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NUMERICAL FORWARD MODELING OF GRAVITY SIGNALS CAUSED BY GLACIER MASS CHANGES IN NOVAYA ZEMLYA

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INTRODUCTION

Novaya Zemlya is an island in the Barents Sea that contains the world's third largest ice sheet of about 22.000 km² and was therefore chosen as study region for ice mass change investigation using gravity signals, cf. Figure 1. Within the frame of project ICEAGE (Modeling Snow-Ice cover Evolution and Associated Gravitational Effects with GOCE constraints) a forward modeling approach for computing gravity signals has been developed using a detailed Digital Terrain Model (DTM) and a 3D density distribution inside. Modification of the model parameters allows their individual analysis by comparing the impact on the gravity field solutions with special emphasis on the Snow and Ice Resources (SIR).

The research process consists of two parts: part one deals with the model properties ice density distribution, bedrock topography and ice thickness to investigate their individual contribution in terms of gravity field changes. In part two these results are compared to satellite based solutions for an assessment of the satellite capabilities in detecting such local ice mass variations with focus on global gravity models and (as soon as data become available) on ESA's mission GOCE. In the case of ice, interior structures and the related changes in density distributions, gravity field variations take place in submilligal range at relatively short wavelengths. Therefore, this type of mass changes is at the edge of being detectable by today's gravity field satellites.

Figure 1. *Study region Novaya Zemlya. (source: marble)*

NUMERICAL FORWARD MODELING

Key element of numerical forward modeling is the rigid solution of the Newton Integral for a rectangular prism.

$$\Delta g = G\,\Delta\rho \iiint \frac{z}{\sqrt{(x^2+y^2+z^2)^3}}\,dx\,dy\,dz$$

It can be expressed in terms of gravity anomalies Δg using a density contrast $\Delta\rho$. Analytical integration yields a closed formula for a prism's gravity effect on an arbitrary computation point, practically carried out by solving the equation for all prism corner combinations:

$$\Delta g = G\,\Delta\rho \cdot \left|\left|\left|\left[x\log(y+r) + y\log(x+r) - z\arctan\frac{xy}{zr} \right]\right|_{x1}^{x2}\right|_{y1}^{y2}\right|_{z1}^{z2}$$

By integrating every 3D prism element of the digital terrain density model (DTDM) for one particular computation point (cf. Figure 2), the sum yields the model's gravity effect on this point. By defining a series of computation points situated on the prism tops, a synthetic gravity field solution can be calculated representing the gravitational effect of the underlying DTDM.

