUL itself are described in detail by Kreuzer (1979). A technical description and examples of application can be found in Kreuzer *et al.* (1986).

3. Model of a Three-axes Robot

Let us consider a six-degrees-of-freedom robot, consisting of three rigid links and three actuating drives (Fig. 1). The links are kinematically connected by rotational joints, actuated by applied torques exerted by servomotors. The motion of the hand itself is neglected in this case. The nominal position of the system is defined by its internal joint coordinates

$$\mathbf{q}_s(t) = [TH_1(t), TH_2(t), TH_3(t)]^T, \tag{1}$$

in particular by the base angle $TH_1(t)$ around the vertical axis and by the angles $TH_2(t)$ and $TH_3(t)$ around the horizontal axes. There is no restriction to the geometric position of the link axes with respect to the joint axes.

The robot is modeled as a multibody system consisting of three rigid bodies representing the links and three rigid bodies modeling the actuating rotors, which are interconnected by flexible shafts. Arbitrary gear ratios can be taken into account. The position of the system is uniquely determined by the six coordinates of the 6×1-position vector $\mathbf{z}(t)$. In this case, six generalized coordinates of relative motion

$$\mathbf{y}(t) = [GA_1, GA_2, GA_3, PH_1, PH_2, PH_3]^T, \tag{2}$$

with reference to the large nominal motion are used for the system's kinematic description, applying the relations

$$\mathbf{y} = \mathbf{z} - \mathbf{z}_s, \quad \text{where} \quad \mathbf{z}_s = \mathbf{z}_s(\mathbf{q}_s(t)). \tag{3}$$

Thus, the position vector $\mathbf{y}$ defines the deflection from the nominal position $\mathbf{z}_s$.

In order to design the control-laws for the actuating system, NEWEUL was used to generate the equations of motion linearized with respect to the prescribed large nominal motion of the robot's links.

A detailed description of the problem can be found in Hirschberg (1985).

