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Predistortion of Hammerstein and Wiener systems using the Nonlinear Filtered-x Prediction Error Method algorithm

E. Abd-Elrady, L. Gan, G. Kubin

Adaptive predistortion of nonlinear systems described using Hammerstein and Wiener models is considered in this paper. The adaptive predistorter is modeled as a Wiener or Hammerstein system, respectively. The parameters of the linear and nonlinear blocks of the predistorter are estimated simultaneously using the Nonlinear Filtered-x Prediction Error Method (NFxPEM) algorithm. The NFxPEM algorithm is derived under the assumption that the parameters of the Wiener and Hammerstein predistorters are changing slowly during adaptation. Simulation study shows that the suggested predistorter using the NFxPEM algorithm can well compensate nonlinear distortion and effectively reduce spectral regrowth. Moreover, the suggested NFxPEM algorithm achieves much better results as compared to the Nonlinear Filtered-x Least Mean Squares (NFxLMS) algorithm.

Keywords: distortion; nonlinear systems; parameter estimation; prediction methods

Vorverzerrung von Hammerstein- und Wiener-Systemen unter Anwendung des Nichtlineare Filter-x Prediction Error-Methode-Algorithmus.

Dieser Artikel behandelt die adaptive Vorverzerrung von nichtlinearen Systemen. Die Nichtlinearität wird als Hammerstein- bzw. Wiener-Modell dargestellt und die adaptive Vorverzerrung entsprechend mit einem Wiener- bzw. Hammerstein-System durchgeführt. Der „Nonlinear Filtered-x Prediction Error Method (NFxPEM)"-Algorithmus schätzt gleichzeitig die Parameter des linearen und des nichtlinearen Blocks des Vorverzerrungssystems. Der NFxPEM-Algorithmus wird unter der Bedingung, dass sich die Parameter des Vorverzerrungssystems während der Adaptation nur langsam ändern, hergeleitet. Simulationen zeigen, dass der vorgeschlagene NFxPEM-Algorithmus in der Lage ist, nichtlineare Verzerrungen zu kompensieren und die spektrale Fortpflanzung effektiv zu reduzieren. Zusätzlich wird gezeigt, dass der vorgeschlagene NFxPEM-Algorithmus wesentlich bessere Ergebnisse als der „Nonlinear Filtered-x Least Mean Squares (NFxLMS)"-Algorithmus erzielt.

Schlüsselwörter: Verzerrung; nichtlineare Systeme; Parameter-Schätzung; Prediktionsmethoden

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1. Introduction

Cancelling or reducing nonlinear distortion due to nonlinearity characteristic of some electronic devices is essential requirement in many areas. In wireless communication systems, the nonlinearity of the high power amplifiers is an obstacle to increase the transfer data rate and mobility. In Hi-Fi systems, small distortion produced by nonlinear components dominates the overall performance. Examples can be found in communication systems, speech processing and control engineering, see (*Gao, Snelgrove, 1990; Lim et al., 1998; Gan, 2009*).

Several adaptive predistortion techniques based on using Volterra series as a model for the nonlinear system have been proposed (*Gao, Snelgrove, 1990; Lim et al., 1998*). However, since these techniques are based on using Volterra models, high computation complexity and slow convergence speed are expected problems during real-time implementation. Recently, an approach based on polyphase representation for Volterra filters that helps to reduce the computation complexity has been introduced in (*Schwingshackl, Kubin, 2007*). Therefore, block-structured models such as the Wiener and Hammerstein model structures, see (*Abd-Elrady, 2004, 2005*), are considered in order to decrease the number of parameters to be estimated – hence decreasing computational complexity and convergence time. The Wiener model structure consists of a linear dynamic system followed by a static nonlinearity block. On the other hand, in the Hammerstein model structure, the static nonlinearity block precedes the linear dynamic system. The Wiener and Hammerstein model structures are shown in Fig. 1.

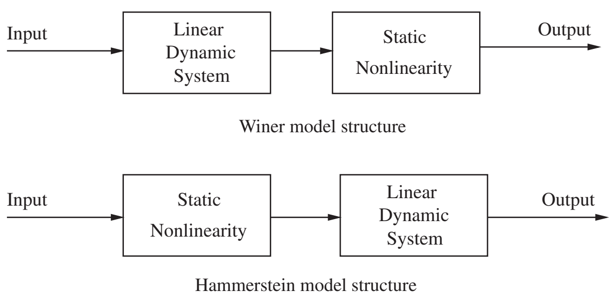

Fig. 1. The Wiener and Hammerstein model structures

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According to (*Gilabert, Montoro, Bertran, 2005*), a power amplifier can be modeled as a FIR Wiener system or an IIR Hammerstein system. The predistortion technique for the FIR Wiener system has been proposed in (*Kang, Cho, Youn, 1998, 1999*). The idea is to connect a FIR Hammerstein predistorter in tandem with the nonlinear system. Then the Nonlinear Filtered-x Least Mean Squares (NFxLMS) algorithm is used for adaptively adjusting the parameters of the predistorter. The approach of (*Kang, Cho, Youn, 1998, 1999*) requires that the FIR Wiener system to be identified first or assumed to be known a priori as well as the method of this paper. For more general results, the IIR Hammerstein and Wiener model structures are considered as a model for the nonlinear system, hence the predistorter is modeled as an IIR Wiener and Hammerstein system, respectively (*Gan, 2009; Gan, Abd-Elrady, 2008; Ibnkahla, 2002*). The results for the FIR Wiener model case follow as a special case.

