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Preparing Thin Layer Models for Application to the Finite Element Method

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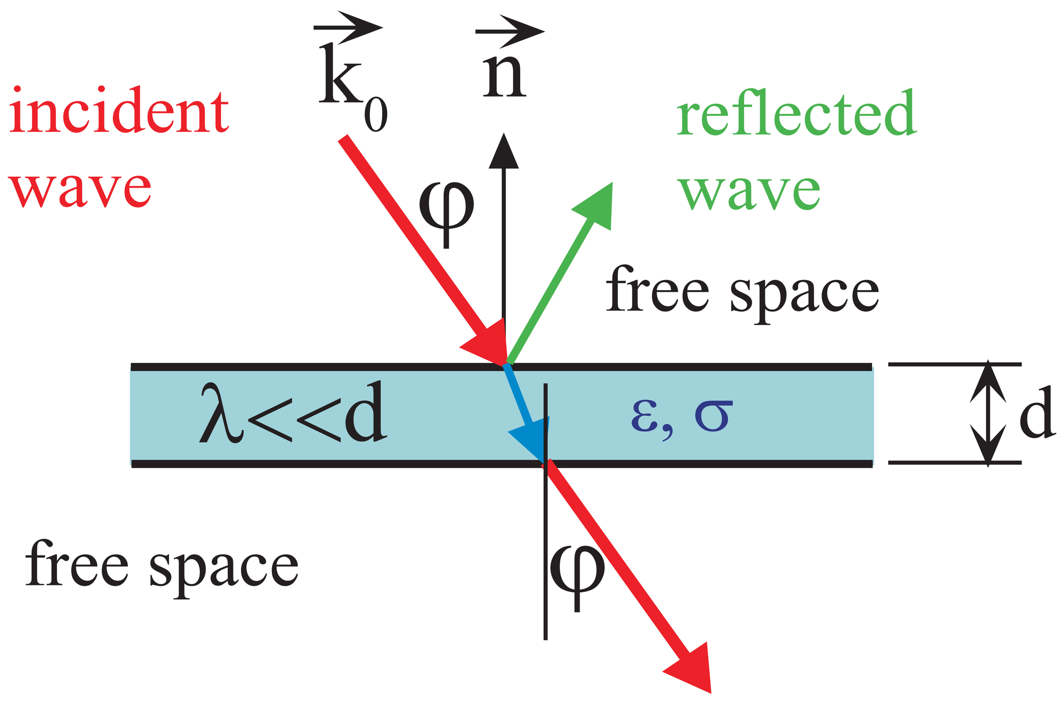
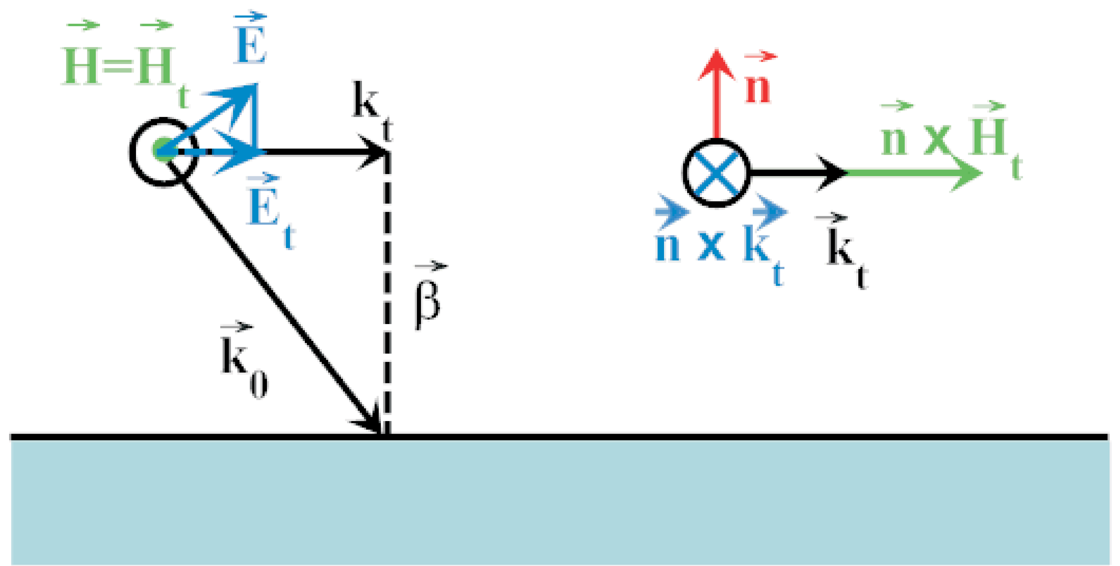
Abstract: Since the FEM is widely used and allows for reliable results in numerical modeling electromagnetic field problems, it will be used as a framework for implementation of analytically modeled layers of any material like metal sheets, conductive paint and the like.
Based on analytical work done by [Tretyakov] a model was developed to determine attenuation of electromagnetic waves due to reflection and absorption when traveling through thin layers of either dielectric or conductive and lossy media under different angles of incidence. The modeling is based on the transversal incident electric and magnetic field components of the incident plane wave and is adapted to the well known **A**,v formulation for application to the Galerkin method.





