Extracting Independent Components with Spiking Neurons

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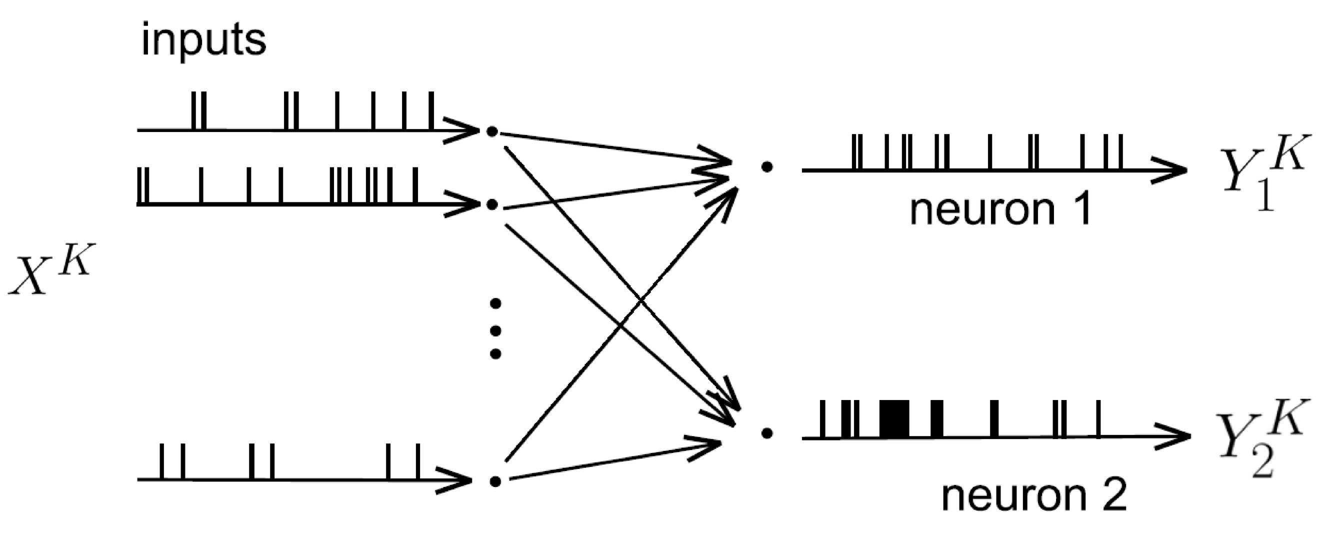
Stefan Klampfl, Robert A. Legenstein, and Wolfgang Maass
Institute for Theoretical Computer Science, Graz University of Technology

1 Introduction:

ICA: Independent Component Analysis (ICA) [1] is a well-known statistical technique for decomposing complex data into statistically independent parts, thereby providing a less redundant representation. Although ICA is one major candidate for unsupervised learning in nervous systems, an application to spiking neurons has still been missing.
Our approach: We minimize the mutual information between the outputs of two spiking neurons receiving the same input. Simultaneously, we want to maximize the information transmission of both neurons while maintaining a constant target firing rate, thereby extending the approach in [2].



2 Neuron Model:

We use a stochastically spiking neuron with refractoriness where the membrane potential of neuron i at time $t^k = k\Delta t$ is given as the sum over all postsynaptic potentials at synapses $j = 1, \ldots, N$:

$$u_i(t^k) = u_r + \sum_{j=1}^{N} \sum_{n=1}^{k} w_{ij} \epsilon(t^k - t^n) x_j^n, \tag{1}$$

where $u_r = -70$mV is the resting potential and w_{ij} is the weight of synapse j. $x_j^n \in \{0, 1\}$ denotes the presence of a input spike at synapse j at time t^n, which evokes a postsynaptic potential (PSP) with time course $\epsilon(t^k - t^n)$.
At each time step the neuron fires with a certain probability that depends on the current membrane potential and refractory state. This neuron model is a stochastic version of the integrate-and-fire model [3].