In (*Costa, Bermudez, Bershad, 2002*), it was shown that the steady-state Mean Square Error (MSE) of the FxLMS algorithm highly depends on the degree of nonlinearity of the system cascaded with the adaptive filter. Also, the steady state error increases monotonically with the degree of nonlinearity. Therefore, the NFxLMS algorithm of (*Lim et al., 1998*) is expected to provide biased estimates. Also, LMS type algorithms usually have slow convergence since increasing the step size parameter leads to instability problems (*Haykin, 2002*).

In this paper, the coefficients of the predistorter are estimated recursively using the Recursive Prediction Error Method (RPEM) algorithm, see (*Ljung, 1999; Söderström, Stoica, 1989*). The RPEM algorithm gives consistent parameter estimates under weak conditions in case the asymptotic loss function has a unique stationary point which represents the true parameter vector (*Ljung, 1999; Söderström, Stoica, 1989; Ljung, Söderström, 1983*). Therefore, using the RPEM algorithm is expected to reduce the steady-state MSE and hence to minimize the total nonlinear distortion at the output of the nonlinear system. Moreover, the RPEM algorithm is known of its high convergence speed.

This paper is organized as follows. In Sect. 2, predistortion of IIR Hammerstein system using an IIR Wiener predistorter is considered. In Sect. 3, the NFxPEM algorithm is derived for adaptively estimating the parameters of the Wiener predistorter. The results for predistortion of IIR Wiener system using an IIR Hammerstein predistorter and the NFxPEM algorithm are discussed in Sect. 4. In Sect. 5, the validity of the proposed algorithm is demonstrated via computer simulations. Section 6 comes to conclusions.

2. Predistortion of Hammerstein systems

According to (*Gan, 2009; Gan, Abd-Elrady, 2008; Ibnkahla, 2002; Lashkari, Puranik, 2005*), the IIR Hammerstein system shown in Fig. 2 is to be precompensated using an IIR Wiener predistorter. The output of the Hammerstein system is given by

$$\begin{aligned} z(n) &= h(z^{-1})y_2(n) = \frac{B(z^{-1})}{1-A(z^{-1})}y_2(n) \\ &= \sum_{m=0}^{m_b} b_m y_2(n-m) + \sum_{m=1}^{m_a} a_m z(n-m) \end{aligned} \quad (1)$$

where $h(z^{-1}) = \frac{B(z^{-1})}{1-A(z^{-1})}$ and the polynomials $A(z^{-1})$ and $B(z^{-1})$ are defined as

$$\begin{aligned} A(z^{-1}) &= \sum_{m=1}^{m_a} a_m z^{-m} \\ B(z^{-1}) &= \sum_{m=0}^{m_b} b_m z^{-m}. \end{aligned} \quad (2)$$

Here z^{-1} is the delay operator such that $z^{-m}x(n) = x(n-m)$. The intermediate signal $y_2(n)$ is defined as

$$y_2(n) = g_1 y(n) + g_2 y^2(n) + \cdots + g_{m_g} y^{m_g}(n) = \theta_g^T \mathbf{y}(n) \quad (3)$$

where

$$\theta_g = \begin{pmatrix} g_1 & g_2 & \cdots & g_{m_g} \end{pmatrix}^T \quad (4)$$

$$\mathbf{y}(n) = \begin{pmatrix} y(n) & y^2(n) & \cdots & y^{m_g}(n) \end{pmatrix}^T. \quad (5)$$

Similarly, the output of the Wiener predistorter is given as

$$y(n) = f_1(n)x_2(n) + f_2(n)x_2^2(n) + \cdots + f_{m_f}x_2^{m_f}(n) = \boldsymbol{\theta}_f^T(n)\mathbf{x}_2(n) \quad (6)$$

where

$$\boldsymbol{\theta}_f(n) = (f_1(n) \quad f_2(n) \quad \cdots \quad f_{m_f}(n))^T \quad (7)$$

$$\mathbf{x}_2(n) = \Big(x_2(n) \quad x_2^2(n) \cdots x_2^{m_f}(n) \Big)^T. \quad (8)$$

The intermediate signal $x_2(n)$ is given by

$$\begin{aligned} x_2(n) &= p(n,z^{-1})x(n) = \frac{D(n,z^{-1})}{1-C(n,z^{-1})}x(n) \\ &= \sum_{m=0}^{m_d} d_m(n)x(n-m) + \sum_{m=1}^{m_c} c_m(n)x_2(n-m) \end{aligned} \quad (9)$$

where $p(n,z^{-1}) = \frac{D(n,z^{-1})}{1-C(n,z^{-1})}$ and the polynomials $C(n,z^{-1})$ and $D(n,z^{-1})$ are defined as

$$\begin{aligned} C(n,z^{-1}) &= \sum_{m=1}^{m_c} c_m(n) z^{-m} \\ D(n,z^{-1}) &= \sum_{m=0}^{m_d} d_m(n) z^{-m}. \end{aligned} \quad (10)$$