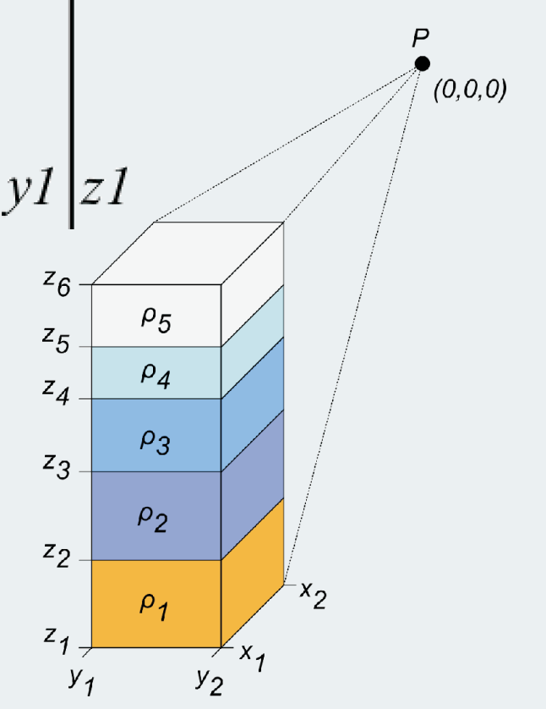
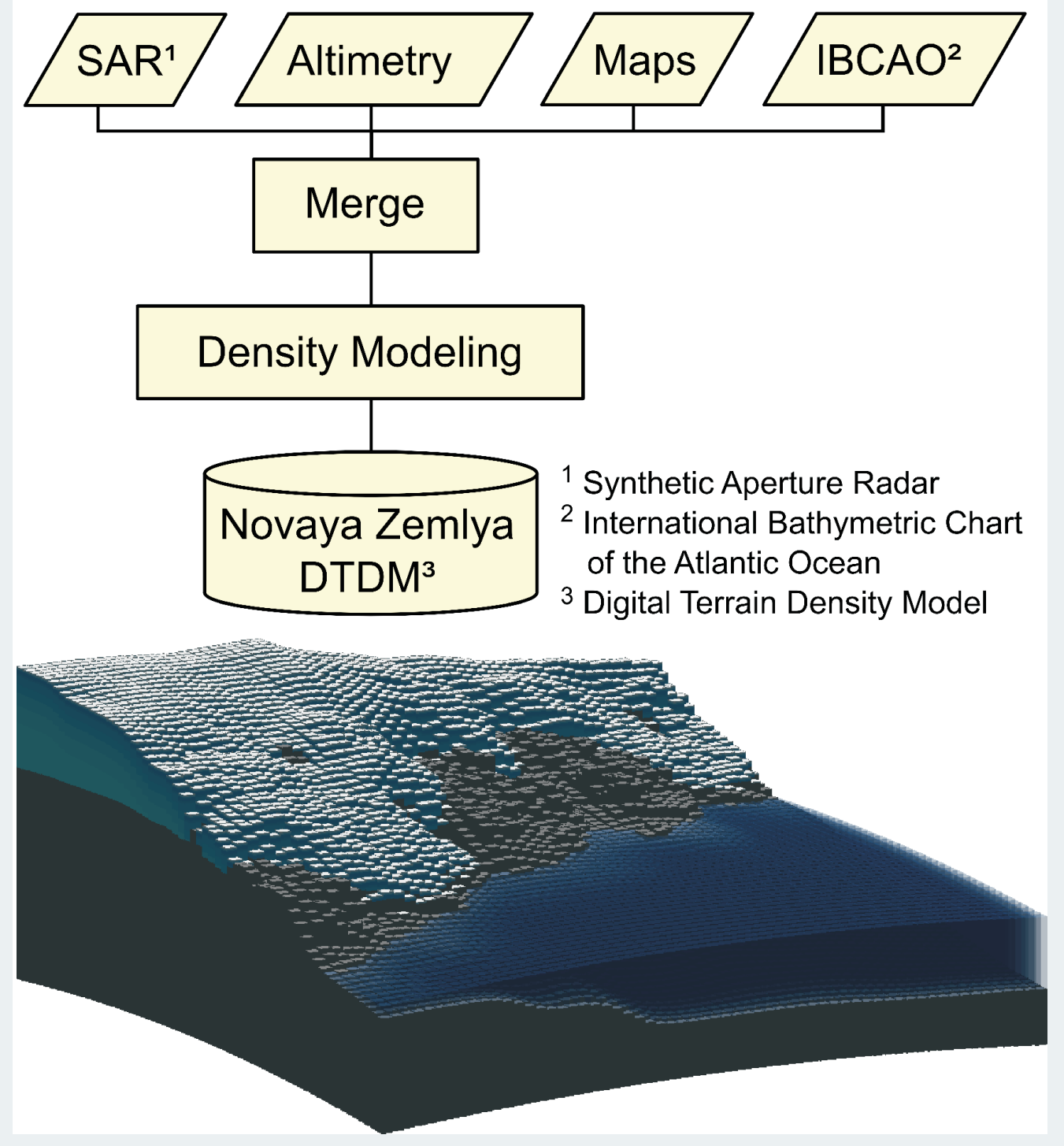


Figure 2. *Integration of multiple density prisms.*

MODEL BUILDING

The geometrical representation of Novaya Zemlya is a combination of different data sources. SAR, altimetry and various maps yielded a digital terrain model of the island itself whereas the International Bathymetric Chart of the Atlantic Ocean (IBCAO) was used for modeling the underwater topography. By merging both, island DTM and bathymetric data, a detailed geometric model of Novaya Zemlya and its surroundings could be generated, cf. Figure 3. After separating ice, bedrock and ocean, each component was filled with density individually. Ocean and the underlying bedrock were kept constant to serve as background for the ice investigation. Therefore, the model has three different parameters that can be modified and analyzed: bedrock height, ice geometry, and ice density.



[1] Synthetic Aperture Radar
[2] International Bathymetric Chart of the Atlantic Ocean
[3] Digital Terrain Density Model

Figure 3. *Combination of different data sources to generate a digital terrain density model (DTDM).*

ICE DENSITY MODELING

In order to achieve a realistic density distribution within the ice body, the empirical depth-density relation published by Schytt (1956) is used.

$$\rho(z) = \rho_i - (\rho_i - \rho_s)\exp[-(C\,z)]$$

The different parameters were defined in accordance with in situ measurements carried out by Joanneum Research in 2008: $\rho_i = 917$ kg/m³ is the empirical density of ice, $\rho_s = 550$ kg/m³ is the surface density and $C = 1.9/z_t$ is a site dependent value, governed by the firn-ice transition depth $z_t = 10$ m.