3 Learning Rule:

Consider input spike trains X^K and output spike trains Y_1^K and Y_2^K of length $K\Delta t$. The objective function to be maximized for neuron i ($i = 1, 2$) is

$$L_i = I(\mathbf{X}^K; \mathbf{Y}_i^K) - \beta I(\mathbf{Y}_1^K; \mathbf{Y}_2^K) - \gamma D_{KL}(P(Y_i^K) || \tilde{P}(Y_i^K)), \tag{2}$$

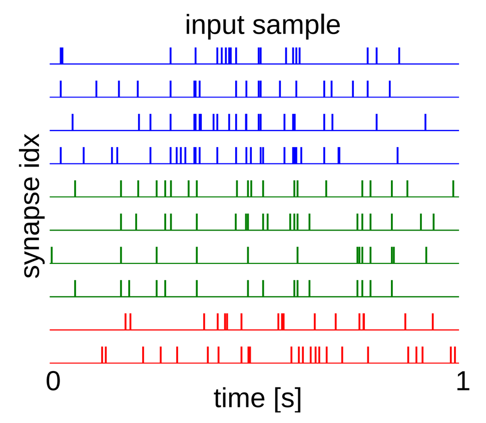
where

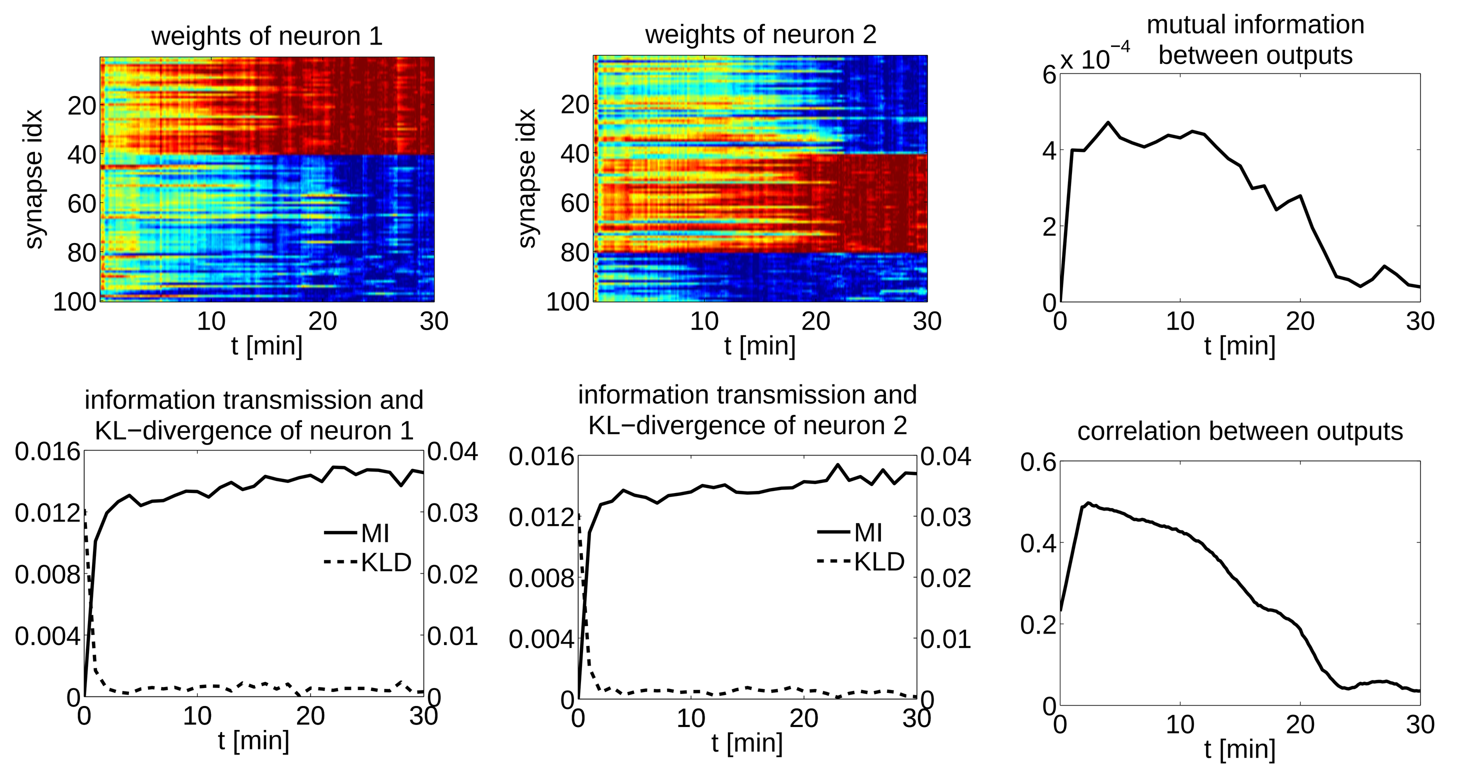
$I(\mathbf{X}^K; \mathbf{Y}_i^K)$	mutual information between input spike trains X^K and output spike train Y_i^K of neuron i
$I(\mathbf{Y}_1^K; \mathbf{Y}_2^K)$	mutual information between output spike train Y_1^K and output spike train Y_2^K
$D_{KL}(P(Y_i^K) \|\| \tilde{P}(Y_i^K))$	Kullback-Leibler divergence between current output distribution $P(Y_i^K)$ and desired target output distribution $\tilde{P}(Y_i^K)$ (constant target firing rate of 30Hz)
β, γ	optimization constants

We have derived a learning rule which performs gradient ascent on the objective function L_i (2).