Let us define the parameter vector $\boldsymbol{\theta}$ of the predistorter as follows

$$\begin{aligned} &\boldsymbol{\theta} = \begin{pmatrix} \boldsymbol{\theta}_f^T & \boldsymbol{\theta}_d^T & \boldsymbol{\theta}_c^T \end{pmatrix}^T, \quad \boldsymbol{\theta}_f = (f_1 \quad f_2 \quad \cdots f_{m_f})^T \\ &\boldsymbol{\theta}_d = (d_0 \quad d_1 \quad \cdots \quad d_{m_d})^T, \quad \boldsymbol{\theta}_c = (c_1 \quad c_2 \quad \cdots \quad c_{m_c})^T. \end{aligned} \quad (11)$$

The goal of this paper is to develop the NFxPEM algorithm in order to estimate the parameter vector $\boldsymbol{\theta}$. This algorithm is introduced in the next section.

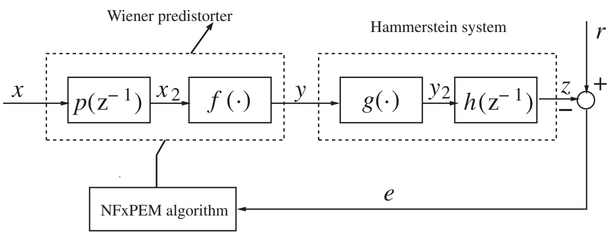

Fig. 2. Predistortion of Hammerstein systems

3. The NFxPEM algorithm

Prediction Error Methods (PEMs) are a family of parameter estimation methods that can be applied to a wide spectrum of model parameterizations. PEM has a close relationship with the Maximum Likelihood (ML) method. Therefore, it gives models with excellent asymptotic properties, see Chapter 7 in (*Söderström, Stoica, 1989*), Sect. 4.4 and Chapter 5 in (*Ljung, Söderström, 1983; Ljung, 2002; Wigren, 1994*). The basic idea behind the prediction error approach is to describe the model as a predictor of the next output. Then, this predictor is parameterized in terms of a finite-dimensional parameter vector $\boldsymbol{\theta}$. Hence, a consistent estimate of $\boldsymbol{\theta}$ is determined from the model parameterization and the observed data set.

The Gauss-Newton NFxPEM algorithm is derived by the minimization of the cost function (*Ljung, Söderström, 1983*)

$$V(\boldsymbol{\theta}) = \lim_{\mathcal{N}\to\infty} \frac{1}{\mathcal{N}} \sum_{n=1}^{\mathcal{N}} \mathrm{E}[e^2(n,\boldsymbol{\theta})]. \tag{12}$$

Here E{.} denotes the Expectation and $e(n,\ \boldsymbol{\theta})$ is the prediction error which is defined as

$$e(n,\ \boldsymbol{\theta}) = r(n) - z(n,\boldsymbol{\theta}) \tag{13}$$

where $r(n)$ is the reference signal which is defined as

$$r(n) = x(n-\tau) + \upsilon(n). \tag{14}$$

Here τ is the delay time and $\upsilon(n)$ is zero-mean Additive White Gaussian Noise (AWGN).

Remark 1: The delay time τ equals to zero in case the system to be compensated is minimum phase (*Lim et al., 1998*).

The formulation of the NFxPEM algorithm requires the negative gradient of $e(n,\ \boldsymbol{\theta})$ w.r.t. $\boldsymbol{\theta}$ which is defined as

$$\psi(n) = \frac{\mathrm{d}e(n,\ \boldsymbol{\theta})}{\mathrm{d}\boldsymbol{\theta}} = \frac{\mathrm{d}z(n,\ \boldsymbol{\theta})}{\mathrm{d}\boldsymbol{\theta}}. \tag{15}$$

Using Eq. (1), $\frac{\mathrm{d}z(n)}{\mathrm{d}\boldsymbol{\theta}(n)}$ can be derived as

$$\frac{\mathrm{d}z(n)}{\mathrm{d}\boldsymbol{\theta}(n)} = \sum_{m=0}^{m_b} b_m \frac{\mathrm{d}y_2(n-m)}{\mathrm{d}\boldsymbol{\theta}(n)} + \sum_{m=1}^{m_a} a_m \frac{\mathrm{d}z(n-m)}{\mathrm{d}\boldsymbol{\theta}(n)}. \tag{16}$$

Assuming that the parameter vector $\boldsymbol{\theta}$ is changing slowly (*Lim et al., 1998; Johnson, 1984*), the following approximations can be made:

$$\begin{aligned}\frac{\mathrm{d}y_2(n-m)}{\mathrm{d}\boldsymbol{\theta}(n)} &\approx \frac{\mathrm{d}y_2(n-m)}{\mathrm{d}\boldsymbol{\theta}(n-m)}, \quad m = 0,1,\ldots,m_b \\ \frac{\mathrm{d}z(n-m)}{\mathrm{d}\boldsymbol{\theta}(n)} &\approx \frac{\mathrm{d}z(n-m)}{\mathrm{d}\boldsymbol{\theta}(n-m)}, \quad m = 1,2,\ldots,m_a.\end{aligned} \tag{17}$$