Figure 4 shows the resulting depth-density relation function. Subdivision into six discrete levels allowed the top down density modeling within the ice prisms.

Due to its low firn-ice transition depth the model has only a thin hull of lighter snow and ice resources (SIR) and a solid ice core with constant density. This model was compared to other parametrizations.

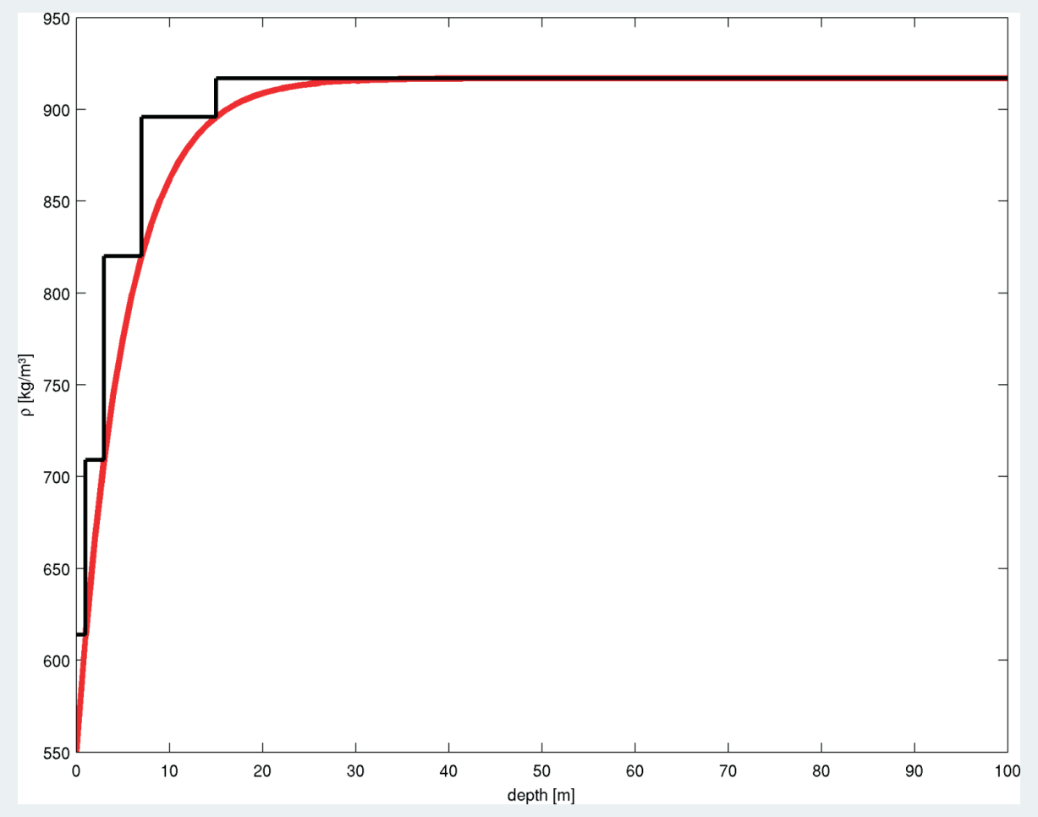


Figure 4. *Empirical depth-density relation with Novaya Zemlya parameters (red) with 6 quantization steps (black).*

INVESTIGATION OF MODEL PARAMETERS

For every result, the general model setup consists of:
- Spatial resolution of mass model: 0.5 x 0.5 km
- Computation points at horizontal prism centers
- Cross section profile at longitude 65° 15'

All results are gravity anomalies Δg, in mgal.