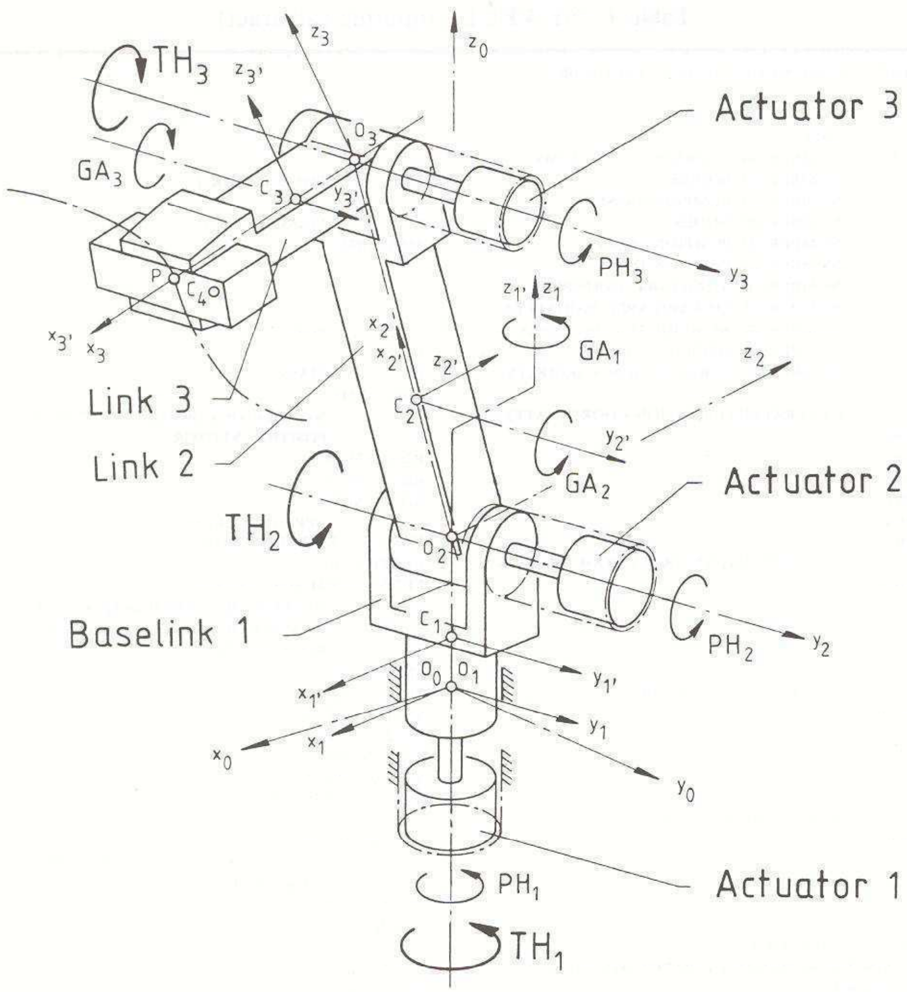

Fig. 1. Robot with three links and flexible actuating drives.

4. Computer Aided Generation of Equations

The program system NEWEUL requires the input of physical and geometrical model-parameters (masses, moments of inertia, positions of mass-centers and connection-joint locations) of the system's bodies in fully symbolic form (Table 1). This work can be supported by an interactive algebraic entry program. However, even when using this entry program, there is some work left to be done by the user.

Once the completed input data is read into the computer, NEWEUL is ready to generate the equations of motion. The symbolic equations are produced as FORTRAN-code in batch mode. A typical problem which occurs occasionally when using NEWEUL for complicated multibody systems is the large size of the generated symbolic equations caused by appearence of trigonometric functions. This shortcoming can be dealt with by:

The substitution of selected parameters by numerical values, thus creating symbolic–numeric equations (see item ① in Table 1);

Table 1. NEWEUL—inputfile (abstract)

```
ROBF6.INP    6 DOF-MODEL: RIGID 3-AXES ROBOT
-------------------------------------------

        1   TASK
        6   NUMBER OF COORDINATE SYSTEMS
        6   NUMBER OF BODIES
        0   NUMBER OF LUMPED MASSES
        0   NUMBER OF NODES
        6   NUMBER OF POSITION DOS'S
        6   NUMBER OF POSITION DOF'S
       13   NUMBER OF AUXILIARY VARIABLES
       22   NUMBER OF LINEARIZABLE VARIABLES
       18   NUMBER OF NUMERICAL CONSTANTS
       35   NUMBER OF SIMPLIFICATIONS
       70   NUMBER OF SUBSTITUTION VARIABLES

C           GENERALIZED POSITION COORDINATES
Y(1) = GA1
Y(2) = GA2
Y(3) = GA3
Y(4) = PHI1
Y(5) = PHI2
Y(6) = PHI3
C           GENERALIZED VELOCITY COORDINATES
Z(1) = GA11
Z(2) = GA21
  .
  .
  .
C           LINEARIZABLE VARIABLES
LIN1 = GA1
LIN2 = GA2
  .
  .
  .
C           NUMERICAL CONSTANTS
RM1 = 15.0
RM2 =  8.2
  .
  .
  .
C           SIMPLIFICATIONS
VF(1, 1) = SIN(TH2)*COS(TH3) + COS(TH2)*SIN(TH3)
VF(1, 2) = SIN(BD)
VF(2, 1) = COS(TH2)*COS(TH3) - SIN(TH2)*SIN(TH3)
VF(2, 2) = COS(BD)
  .
  .
  .
C           SUBSTITUTION VARIABLES
SU1 = UO2*COS(TH1) - VO2*SIN(TH1)
SU2 = UO2*SIN(TH1) + VO2*COS(TH1)
SU3 = WO2
  .
  .
  .
```

```
      .
      .
      .
      1         BODY 1, LINK 1
      C         -------
      C         MASS
      MA1 = RM1
      .
      .
      .
      1         BODY 2, LINK 2
      C         -------
      C         MASS
      MA2 = RM2
      2         NUMBER OF COORDINATE SYSTEM REF
      C         POSITION VECTOR
      R2(1) = SU28
      R2(2) = SU29
      R2(3) = SU30
      C         APPLIED FORCES
      FE2(1) =   RM2*GH*SIN(TH2)
      FE2(2) =   0
      FE2(3) = - RM2*G*COS(TH2)
      2         NUMBER OF COORDINATE SYSTEM REF
      1         NUMBER OF PARTIAL ROTATIONS
      C         ROTATION MATRIX
      S12(1, 1) = SU19
      S12(1, 2) = SU20
②     S12(1, 3) = SU21
      S12(2, 1) = SU22
      S12(2, 2) = SU23
      S12(2, 3) = SU24
      S12(3, 1) = SU25
      S12(3, 2) = SU26
①     S12(3, 3) = SU27
      1         TRANSFORMATION CONTROL PARAMETER
      C         INERTIA TENSOR
      I2(1, 1) = T2X
      I2(1, 2) = 0
      I2(1, 3) = 0
      I2(2, 1) = 0
③     I2(2, 2) = T2Y
      I2(2, 3) = 0
      I2(3, 2) = 0
      I2(3, 3) = T2Z
      C         APPLIED MOMENTS
      LE2(1) = 0
      LE2(2) = SU57 + SU64 - SU69 + SU70 - RMO3
      LE2(3) = 0
      .
      .
      .
      C         END
```