Consequently, Eq. (16) can be written as

$$\begin{aligned}\frac{\mathrm{d}z(n)}{\mathrm{d}\boldsymbol{\theta}(n)} &\approx \sum_{m=0}^{m_b} b_m \frac{\mathrm{d}y_2(n-m)}{\mathrm{d}\boldsymbol{\theta}(n-m)} + \sum_{m=1}^{m_a} a_m \frac{\mathrm{d}z(n-m)}{\mathrm{d}\boldsymbol{\theta}(n-m)} = \frac{B(\mathrm{z}^{-1})}{1-A(\mathrm{z}^{-1})}\frac{\mathrm{d}y_2(n)}{\mathrm{d}\boldsymbol{\theta}(n)}. \\ &\approx \widehat{h}(\mathrm{z}^{-1})\frac{\mathrm{d}y_2(n)}{\mathrm{d}\boldsymbol{\theta}(n)}\end{aligned} \tag{18}$$

where $\widehat{h}(\mathrm{z}^{-1})$ is the estimate of $h(\mathrm{z}^{-1})$. From Eqs. (3) – (5), we have

$$\frac{\mathrm{d}y_2(n)}{\mathrm{d}\boldsymbol{\theta}(n)} = \boldsymbol{\theta}_g^{\mathrm{T}} \frac{\mathrm{d}\boldsymbol{y}(n)}{\mathrm{d}\boldsymbol{\theta}(n)} = \boldsymbol{\theta}_g^{\mathrm{T}} \frac{\mathrm{d}\boldsymbol{y}(n)}{\mathrm{d}y(n)}\frac{\mathrm{d}y(n)}{\mathrm{d}\boldsymbol{\theta}(n)} \approx s_1(n)\frac{\mathrm{d}y(n)}{\mathrm{d}\boldsymbol{\theta}(n)} \tag{19}$$

where

$$s_1(n) = \widehat{\boldsymbol{\theta}}_g^{\mathrm{T}} \frac{\mathrm{d}\boldsymbol{y}(n)}{\mathrm{d}\boldsymbol{\theta}(n)} = \widehat{\boldsymbol{\theta}}_g^{\mathrm{T}} \big(1 \quad 2y(n) \quad \cdots \quad m_g y^{m_g-1}(n)\big)^{\mathrm{T}}. \tag{20}$$

Here $\widehat{\boldsymbol{\theta}}_g$ is the estimate of $\boldsymbol{\theta}_g$. Using Eqs. (11) and (19), Eq. (18) becomes

$$\begin{aligned}\frac{\mathrm{d}z(n)}{\mathrm{d}\boldsymbol{\theta}(n)} &\approx \widehat{h}(\mathrm{z}^{-1})s_1(n)\frac{\mathrm{d}y(n)}{\mathrm{d}\boldsymbol{\theta}(n)} \\ &\approx \widehat{h}(\mathrm{z}^{-1})s_1(n)\left(\frac{\partial y(n)}{\partial\boldsymbol{\theta}_f(n)}\ \frac{\partial y(n)}{\partial\boldsymbol{\theta}_d(n)}\ \frac{\partial y(n)}{\partial\boldsymbol{\theta}_c(n)}\right).\end{aligned} \tag{21}$$

Considering Eqs. (6–9), $\frac{\partial y(n)}{\partial\boldsymbol{\theta}_f(n)}$ can be derived as

$$\begin{aligned}\frac{\partial y(n)}{\partial\boldsymbol{\theta}_f(n)} &= \frac{\partial\boldsymbol{\theta}_f^{\mathrm{T}}(n)\boldsymbol{x}_2(n)}{\partial\boldsymbol{\theta}_f(n)} = \boldsymbol{x}_2^{\mathrm{T}}(n) \\ &= \big(p(n,\mathrm{z}^{-1})x(n)\ \ [p(n,\mathrm{z}^{-1})x(n)]^2\ \cdots\ [p(n,\mathrm{z}^{-1})x(n)^{m_f}\big)^{\mathrm{T}}.\end{aligned} \tag{22}$$

Note that the intermediate signal $x_2(n)$ should be estimated since it is usually not measurable. Again, using Eqs. (6–9), $\frac{\partial y(n)}{\partial\boldsymbol{\theta}_d(n)}$ and $\frac{\partial y(n)}{\partial\boldsymbol{\theta}_c(n)}$ can be derived as

$$\begin{aligned}\frac{\partial y(n)}{\partial\boldsymbol{\theta}_d(n)} &= \boldsymbol{\theta}_f^{\mathrm{T}}(n)\frac{\mathrm{d}\boldsymbol{x}_2(n)}{\mathrm{d}x_2(n)}\frac{\partial x_2(n)}{\partial\boldsymbol{\theta}_d(n)} = s_2(n)\frac{\partial x_2(n)}{\partial\boldsymbol{\theta}_d(n)} \\ \frac{\partial y(n)}{\partial\boldsymbol{\theta}_c(n)} &= \boldsymbol{\theta}_f^{\mathrm{T}}(n)\frac{\mathrm{d}\boldsymbol{x}_2(n)}{\mathrm{d}x_2(n)}\frac{\partial x_2(n)}{\partial\boldsymbol{\theta}_c(n)} = s_2(n)\frac{\partial x_2(n)}{\partial\boldsymbol{\theta}_c(n)}\end{aligned} \tag{23}$$

