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Covariance Propagation of Latitude-dependent Orbit Errors within the Energy Integral Approach

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Introduction

The satellite mission GOCE has the demanding task to map the Earth's gravity field with unprecedented accuracy by using state-of-the-art observation technologies. The processing strategy of the orbit data is based on the energy integral approach to determine the long wavelength structure of the gravity field. The velocity of the satellite serves as basic input of the energy integral and is derived from the measured satellite position by applying a suitable numerical differentiation technique. The final product will consist of the gravity field model in terms of estimated spherical harmonic (SH) coefficients and the corresponding error description. Therefore, the adjustment procedure is extended by a covariance propagation starting from orbit errors.
The study about covariance propagation of latitude-dependent orbit errors is driven by the fact that the GPS receiver used for GOCE might not have full performance in the case of low-elevation GPS satellites, which might lead to a reduced number of observable satellites in higher latitudes.

Mathematical Formulation

To derive the velocity of the satellite, the Taylor-MacLaurin differentiator is used, which is based on a Taylor series expansion in each position. Let us consider a set of differentials Δ^j at a certain sampling point k,

$$\boxed{\Delta_k^j = \frac{x_{k+j} - x_{k-j}}{2j\delta}} \qquad t_k = t_{k-1} + \delta \qquad k = 1, 2, \ldots n \tag{1}$$

where x indicates one of the three coordinates of the satellite's position at a certain epoch t_k. δ is a constant sampling interval. Inserting a Taylor series at position *(k-j)* and *(k+j)* in Eq. (1) yields

$$\Delta_k^j = x_k' + j^2 \frac{1}{6} \delta^2 x_k''' + j^4 \frac{1}{120} \delta^4 x_k^V + \ldots \tag{2}$$

$$\boldsymbol{l} = \boldsymbol{A} \cdot \boldsymbol{u}$$

$$\begin{bmatrix} \Delta_k^1 \\ \Delta_k^2 \\ \Delta_k^3 \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{6} & \frac{1}{120} \\ 1 & \frac{4}{6} & \frac{16}{120} \\ 1 & \frac{9}{6} & \frac{81}{120} \end{bmatrix} \cdot \begin{bmatrix} x_k' \\ \delta^2 x_k''' \\ \delta^4 x_k^V \end{bmatrix} \tag{3}$$

To solve for the derivatives of the function x_k, the equation system Eq. (3) can be established.

It is an over-determined system, if the order of derivatives is smaller than the number of differences, which can be solved by a standard least squares adjustment (LS) approach (Eq. (4)).

$$\boxed{\hat{\boldsymbol{u}} = \left(\boldsymbol{A}^T \boldsymbol{P} \boldsymbol{A}\right)^{-1} \boldsymbol{A}^T \boldsymbol{P} \boldsymbol{l}} \tag{4}$$

$$\sigma^2_{\Delta_k^j} = \frac{1}{4j^2\delta^2} \left(\sigma^2_{k+j} + \sigma^2_{k-j}\right) \tag{5}$$

To compose the weight matrix P, the weight of each Δ_k^j is computed by its standard deviation. Since the differentials are not directly measured, but rather derived from the positions, the variance $\sigma^2_{\Delta_k^j}$ has to be computed from the known orbit errors. Therefore, the pythagorean sum of the basic observation equation (cf. Eq. (1)) can be formed to determine the propagated errors of the differentials. σ^2_{k+j} indicates the standard deviation of the satellite position in each direction at a certain epoch $(k \pm j)$.

$$\boldsymbol{P} = \begin{bmatrix} \frac{1}{\sigma^2_{\Delta_k^1}} & 0 & \cdots & 0 \\ 0 & \frac{1}{\sigma^2_{\Delta_k^2}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{1}{\sigma^2_{\Delta_k^n}} \end{bmatrix} \tag{6}$$

$$\boxed{\boldsymbol{\Sigma}\left(\hat{\boldsymbol{u}}\right) = \left(\boldsymbol{A}^T \boldsymbol{P} \boldsymbol{A}\right)^{-1}} \tag{7}$$

Finally, the variance of the derived velocity can be found in element (1,1) of the covariance matrix (cf. Eq. (7)), which is further used to set up a weight matrix for the LS adjustment of the SH coefficients in the energy integral software.