4 Results:

In this experiment two neurons receive the same input at 100 synapses, consisting of Poisson spike trains at a constant rate of 20Hz. The input is divided into two groups of 40 spike trains each, such that synapses 1 to 40 and 41 to 80 receive correlated input with a correlation coefficient of 0.5 within each group, however, any spike trains belonging to different input groups are uncorrelated. The remaining 20 synapses receive uncorrelated Poisson input. Each neuron develops strong weights for different correlated groups.





5 Discussion:

Independent Component Analysis (ICA) has been proposed as a general principle for unsupervised learning, however, learning rules that can implement this principle with spiking neurons have still been missing. In this work we have derived a learning rule from abstract information theoretic principles that allows two neurons to extract statistically independent components from a common spiking input.

References

[1] A. Hyvärinen and E. Oja. Independent component analysis: Algorithms and applications. *Neural Networks*, 13:411–430, 2000.

[2] T. Toyoizumi, J.-P. Pfister, K. Aihara, and W. Gerstner. Generalized Bienenstock-Cooper-Munro rule for spiking neurons that maximizes information transmission. *Proc. Natl. Acad. Sci. USA*, 102:5239–5244, 2005.

[3] W. Gerstner and W. M. Kistler. *Spiking Neuron Models*. Cambridge University Press, Cambridge, 2002.

Appendix: Gradient ascent rule

Performing gradient ascent on L_i (2) and taking the limit $\Delta t \to 0$ yields an online learning rule for the weights of neuron i, w_{ij},

$$\frac{dw_{ij}}{dt} = \alpha C_{ij}(t) \left[B_i^{post}(t) - \beta B_{12}^{post}(t) \right], \tag{3}$$

with a learning rate $\alpha > 0$.
The correlation term C_{ij} measures coincidences between postsynaptic spikes at neuron i and PSPs generated by presynaptic action potentials arriving at synapse j:

$$\frac{dC_{ij}(t)}{dt} = -\frac{C_{ij}(t)}{\tau_C} + \sum_f \epsilon(t - t_j^{(f)}) \frac{g'(u_i(t))}{g(u_i(t))} [\delta(t - \hat{t}_i) - g(u_i(t)) R_i(t)] \tag{4}$$

τ_C	time constant of correlation window (1s)
$g'(u)$	derivative of $g(u)$ with respect to u
$\delta(t)$	Dirac-δ function
$\hat{t}_i$	time of last spike of neuron i
$t_j^{(f)}$	time of f-th presynaptic spike at synapse j

The term B_i^{post} maximizes information transmission and maintains the constant target firing rate for neuron i. It compares the current firing rate $g(u_i(t))$ with its running average $\bar{g}_i(t)$, and simultaneously the running average $\bar{g}_i(t)$ with the constant target rate $\tilde{g}$:

$$\begin{aligned} B_i^{post}(t) = \delta(t - \hat{t}_i) \log &\left[\frac{g(u_i(t))}{\bar{g}_i(t)} \left(\frac{\tilde{g}}{\bar{g}_i(t)} \right)^{\gamma} \right] \\ &- R_i(t)[g(u_i(t)) - (1+\gamma)\bar{g}_i(t) - \gamma\tilde{g}] \end{aligned} \tag{5}$$

The term B_{12}^{post} measures the mutual information between the output spike trains of neurons 1 and 2. It basically compares the average product of firing rates $\bar{g}_{12}(t)$ with the product of average firing rates $\bar{g}_1(t)\bar{g}_2(t)$:

$$\begin{aligned} B_{12}^{post}(t) = \delta(t - \hat{t}_1) &\left\{ \delta(t - \hat{t}_2) \log \frac{\bar{g}_{12}(t)}{\bar{g}_1(t)\bar{g}_2(t)} - R_2(t) \left[\frac{\bar{g}_{12}(t)}{\bar{g}_1(t)} - \bar{g}_2(t) \right] \right\} - \\ - R_1(t) &\left\{ \delta(t - \hat{t}_2) \left[\frac{\bar{g}_{12}(t)}{\bar{g}_2(t)} - \bar{g}_1(t) \right] - R_2(t) \left[\bar{g}_{12}(t) - \bar{g}_1(t)\bar{g}_2(t) \right] \right\} \end{aligned} \tag{6}$$