where

$$\begin{aligned}s_2(n) &= \boldsymbol{\theta}_f^{\mathrm{T}}(n)\frac{\mathrm{d}\boldsymbol{x}_2(n)}{\mathrm{d}x_2(n)} = \boldsymbol{\theta}_f^{\mathrm{T}}(n)\left(1 \quad 2x_2(n) \quad \cdots \quad m_f x_2^{m_f-1}(n)\right)^{\mathrm{T}} \\ &= \boldsymbol{\theta}_f^{\mathrm{T}}(n)\big(1 \quad 2[p(n,\mathrm{z}^{-1})x(n)] \quad \cdots \quad m_f[p(n,\mathrm{z}^{-1})x(n)]^{m_f-1}\big)^{\mathrm{T}}.\end{aligned} \tag{24}$$

Now, it remains to derive $\frac{\partial x_2(n)}{\partial\boldsymbol{\theta}_d(n)}$ and $\frac{\partial x_2(n)}{\partial\boldsymbol{\theta}_c(n)}$. Differentiating both sides of Eq. (9) with respect to $d_k(n)$ and $c_k(n)$ gives

$$\begin{aligned}\frac{\partial x_2(n)}{\partial d_k(n)} &= x(n-k) + \sum_{m=1}^{m_c} c_m(n)\frac{\partial x_2(n-m)}{\partial d_k(n)} \\ \frac{\partial x_2(n)}{\partial c_k(n)} &= x_2(n-k) + \sum_{m=1}^{m_c} c_m(n)\frac{\partial x_2(n-m)}{\partial c_k(n)}.\end{aligned} \tag{25}$$

Since the parameter vector $\boldsymbol{\theta}$ is assumed to be changing slowly, we can write

$$\begin{aligned}\frac{\partial x_2(n-m)}{\partial d_k(n)} &\approx \frac{\partial x_2(n-m)}{\partial d_k(n-m)}, \\ \frac{\partial x_2(n-m)}{\partial c_k(n)} &\approx \frac{\partial x_2(n-m)}{\partial c_k(n-m)}, \quad m = 1,2,\ldots,m_c.\end{aligned} \tag{26}$$

Hence, Eq. (25) can be rewritten as

$$\begin{aligned}\frac{\partial x_2(n)}{\partial d_k(n)} &\approx x(n-k) + \sum_{m=1}^{m_c} c_m(n)\frac{\partial x_2(n-m)}{\partial d_k(n-m)} \\ &= \frac{\mathrm{z}^{-k}}{1-C(n,\mathrm{z}^{-1})}x(n), \quad k = 0,1,\ldots,m_d\end{aligned} \tag{27}$$

$$\begin{aligned}\frac{\partial x_2(n)}{\partial c_k(n)} &\approx x_2(n-k) + \sum_{m=1}^{m_c} c_m(n)\frac{\partial x_2(n-m)}{\partial c_k(n-m)} \\ &= \frac{\mathrm{z}^{-k}}{1-C(n,\mathrm{z}^{-1})}[p(n,\mathrm{z}^{-1})x(n)], \quad k = 1,\ldots,m_c.\end{aligned}$$

Therefore, from Eq. (23), we have

$$\frac{\partial y(n)}{\partial\boldsymbol{\theta}_d(n)} = s_2(n)\begin{pmatrix}\frac{\partial y(n)}{\partial d_0(n)} \\ \frac{\partial y(n)}{\partial d_1(n)} \\ \vdots \\ \frac{\partial y(n)}{\partial d_{m_d}(n)}\end{pmatrix}^{\mathrm{T}} \approx s_2(n)\begin{pmatrix}\frac{1}{1-C(n,\ \mathrm{z}^{-1})}x(n) \\ \frac{\mathrm{z}^{-1}}{1-C(n,\ \mathrm{z}^{-1})}x(n) \\ \vdots \\ \frac{\mathrm{z}^{-m_d}}{1-C(n,\ \mathrm{z}^{-1})}x(n)\end{pmatrix}^{\mathrm{T}} \tag{28}$$

$$\begin{aligned}\frac{\partial y(n)}{\partial\theta_c(n)} &= s_2(n)\begin{pmatrix}\frac{\partial y(n)}{\partial c_1(n)} \\ \frac{\partial y(n)}{\partial c_2(n)} \\ \vdots \\ \frac{\partial y(n)}{\partial C_{m_c}(n)}\end{pmatrix}^{\mathrm{T}} \\ &\approx s_2(n)\begin{pmatrix}\frac{\mathrm{z}^{-1}}{1-C(n,\ \mathrm{z}^{-1})}[p(n,\mathrm{z}^{-1})x(n)] \\ \frac{\mathrm{z}^{-2}}{1-C(n,\ \mathrm{z}^{-1})}[p(n,\mathrm{z}^{-1})x(n)] \\ \vdots \\ \frac{\mathrm{z}^{-m_c}}{1-C(n,\ \mathrm{z}^{-1})}[p(n,\mathrm{z}^{-1})x(n)]\end{pmatrix}^{\mathrm{T}}.\end{aligned} \tag{29}$$