Numerical case studies

The test data set is composed of a noise-free orbit with GOCE characteristics, for a measurement period of 59 days, and with 1 s sampling. The orbit is based on the gravity field model OSU91a complete to degree/order 80. Three noise scenarios, which are illustrated in Fig. 1, shall be investigated.
In noise scenario [NM 1] (cf. Fig. (1) top), a white noise time series with a standard deviation of σ = 1 cm is superposed to the x-, y-, and z-coordinate of the satellite position. Noise model [NM 2] is composed with the assumption that the position accuracy in all three coordinates decreases linearly in dependence of the geographical latitude, starting at $|\varphi| = 60°$. [NM 3] is based on a realistic noise scenario, where the latitude-dependent accuracy of the three coordinates behaves differently (cf. Fig. (1) bottom).

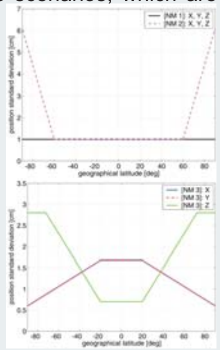

Fig. 1 Noise models for the three case study scenarios.

Gravity Field Recovery

To show the impact of the three test scenarios on the coefficient solutions more clearly, the cumulative geoid height errors are plotted in Fig. 2 (top). The bottom row shows that the statistical error description reflects the geoid height errors in relation to the different noise models. In the case of [NM 1], the improved accuracy towards the poles is due to meridian convergence of the orbits, and thus a larger number of observations in near-polar regions with homogenous measurement accuracy. Concerning [NM 2], the degraded measurement accuracy towards the poles (cf. Fig. 1)

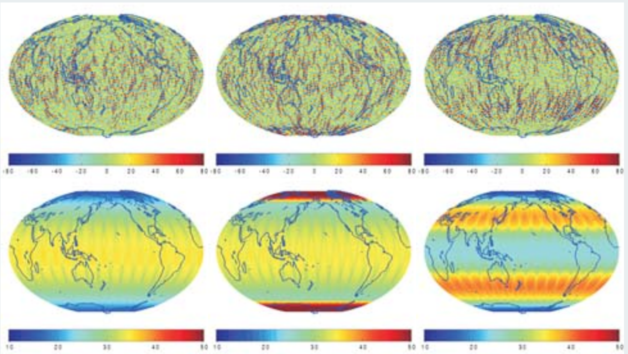

Fig. 2 Cumulative geoid height errors [cm] at degree/order 80 (top) and corresponding statistical error estimates [cm] (bottom) based on the three noise characteristics [NM 1] (left), [NM 2] (center) and [NM 3] (right).

leads to a reduced geoid height accuracy, which is reflected both in the geoid height deviations from the true reference (cf. Fig. 2 (top center)) and the geoid height standard deviations (bottom center). In the case of [NM 3], the spatial error bands in the mid-latitudes, which are also related to the initial orbit error model (cf. Fig. 1), are consistently reflected in the absolute deviations (top right) and the statistical error description (bottom right). The statistical error estimates of [NM 1] will apply for all three solutions, if the studies are based on the white noise assumption. The coefficient solution itself improves only slightly due to the use of the correct stochastic model.

So far, the main conclusion could apparently be that the quality of the solution is quite insensitive to the metric applied in the normal equation system. To demonstrate that the use of a correct stochastic model is indeed important, Fig. 3 shows the standard deviation per geographical latitude from the geoid height differences, as well as a mean standard deviation per geographical latitude computed from the propagated geoid height standard deviations. In the case of the white noise assumption (Euclidean metric), the stochastic error estimate (green curve) corresponds to the one of [NM 1], and is inconsistent with the true geoid height errors per latitude (red curve). If the correct stochastic model is used and consistently propagated, the gravity field solution is only slightly improved (dark blue curve). However, the statistical error information (light blue curve) is now perfectly consistent.

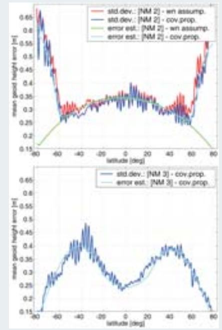

Fig. 3 Mean geoid height errors per degree: simulation based on [NM 2] (top) and [NM 3] (bottom).

Conclusion

Based on several simulations it could be demonstrated, that the consistent propagation of latitude-dependent errors (assuming that these error characteristics are known a-priori) through the gravity field adjustment procedure, where they are used to modify the metrics of the normal equation systems, can not significantly improve the coefficient solution itself, but it leads to a consistent propagation to the covariance information, which is important in the course of the combination with the SGG component. A degraded accuracy of the SST gravity field component in a certain region, which is correctly reflected by the corresponding covariance information, will lead to a modified relative weighting of the SST and the SGG component. Therefore, if in reality there is indeed a significant spatial correlation of the orbit accuracy (due to the elevation cut-off or any other reason), it should be taken into consideration in the stochastic model.