GOCE Quick-Look Gravity Field Analysis

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Abstract

The goal of the GOCE-mission (Gravity Field and Static State Ocean Circulation Explorer) is to determine the static gravity field of the Earth on a global scale, with high spatial resolution from satellite-to-satellite tracking (SST) and satellite gravity gradiometry (SGG) measurements. A strict gravity field analysis is a very demanding task, that has high requirements to computer resources. Hence a quick-look gravity field analysis was included in the WP6000 of the GOCE high-level-processing facility (HPF). Its purpose is to regularly derive gravity field solutions on partial data sets and with a latency of only a few days after the observation, in order to detect any deficiency of the instruments. The quick-look analysis is based on the semi-analytic approach, which allows for a very fast solution.

The semi-analytic approach

The gravity potential and its second derivations of the gravity potential in orbital coordinates can be written as 2D-Fourier series:

$$V_{ij}(u,\Lambda)=\sum_{m=0}^{L}\sum_{k=-L}^{L}A_{mk}^{ij}\cos\psi_{mk}+B_{mk}^{ij}\sin\psi_{mk} \quad , \qquad \text{where}$$

$$\left.\begin{matrix}A_{mk}^{ij}\\ B_{mk}^{ij}\end{matrix}\right\}=\sum_{l=0}^{L}H_{lmk}^{ij}\left\{\begin{matrix}\overline{C}_{lm}\\ \overline{S}_{lm}\end{matrix}\right. \quad \text{with} \quad H_{lmk}^{ij}=\frac{GM}{R}\left(\frac{R}{r}\right)^{l+1}\lambda_{lk}^{ij}\,\overline{F}_{lmk}(I)$$

where $\psi_{mk} = ku - m\Lambda$. Under the assumption of a circular orbit (r = const. and I = const.) the lumped coefficients (A_{km}, B_{km}) are a linear combination of the potential coefficients (C_{lm}, S_{lm}) with the transfer coefficients H_{lmk} as coefficients. Hence, the potential coefficients can be estimated in a least-square adjustment. As only lumped coefficients and potential coefficients of the same order m are linearly dependent, the normal equation matrix becomes block-diagonal and the large adjustment resolves into many small adjustments, which can be solved very efficiently:

$$\left[\overline{C}_{lm},\overline{S}_{lm}\right]=\left(\mathbf{H}_{lmk}^{T}\mathbf{P}_{\psi_{mk}}\mathbf{H}_{lmk}+\alpha\mathbf{R}\right)^{-1}\mathbf{H}_{lmk}^{T}\mathbf{P}_{\psi_{mk}}\left[A_{mk},B_{mk}\right]$$

The design matrix $\mathbf{H}_{lmk}$ is composed of the corresponding transfer coefficients. The lumped coefficients can be obtained by two different spectral approaches: the 1D-FFT approach and the 2D-FFT approach, which is also known as torus—approach.

1D-FFT approach

This approach has the precondition, that the orbit is a repeat orbit, which means, that its ground-track repeats after β revolutions in α nodal days. The rates of the orbital coordinates u and Λ can then be related to α and β. If the measurements along the orbit are transformed to their spectrum a_n and b_n by an FFT, the one-dimensional spectrum can be related to the 2D lumped coefficient spectrum by the following mapping:

$$-\frac{\dot{u}}{\dot{\Lambda}}=\frac{\beta}{\alpha} \qquad (m,k)\mapsto n\,, \qquad \psi_{mk}\mapsto\psi_{n}\,, \qquad n=k\beta-m\alpha$$

2D-FFT (torus-) approach

Under the assumption of a circular orbit, the gravity potential and its derivatives are only dependent on two variables u and Λ. While the Argument of Latitude $u = \omega + \nu$ is the angle between the ascending node and the satellite in the orbital plane, $\Lambda = \Omega + \theta_g$ can be regarded as the right ascension of the ascending node in the Earth-fixed frame. They both range periodically from 0° to 360° forming a torus, which is the spatial domain of the 2D-Fourier Transform. The orbit can be transformed to the (u, Λ) domain via the Keplerian elements. It can be imagined to be wrapped around a torus and the measurement values can be interpolated to a regular grid on the torus surface:

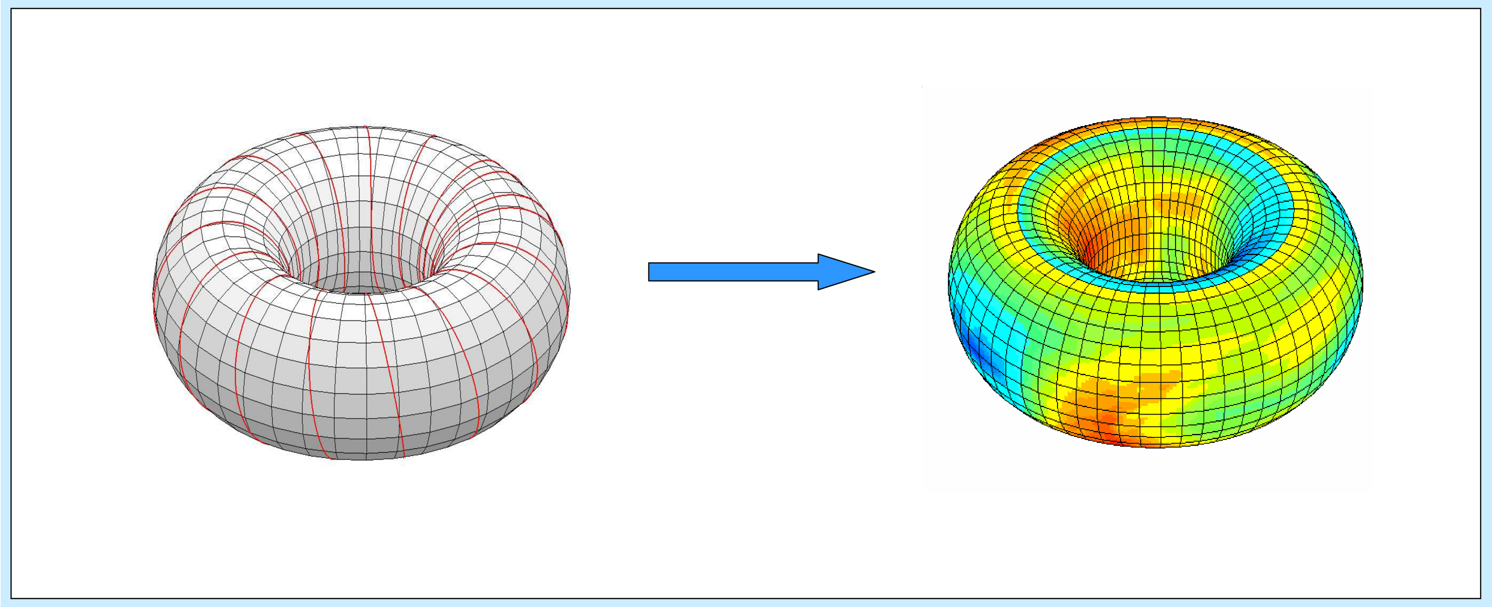

Fig.1: Interpolation of observations from the orbit on the (u, Λ) torus

Iterative Solution Strategy

All deviations from the assumed perfect circular orbit will lead to a degraded solution. The deviations are in particular, variations in orbit height and inclination, misalignment of the gravity tensor, non-closure of the repeat orbit and data gaps. This degradation can be partially overcome by an iterative strategy. Residuals are computed in a synthesis step (without approximation) and entered in the whole computation until convergence is achieved. During each iteration the weight factors between the different components can be improved by variance component estimation (VCE). Filtering can be applied in the spectral domain and the noise model can as well be estimated improving the filtering iteratively.