Now, we have completely derived the components of $\frac{\mathrm{d}z(n,\boldsymbol{\theta})}{\mathrm{d}\boldsymbol{\theta}(n)}$ in Eq. (21) and hence the gradient vector $\psi(n)$. Therefore, the NFxPEM algorithm follows as (*cf. Söderström, Stoica, 1989; Ljung, Söderström, 1983; Ljung, 2002; Wigren, 1994*)

$$\begin{aligned} e(n,\boldsymbol{\theta}) &= r(n) - z(n,\boldsymbol{\theta}) \\ \lambda(n) &= \lambda_o\lambda(n-1) + 1 - \lambda_o \\ S(n) &= \psi^{\mathrm{T}}(n)\mathbf{P}(n-1)\psi(n) + \lambda(n) \\ \mathbf{P}(n) &= (\mathbf{P}(n-1) - \mathbf{P}(n-1)\psi(n)S^{-1}(n)\psi^{\mathrm{T}}(n) \\ &\quad \times \mathbf{P}(n-1))/\lambda(n) \\ \boldsymbol{\theta}(n) &= \boldsymbol{\theta}(n-1) + \mathbf{P}(n)\psi(n)e(n,\boldsymbol{\theta}). \end{aligned} \tag{30}$$

Here $\lambda(n)$ is a forgetting factor that grows exponentially to 1 as $n \to \infty$ where the rate λ_o and the initial value $\lambda(0)$ are design variables. The numerical values $\lambda_o = 0.99$ and $\lambda(0) = 0.95$ have proven to be useful in many applications (*Ljung, Söderström, 1983*). Also, $\mathbf{P}(n) = n\mathbf{R}^{-1}(n)$ where $\mathbf{R}(n)$ is the Hessian approximation in the Gauss-Newton algorithm, see (*Söderström, Stoica, 1989; Ljung, Söderström, 1983*). The most common choice for the initial condition of $\mathbf{P}(n)$ is $\mathbf{P}(0) = \rho\mathbf{I}$ where $\mathbf{I}$ is the identity matrix and ρ is a constant that reflects our trust in the initial parameter vector $\theta(0)$. In case of no prior knowledge, $\boldsymbol{\theta}(0) = \mathbf{0}$ and ρ is large to speed up convergence to the true parameter vector.

4. The NFxPEM algorithm for predistortion of Wiener systems

In this section, the situation of predistortion of IIR Wiener system using an IIR Hammerstein predistorter is considered, see Fig. 3. Straightforward analysis similar to Sect. 3, see (*Gan, 2009*) for details, to calculate the gradient vector $\psi(n)$ in this case gives

$$\frac{dz(n)}{d\boldsymbol{\theta}(n)} \approx s_3(n)\widehat{h}(z^{-1})\left(\frac{\partial y(n)}{\partial\boldsymbol{\theta}_f(n)}\ \frac{\partial y(n)}{\partial\boldsymbol{\theta}_d(n)}\ \frac{\partial y(n)}{\partial\boldsymbol{\theta}_c(n)}\right) \tag{31}$$

where

$$s_3(n) = \widehat{\boldsymbol{\theta}}_g^{\mathrm{T}}\left(1 \quad 2[\widehat{h}(z^{-1})y(n)] \quad \cdots \quad m_g\,[\widehat{h}(z^{-1})y(n)]^{m_g-1}\right)^{\mathrm{T}} \tag{32}$$

$$\frac{\partial y(n)}{\partial\boldsymbol{\theta}_f(n)} = \left(p(n,z^{-1})x(n)\ \ p(n,z^{-1})x^2(n)\ \cdots\ p(n,z^{-1})x^{m_f}(n)\right)^{\mathrm{T}} \tag{33}$$

$$\frac{\partial y(n)}{\partial\boldsymbol{\theta}_d(n)} = \begin{pmatrix} \frac{\partial y(n)}{\partial d_0(n)} \\ \frac{\partial y(n)}{\partial d_1(n)} \\ \vdots \\ \frac{\partial y(n)}{\partial d_{m_d}(n)} \end{pmatrix}^{\mathrm{T}} \approx \begin{pmatrix} \frac{1}{1-C(n,\,z^{-1})}\boldsymbol{\theta}_f^{\mathrm{T}}\boldsymbol{x}(n) \\ \frac{z^{-1}}{1-C(n,\,z^{-1})}\boldsymbol{\theta}_f^{\mathrm{T}}\boldsymbol{x}(n) \\ \vdots \\ \frac{z^{-m_d}}{1-C(n,\,z^{-1})}\boldsymbol{\theta}_f^{\mathrm{T}}\boldsymbol{x}(n) \end{pmatrix}^{\mathrm{T}} \tag{34}$$

$$\boldsymbol{x}(n) = \left(x(n) \quad x^2(n) \quad \cdots \quad x^{m_g}(n)\right)^{\mathrm{T}}. \tag{35}$$

$$\frac{\partial y(n)}{\partial\boldsymbol{\theta}_c(n)} = \begin{pmatrix} \frac{\partial y(n)}{\partial c_1(n)} \\ \frac{\partial y(n)}{\partial c_2(n)} \\ \vdots \\ \frac{\partial y(n)}{\partial c_{m_c}(n)} \end{pmatrix}^{\mathrm{T}} \approx \begin{pmatrix} \frac{z^{-1}}{1-C(n,\,z^{-1})}y(n) \\ \frac{z^{-2}}{1-C(n,\,z^{-1})}y(n) \\ \vdots \\ \frac{z^{-m_c}}{1-C(n,\,z^{-1})}y(n) \end{pmatrix}^{\mathrm{T}} \tag{36}$$