$$\vec{E}_t = \vec{E}_0 \cos\varphi$$

$$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

$$Z_m = \sqrt{\frac{\mu_0\mu_r}{\epsilon_0\epsilon_r - j\frac{\sigma}{\omega}}}$$

$$R = \frac{Z_m \frac{\beta}{k_0} - Z_0\cos(\varphi)}{Z_m \frac{\beta}{k_0} + Z_0\cos(\varphi)}$$

$$k_0 = |\vec{k_0}| = \omega\sqrt{\epsilon_0\mu_0}$$

$$\vec{E_t} = \vec{E_0}(1+R)\cos\varphi, \quad \vec{H_t} = \frac{\vec{E_0}}{Z_0}(1-R)$$

$$S = \frac{1}{2}\mathrm{Re}(\vec{E}\times\vec{H}^*)$$

$$a = 10\log\frac{S_{out}}{S_{in}}$$

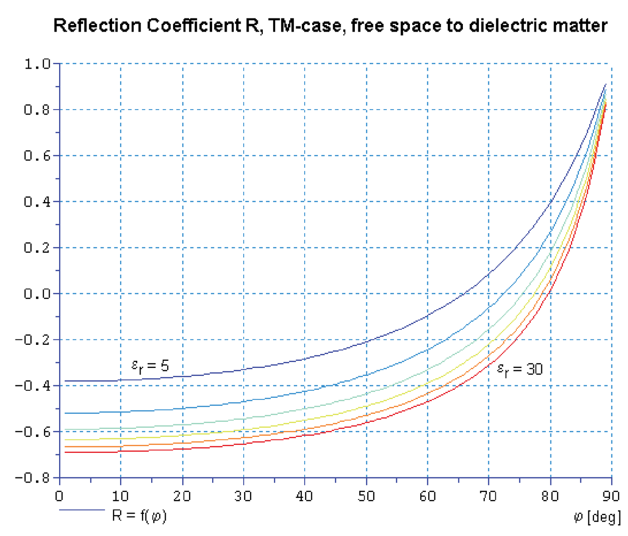
Galerkin, A,v-formulation:

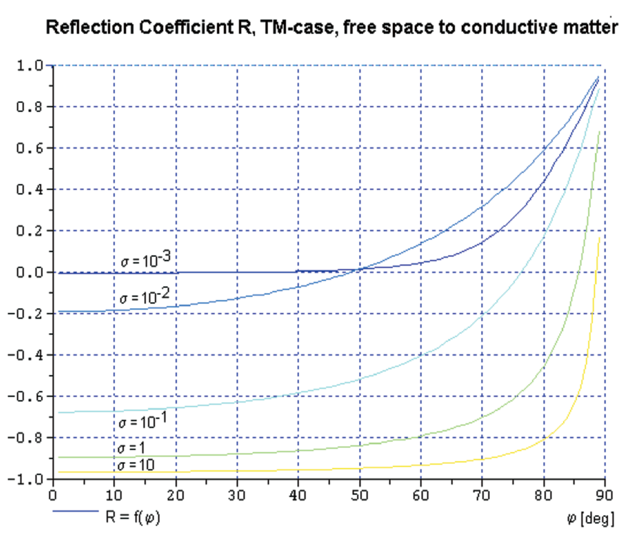
$$-\int_\Omega \nabla\times\vec{N_i}\cdot\frac{1}{\mu}\nabla\times\vec{A}d\Omega + \int_{\Gamma_H}\vec{N_i}\cdot(\vec{n}\times(\frac{1}{\mu}\nabla\times\vec{A}))d\Gamma$$
$$+\int_\Omega \vec{N_i}\cdot(\sigma+j\omega\epsilon)j\omega(\vec{A}+\nabla v)d\Omega = 0$$