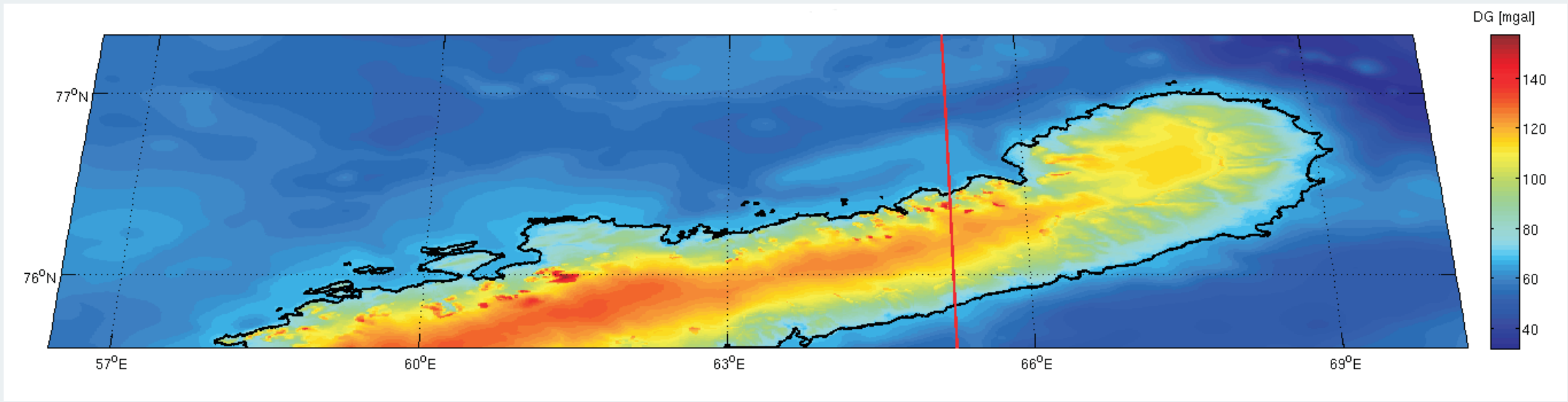


Figure 5. *Absolute gravity field solution expressed in gravity anomalies. The red line indicates the position of the cross section profile (for Figs. 8, 10, and 11).*

CHANGES IN ICE GEOMETRY

An assumed ice loss of 10% (about 40 to 50 meters) at the main ice shield, results in a gravity field change in the range of 2 mgal, cf. Figure 6.

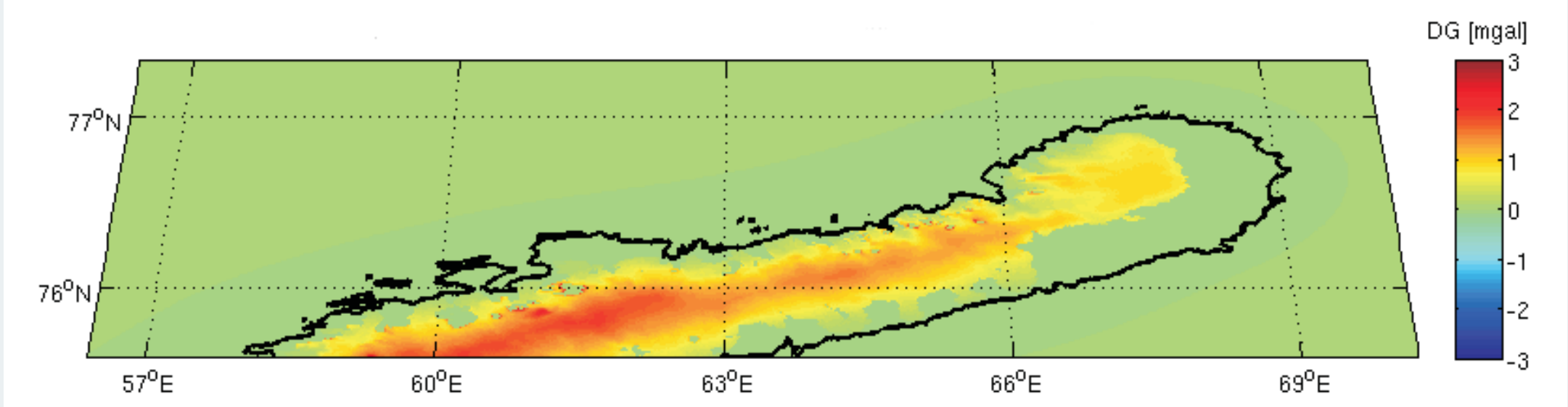


Figure 6. *Impact of simulated 10% ice mass loss on the gravity field solution.*

BEDROCK HEIGHT

The bedrock height is obtained via a look up table (LUT) remapping ice heights to surface correlated bedrock topography. Two different LUT settings were compared in Figure 7 to analyze the consequence of a possible uncertainty in the bedrock height.