Expanding the equations of motion about a prescribed motion of the system and retaining only the first-order terms, thus creating linearized equations of motion (see item ② in Table 1)

The introduction of user-defined algebraic simplification rules, which make use of the specific properties of the particular system. This is especially necessary in the case of large non-linear kinematic relations (see item ③ in Table 1).

Table 2. Symbolic equations of motion (abstract) derived by NEWEUL

```
  MASSMATRIX [M]; UPPER TRIANGULAR FORM
  RM(1, 1) = 0.05 + 2.*RM3*W3*SBD*CT2 + 2.*RM3*U3*CBD*CT2 + 0.0064*ST2Q +
+            3.62*CT2Q + RM3*CT2Q + 2.*RM3*W3*U3*CBD*SBD +
+            RMS*W3**2*SBDQ + RM3*V3**2*SBDQ + T3X*SBDQ + RM3*V3**2*CBDQ +
+            RM3*U3**2*CBDQ + T3Z*CBDQ
  RM(1, 2) = RM3*V3*SBD*CT3 - RM3*V3*CBD*ST3
  RM(1, 3) = RM3*V3*U3*SBD - RM3*W3*V3*CBD
  RM(2, 2) = 3.62 + TMA3*R3**2 + RM3*ST3Q + RM3*CT3Q
  RM(2, 3) = RMS*W3*ST3 + RM3*U3*CT3
  RM(2, 6) = TMA3*R3
  RM(3, 3) = RM3*W3**2 + RM3*U3**2 + T3Y
  RM(4, 4) = TMA1
  RM(5, 5) = TMA2
  RM(6, 6) = TMA3

  MATRIX [P] OF VELOCITY DEPENDENT FORCES
  P(1, 1) = -7.2272*CT2*ST2*PY - 2.*RM3*CT2*ST2*PY -
-           2.*RM3*W3*SBD*ST2*PY - 2.*RM3*U3*SBD*CT2*QY -
-           2.*RM3*U3*CBD*ST2*PY + 2.*RM3*W3*CBD*CT2*QY -
-           2.*RM3*U3**2*CBD*SBD*QY + 2.*RMS*W3**2*CBD*SBD*QY +
+           2.*T3X*CBD*SBD*QY - 2.*T3Z*CBD*SBD*QY -
-           2.*RM3*W3*U3 + SBDQ + QY + 2.*RM3*W3*U3*CBDQ*QY + DA1 + DL1
  P(1, 2) = -2.*RM3 + V3 + CT2*T30T + 2.*RM3*V3*CT2*QY -
-           7.2272*CT2*ST2*OZ - 2.*RM3*CT2*ST2*OZ -
-           2.*RM3*W3*SBD*ST2*OZ - 2.*RM3*U3*CBD*ST2*OZ
  P(1, 3) = 2.*RMS*W3*V3*SBD*QY - 2.*RM3*U3*SBD*CT2*OZ +
+           2.*RM3*V3*U3*CBD*QY + 2.*RM3*2E*CBD*CT2*OZ -
-           2.*RM3*U3**2*CBD*SBD*OZ + 2.*RM3*W3**2*CBD*SBD*OZ +
+           2.*T3X*CBD*OZ - 2.*T3Z*CBD*SBD*OZ -
-           2.*RM3*W3*U3*SBDQ*OZ + 2.*RM3*W3*U3*CBDQ*OZ
  P(1, 4) = -DA1*RR1
  P(2, 1) = 7.2272*CT2*OZ + Z.*RM3*CT2*ST2*OZ +
+P          2.*RM3*W3*SBD*ST2*OZ + 2.*RM3*U3*CBD*ST2*OZ
  P(2, 2) = DA2 + DL2 + DL3 + DA3
  P(2, 3) = - 2.*RM3*U3*ST3*QY + 2.*RM3*W3*CT3*QY - DL3 - DA3
```