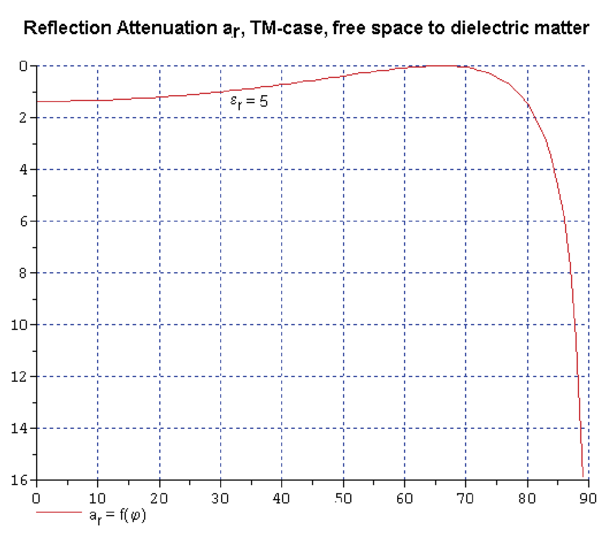
Network model:

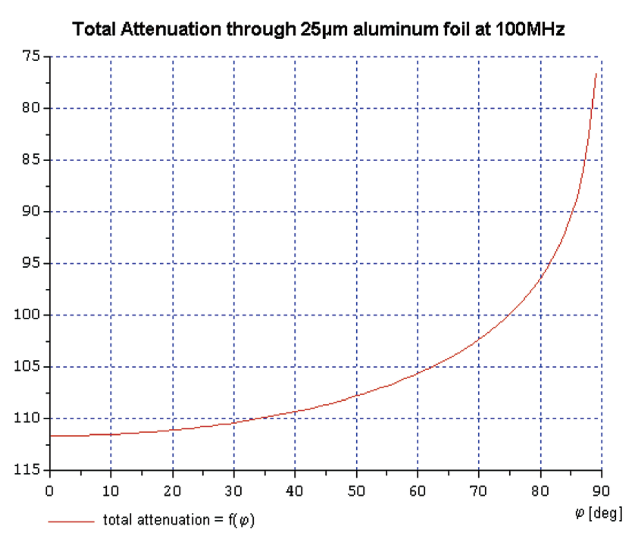
$$\begin{Bmatrix} \vec{E}_{t+} \\ \vec{n}\times\vec{H}_{t+} \end{Bmatrix} = \begin{bmatrix} \bar{a}_{11} & \bar{a}_{12} \\ \bar{a}_{21} & \bar{a}_{22} \end{bmatrix} \cdot \begin{Bmatrix} \vec{E}_{t-} \\ \vec{n}\times\vec{H}_{t-} \end{Bmatrix}$$

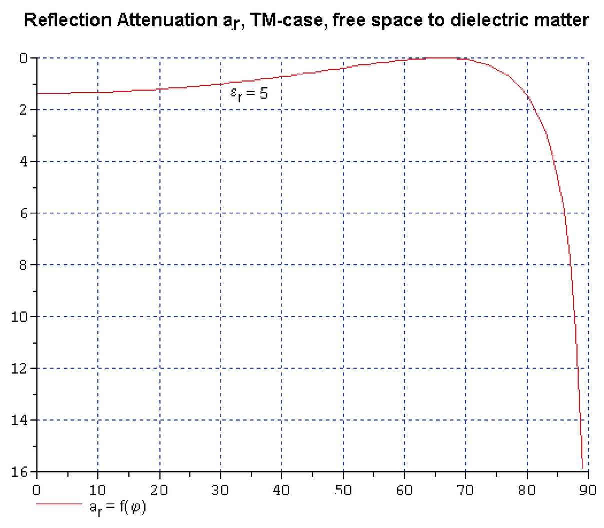
$$\bar{a}_{11} = \bar{a}_{22} = \cos(\beta_m d)\cdot\bar{I}_t$$
$$\bar{a}_{12} = j\frac{k_m Z_m}{\beta_m}\sin(\beta_m d)\cdot\bar{B}$$
$$\bar{a}_{21} = j\frac{k_m}{Z_m\beta_m}\sin(\beta_m d)\cdot\bar{C}$$
$$\bar{B} = \bar{I}_t - \frac{\vec{k_t}\cdot\vec{k_t}}{k^2}$$
$$\bar{C} = \bar{I}_t - \frac{\vec{n}\times\vec{k_t}\cdot\vec{n}\times\cdot\vec{k_t}}{k^2}$$