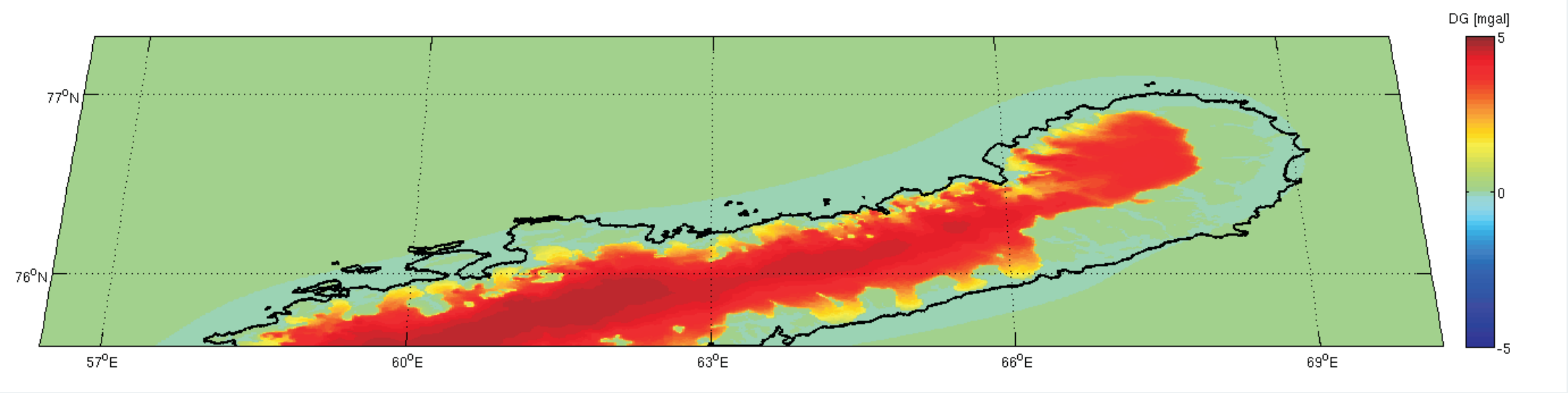


Figure 7. *Impact of two different model situations for the bedrock topography on the gravity field solution.*

A closer look at the cross section profile shows the interior model changes, Figure 8. The impact on the gravity field solution is caused by a bedrock change of about 50 meters.

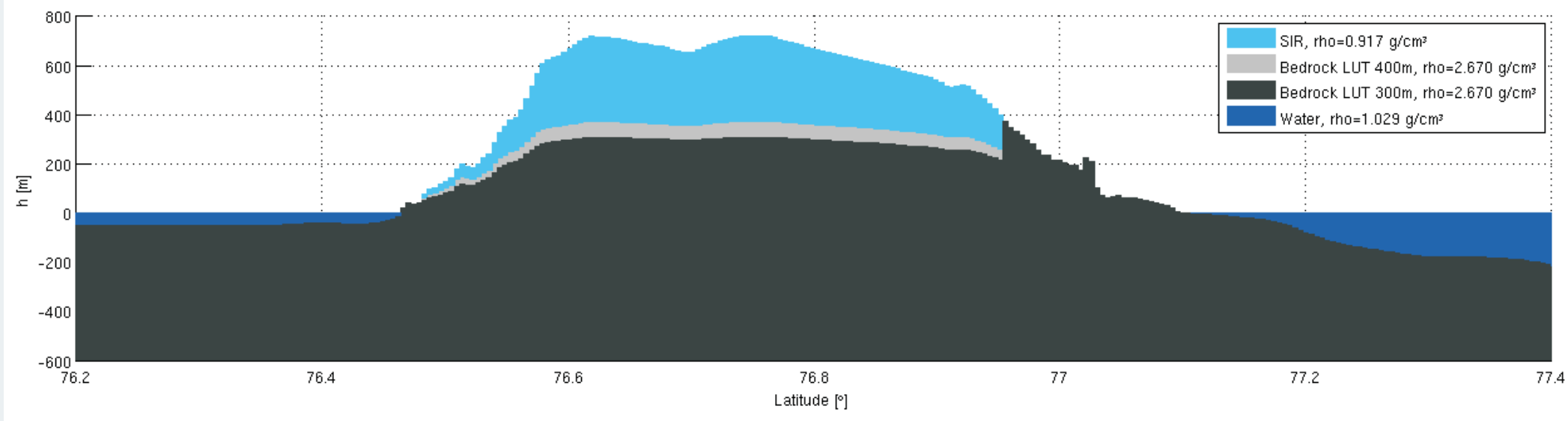


Figure 8. *Cross section profile: interior bedrock difference between models.*

DENSITY MODEL

The comparison (Figure 9) of gravity fields from two different parameter sets for the ice density confirms the expectation of a small impact (not even 1 mgal). Figures 10 and 11 show the influence of different firn-ice transition depths.