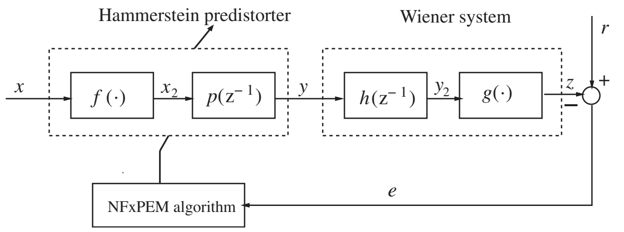

Fig. 3. Predistortion of Wiener systems

Remark 2: For the case of FIR Wiener model, Eqs. (31–36) are modified by choosing $A(z^{-1}) = 0$ in $h(z^{-1})$ and $C(n,z^{-1}) = 0$ in $p(n,z^{-1})$.

5. Simulation study

In this section, simulation examples for the predistortion of Hammerstein and Wiener systems are given. In this study, the performance of the predistorter using the NFxPEM algorithm is compared with the case using the NFxLMS algorithm.

The input signal was a random signal with uniform distribution over $(-1, 1)$ with data length of $2\text{x}10^5$ samples. The bandwidth of the input signal was limited in order to prevent aliasing (*Schetzen, 1989*). The measurement noise was considered as AWGN with a signal to noise ratio (SNR) equal to 40dB.

As a performance measure, the normalized mean-square distortion E_D of the system containing the predistorter and the nonlinear system has been evaluated. E_D is defined as

$$E_D(n) = 10\ \log_{10}\left(\frac{\widehat{\mathrm{E}}\{e^2(n)\}}{\widehat{\mathrm{E}}\{r^2(n)\}}\right) \tag{37}$$

where $\widehat{\mathrm{E}}\{\cdot\}$ is the mean over 200 independent realizations.

Remark 3: In this simulation study we assumed that $\widehat{A}(z^{-1})$, $\widehat{B}(z^{-1})$ and $\widehat{\boldsymbol{\theta}}_g$ are equal to $A(z^{-1})$, $B(z^{-1})$ and $\boldsymbol{\theta}_g$, respectively.

Remark 4: In this simulation study, the initial conditions of the parameter vectors of the predistorter are chosen such that $y = x_2$ and $x_2 = x$, i.e. the predistorter is assumed initially to be a linear system with transfer function equals one.

Example 1: Predistortion of IIR Hammerstein system

The following IIR Hammerstein system was considered:

$$\begin{aligned} z(n) &= \frac{0.72 + 1.51z^{-1} + 1.04z^{-2} + 0.26z^{-3}}{1 + 1.46z^{-1} + 0.89z^{-2} + 0.18z^{-3}}y_2(n) \\ y_2(n) &= y(n) + 0.25y^2(n) + 0.125y^3(n). \end{aligned} \tag{38}$$

The order of the linear and nonlinear blocks of the IIR Wiener predistorter were chosen as $m_c = 3$, $m_d = 3$ and $m_f = 9$, respectively.

Figure 4 shows the E_D comparison between the NFxLMS and NFxPEM algorithms. The step size for the NFxLMS algorithm was set as μ=0.05 and the matrix $\mathbf{P}(0) = \mathbf{I}$ for the NFxPEM algorithm. The nonlinear distortion of the IIR Hammerstein system without the predistorter was −16.85dB. As it is shown in Fig. 4, the NFxPEM

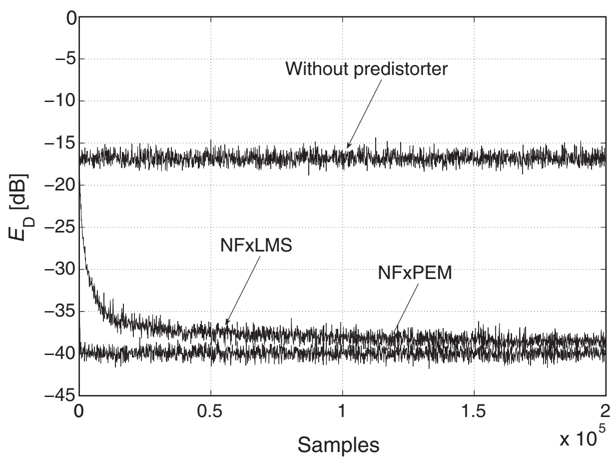

Fig. 4. E_D for the NFxLMS and NFxPEM algorithms in Example 1

algorithm gives a lower distortion than the NFxLMS algorithm. On average, the NFxPEM and NFxLMS algorithms achieve about −40.01 dB and −38.59 dB, respectively. On the other hand, the NFxPEM algorithm converges much faster than the NFxLMS algorithm. Figure 5 shows the power spectral densities (PSDs) of the output signals of the IIR Hammerstein system with and without predistorter. From this figure, we can see that the predistorter using the NFxPEM algorithm can reduce the spectral regrowth more effectively, as compared to using the NFxLMS algorithm.