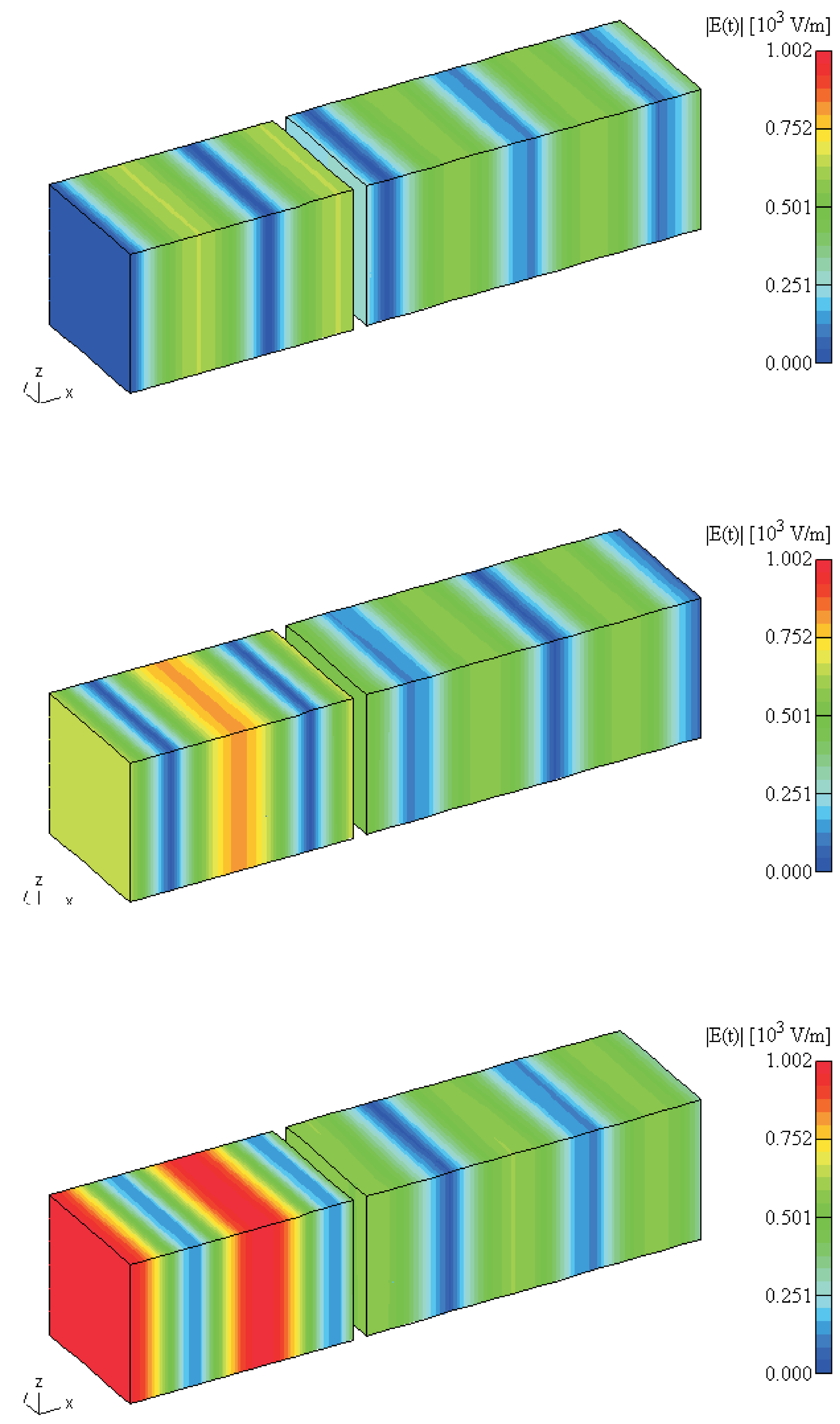












Dielectric slab, ε_r = 10, |E| at different time steps.

Conclusion:

Discussion of thin layer behaviour with a network model

Thin layer network model in FEM-formulation implemented

Up to now plane wave penetrates the layer perpendicularly, only.