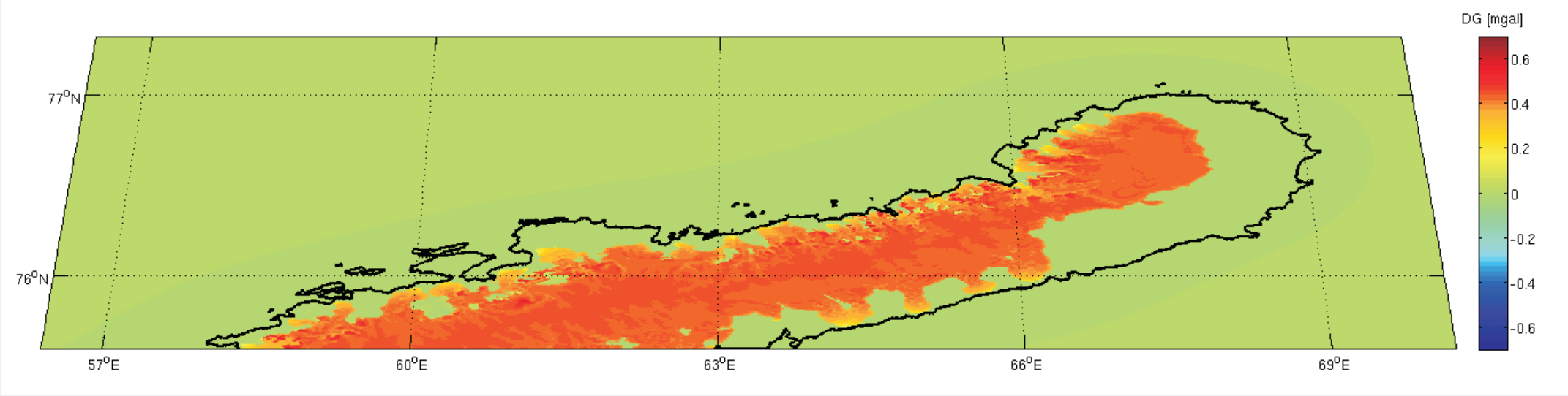


Figure 9. *Comparison of two different ice density parametrization for the Schytt depth-density relation.*

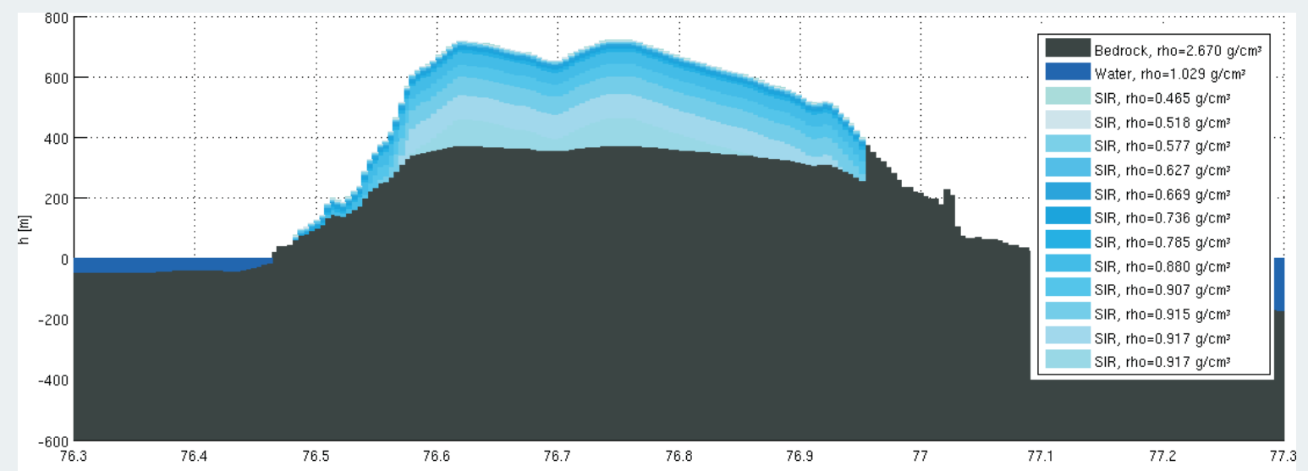


Figure 10. *Cross section profile for a smoother parametrization of the depth-density relation.*