Using the generation formalism documented in Schielen (1986) the set of nonlinear equations of motion written in matrix notation take the following form

$$\bar{\mathbf{M}}(\mathbf{z}, t)\ddot{\mathbf{z}} + \mathbf{k}(\mathbf{z}, \dot{\mathbf{z}}, t) = \mathbf{q}(\mathbf{z}, \dot{\mathbf{z}}, t), \tag{4}$$

where $\bar{\mathbf{M}}$ is the 6×6-symmetric mass matrix, $\mathbf{k}$ is the 6×1-vector of generalized centrifugal and gyroscopic forces and $\mathbf{q}$ is the 6×1-vector of generalized applied forces. Taking notice of equation (3), the equations of motion linearized with respect to the nominal motion can be rewritten as

$$\mathbf{M}(t)\ddot{\mathbf{y}} + \mathbf{P}(t)\dot{\mathbf{y}} + \mathbf{Q}(t)\mathbf{y} = \mathbf{h}(t), \tag{5}$$

where $\mathbf{M}$ is the 6×6-symmetric matrix of generalized masses and $\mathbf{P}$ and $\mathbf{Q}$ are the time-dependent 6×6-matrices of velocity- and position-depending forces, respectively. The generalized excitation forces and the actuating torques are collected in the 6×1-vector $\mathbf{h}(t)$.

The six equations generated by NEWEUL are coded in FORTRAN and are therefore ready for further processing in an extensive simulation program, as well as for the calculation of the system's eigenvalues and eigenmodes in the case of linearized equations under steady-state conditions.

To give an impression of what the equations of the robot produced by NEWEUL look like, some elements of the matrices **M** and **P** are shown in Table 2. The calculation of the symbolic equations of motion took approximately 30 s of CPU-time on a VAX 8600 computer and 110 s of CPU-time on a MICRO-VAX II. the complete set of symbolic equations is documented in Hirschberg (1987) and serves for simulation in robot dynamics.

By introducing the state-vector

$$\mathbf{x} = [\mathbf{y}^T \dot{\mathbf{y}}^T]^T, \tag{6}$$

the differential equations (5) can be transformed to the equivalent notation as dynamic state equation of first order

$$\dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{b}(t). \tag{7}$$

The transformation was performed numerically. This equation is the foundation of the majority of modern methods for the design of control-laws in dynamics. In particular, it is ready for direct numeric integration.

Finally, it should be mentioned, that in this case the created equations of motion appeared to be numerically stiff. Therefore, special numerical integration methods had to be used for an efficient and stable solution. Typically, for robot control, equation stiffness results from the actuator's stiff connections to the nominal motion.

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