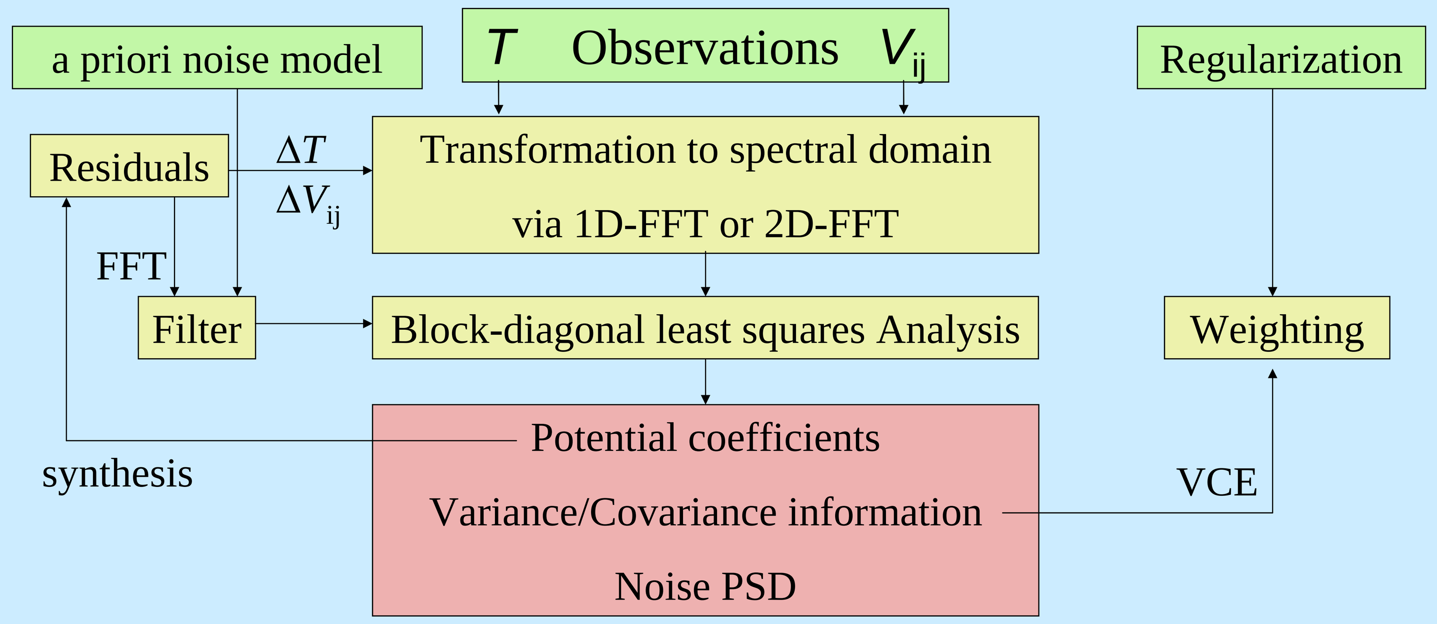

Fig.2: Iterative Solution Strategy

Weighting

For the GOCE-solution different data sets will be combined: the SST observations, the tensor components (only V_{xx}, V_{yy}, V_{zz} and possibly V_{xz} will be used for GOCE) and the regularization matrix $\mathbf{R}$. The components enter with an a priori weight, which can be improved in each iteration by a variance component estimation (VCE). The weight of the i-th component can be determined by: $s_i = v^T\,\mathbf{P}\,v\ /\ (n_i - \mathrm{trace}(\mathbf{N}^{-1}\,\mathbf{N}_i))$. The diagonal of the redundancy matrix $\mathbf{Q}_i = \mathbf{N}^{-1}\,\mathbf{N}_i$ holds the contribution of that component to each unknown coefficient, which is shown in fig. 3.