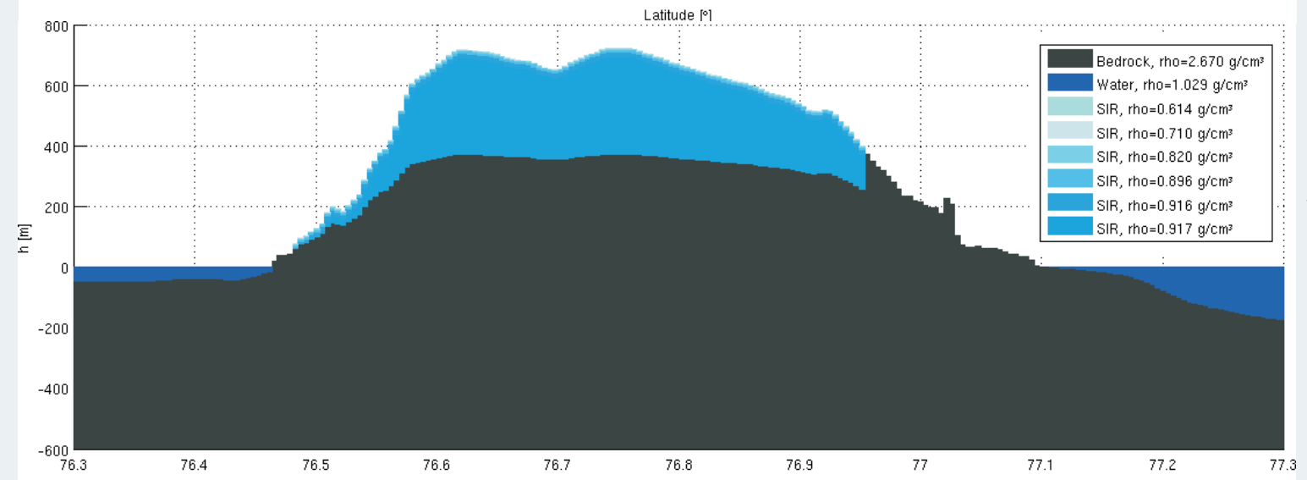


Figure 11. *Cross section profile for the parametrization described in section Ice Density Modeling.*

COMPARISON BETWEEN SYNTHETIC AND MEASURED GRAVITY FIELD SOLUTIONS

Results of this numerical modeling approach are not directly comparable to absolute global gravity field solutions: on the one hand, local modeling of mass prisms is mainly based on relative density contrasts in the upper lithosphere. Also, the modeled area is just a finite part of the whole Earth. On the other hand, the high spatial resolution of the used DTDM surpasses even high-degree solutions, e.g., EGM 2008.

To allow at least a visual comparison, high-frequency results (Figure 12) of the modeling approach had to be band-pass filtered, cf. Figure 13. EGM 2008 gravity anomalies in Figure 14 are also band-pass filtered from full degree 2190.

Here, there are differences to be observed, notably a gravity deficiency (labeled A) – and thence a lack of modeled mass/density – at the northern ice-cap in contrast to the main ice shield. Such phenomena will be further investigated to interactively find answers to questions of local gravity field inversion and interpretation.