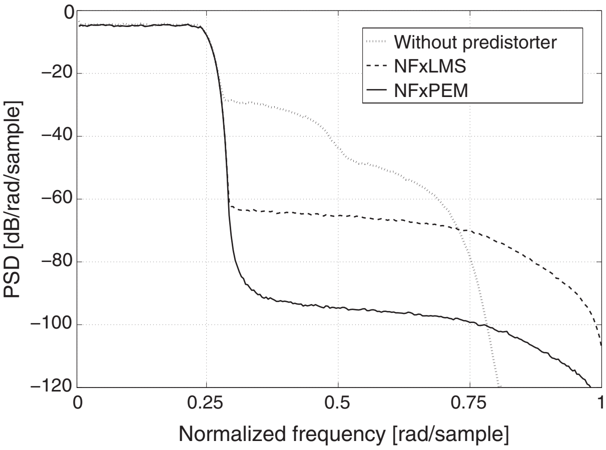

Fig. 5. PSDs for the output signals in Example 1

Example 2: Predistortion of FIR Wiener system

The following FIR Wiener system was considered:

$$\begin{aligned} z(n) &= y_2(n) + 0.5y_2^2(n) + 0.25y_2^3(n) \\ y_2(n) &= 0.75y(n) + 0.375y(n-1) - 0.15y(n-2). \end{aligned} \tag{39}$$

The orders of the linear and nonlinear blocks of the FIR Hammerstein predistorter were chosen as $m_d = 8$ and $m_f = 9$, respectively. The step size for the NFxLMS algorithm was set as μ=0.02. For the NFxPEM algorithm, the matrix $\mathbf{P}(0) = \mathbf{I}$.

The E_D comparison between the two algorithms is given in Fig. 6. The distortion of the FIR Wiener system without predistorter was −12.10 dB. On average, the NFxPEM and NFxLMS algorithms achieve about −39.95 dB and −35.94 dB, respectively. Figure 7 shows the PSDs of the output signals of the FIR Wiener system with and without predistorter. Also, it is concluded from Fig. 7 that the

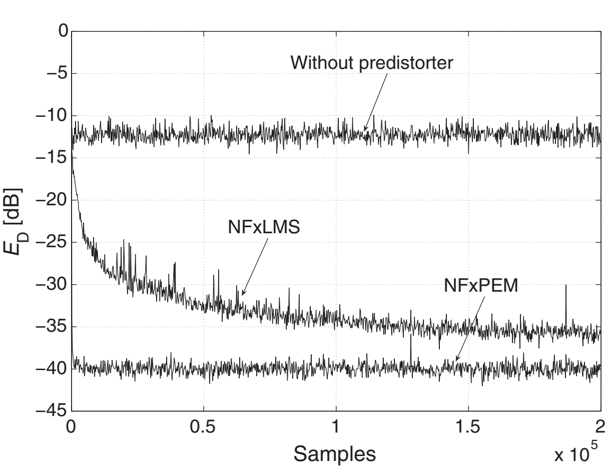

Fig. 6. E_D for the NFxLMS and NFxPEM algorithms in Example 2

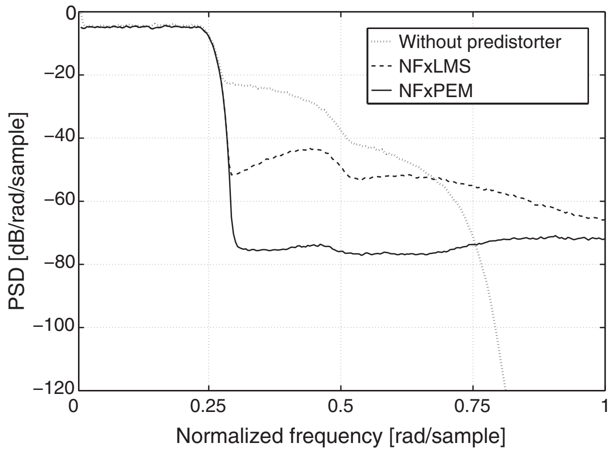

Fig. 7. PSDs for the output signals in Example 2

NFxPEM algorithm more significantly suppress spectral regrowth as compared to the NFxLMS algorithm.

Remark 5: The computation complexity of the NFxPEM algorithm (30) is higher than the NFxLMS algorithm due to the fact that the NFxPEM algorithm requires the recursive computation of the matrix $\mathbf{P}(n)$ in addition to the parameter vector $\boldsymbol{\theta}(n)$. See (*Gan, 2009*) for detailed discussion on the computational complexities of these algorithms.

6. Conclusions

Adaptive predistortion of nonlinear systems described using IIR Hammerstein and Wiener models is considered in this paper. The NFxPEM algorithm has been derived for the estimation of the parameters of the IIR Wiener and Hammerstein predistorters, respectively. This is done under the assumption that the parameters of the predistorter are changing slowly during the adaptation process. The simulation results show that the suggested predistorter using the NFxPEM algorithm can effectively compensate the nonlinear distortion of the nonliner system and more significantly suppress spectral regrowth as compared to the NFxLMS algorithm.

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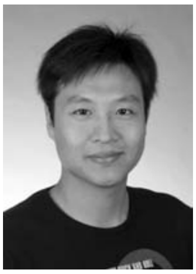

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