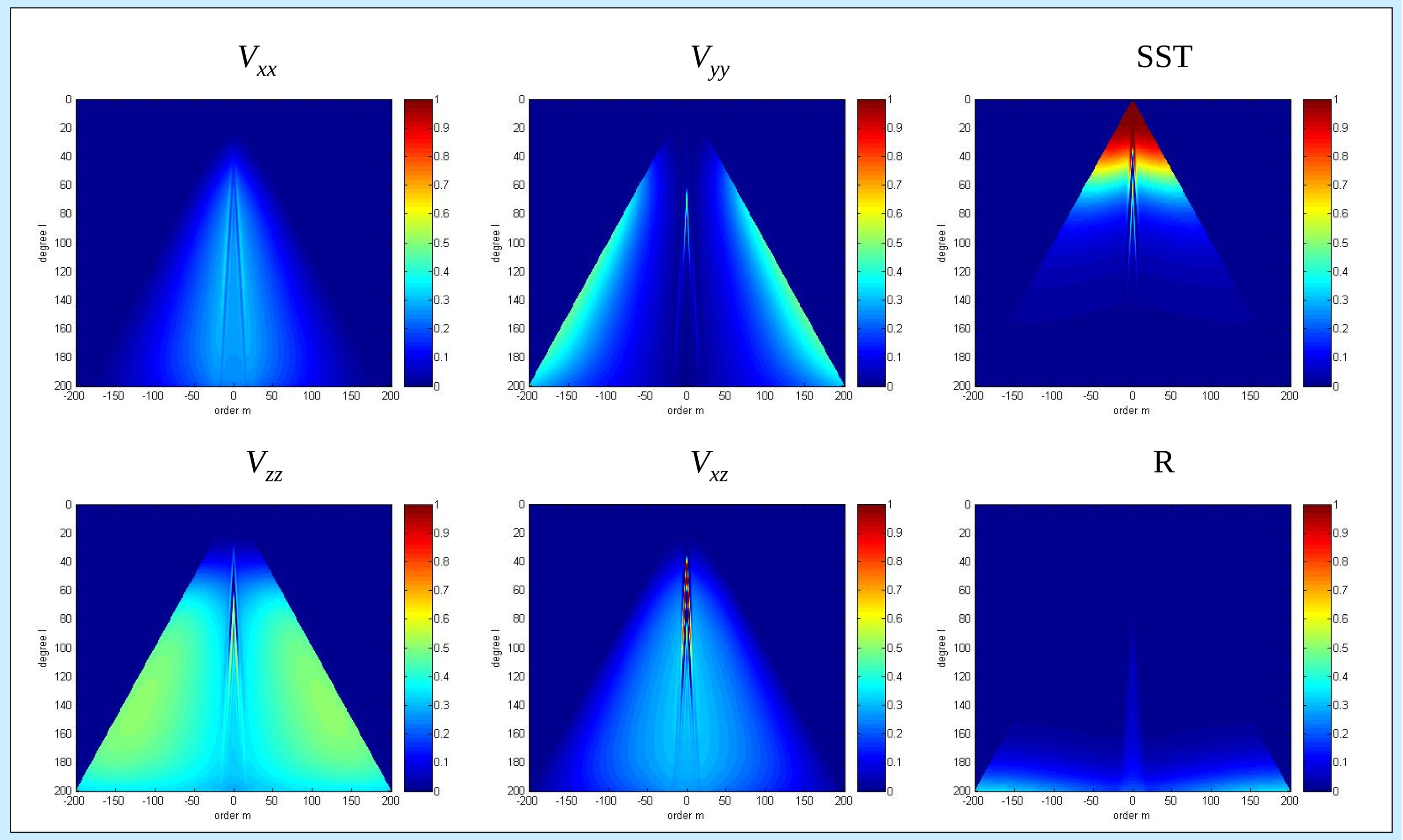

Fig. 3: Contribution of the different components to the solution. Sum = 1.

Solution

Fig. 4 shows the iterative solution of a recent GOCE simulation with realistic instrument noise. Out of a simulation period of 60 days, a repeat-cycle of 25 days and 403 revolutions was chosen. It shows that a converging solution can be found from such a short data set.

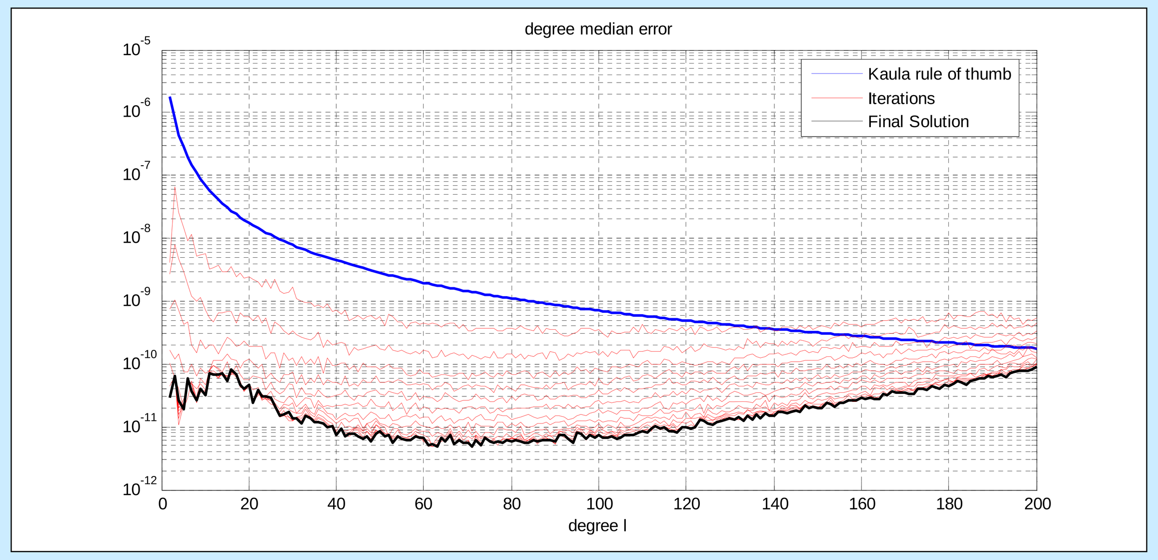

Fig. 4: Degree-median-error of an iterative solution.

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