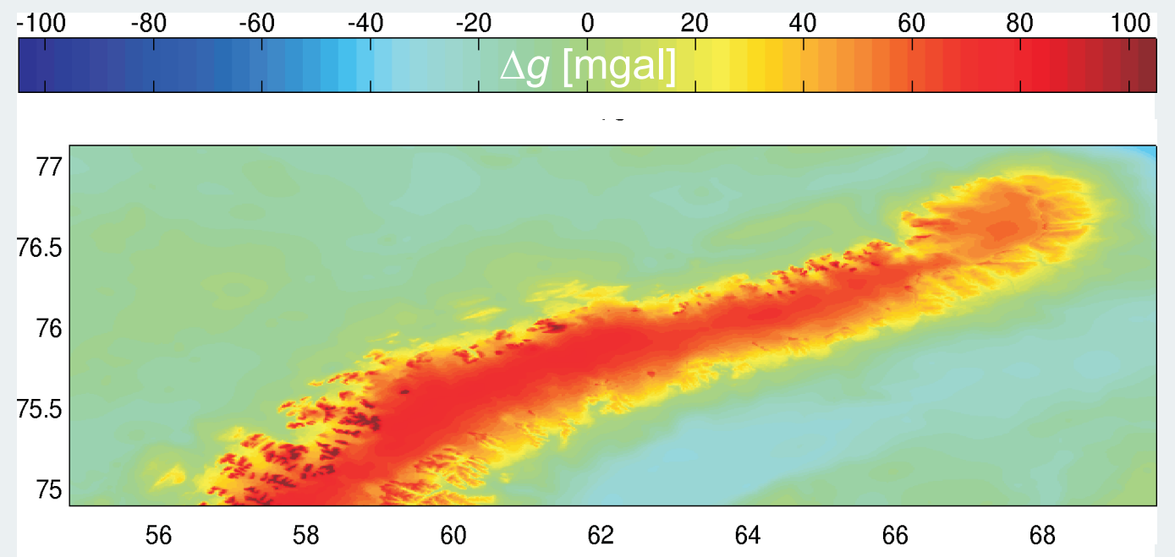


Figure 12. *Unfiltered gravity anomalies from numerical modelling of test area.*

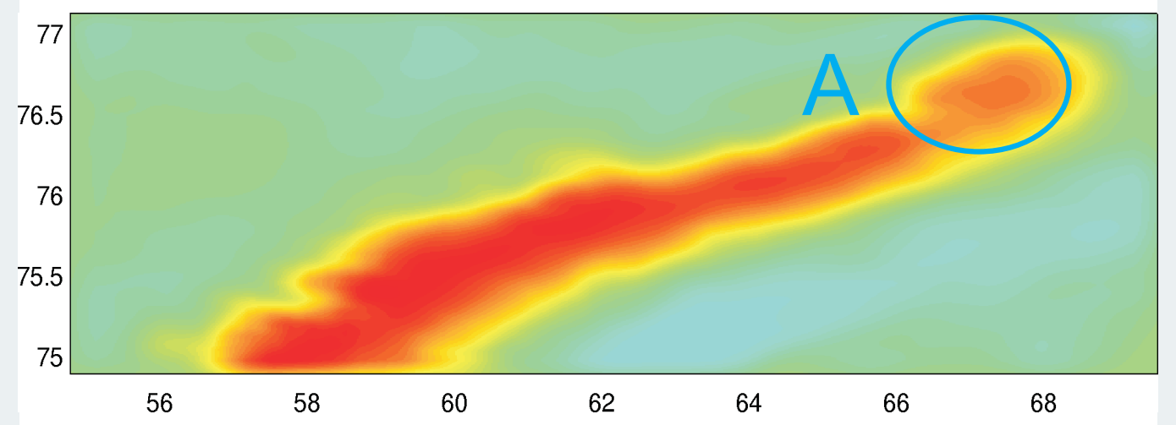


Figure 13. *Band-pass filtered gravity anomalies from numerical modelling.*

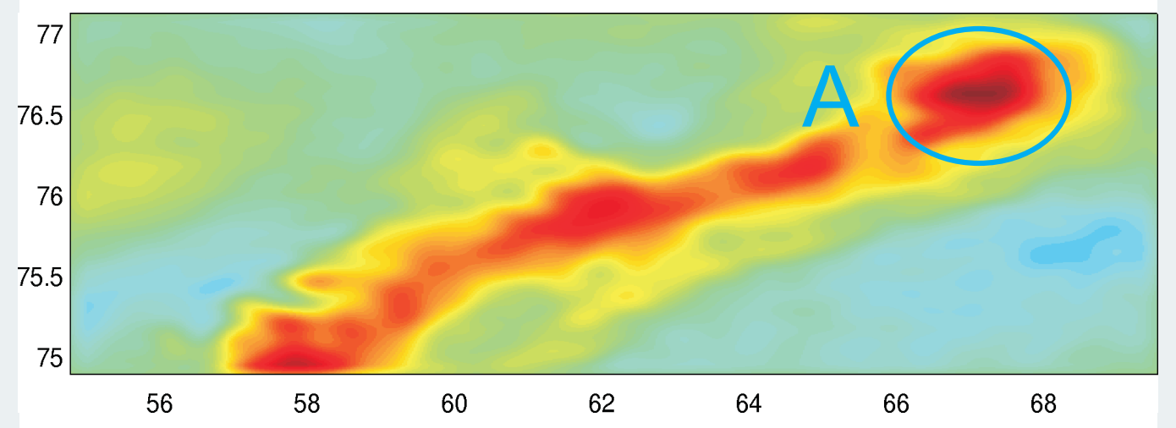


Figure 14. *EGM 2008 gravity field anomalies (filtered to match spatial resolution of Figure 13).*

Website. www.inas.tugraz.at/iceage

Acknowledgement. This work was performed in the frame of the Austrian Space Application Programme (ASAP), Phase 5, funded by the Austrian Research Promotion Agency, project number 817106.