Computing the shielding effectiveness of waveguides using FE-mesh truncation by surface operator implementation

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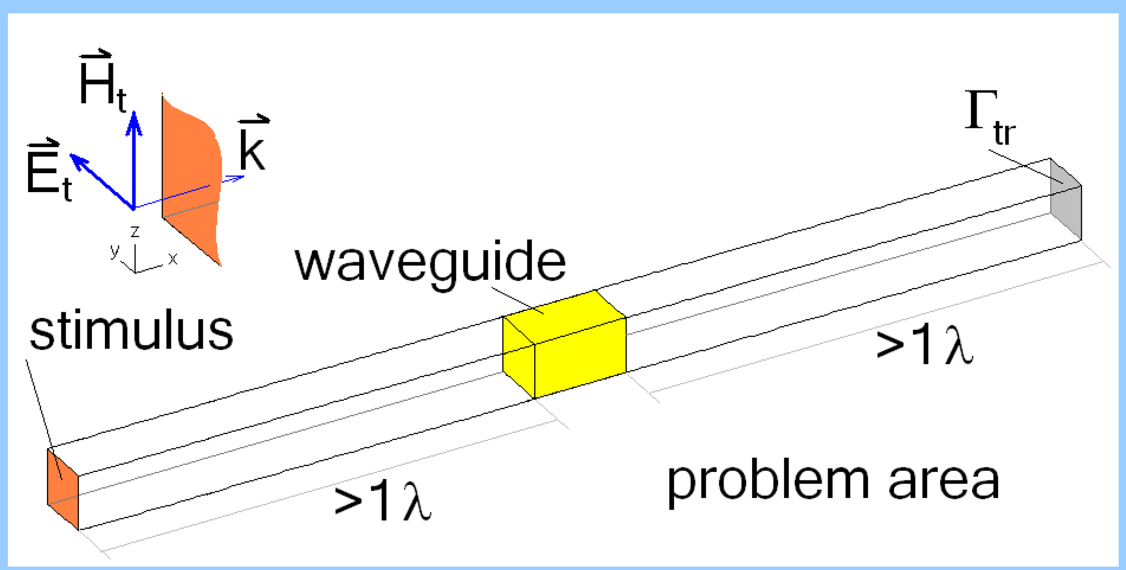
Abstract - A plane wave incident perpendicular to one open end of a conductive tube, as part of a honeycomb-structure, is attenuated on its way through it. In order to calculate its total attenuation for various frequencies the FE-method will be used. This requires a reflectionless truncation of the FE-mesh for which a Surface Operator Boundary Condition (SOBC) will be employed.

Basic arrangement of waveguide and problem area:



$$\nabla \times \vec{E} = -j\omega\mu\vec{H}, \quad \nabla \times \vec{H} = j\omega\epsilon\vec{E}$$

$$\vec{E} = \vec{E}_t + \vec{n}E_n, \vec{H} = \vec{H}_t + \vec{n}H_n, \nabla = \nabla_t + \frac{\partial}{\partial n}\vec{n}$$

Surface Operator Boundary Conditions (SOBC)

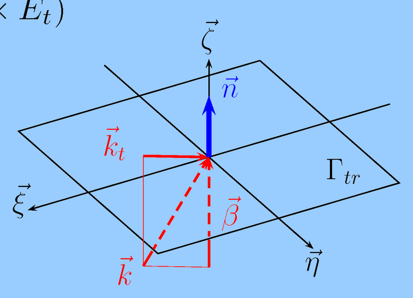
$$\frac{\partial(\vec{n} \times \vec{E}_t)}{\partial n} = -j\omega\mu\vec{H}_t - \frac{1}{j\omega\epsilon}\nabla_t \times (\nabla_t \times \vec{H}_t)$$

$$\frac{\partial(\vec{n} \times \vec{H}_t)}{\partial n} = j\omega\epsilon\vec{E}_t + \frac{1}{j\omega\mu}\nabla_t \times (\nabla_t \times \vec{E}_t)$$

$$\vec{k} = \vec{k}_t + \vec{\beta}$$

$$\beta = \pm\sqrt{k^2 - {k_t}^2}$$

$$k = \omega\sqrt{\mu\epsilon}$$



$$\int_{\zeta=0}^{\infty} \vec{H_{t0}} e^{-j\beta\zeta} d\zeta = \frac{1}{j\beta}\vec{H_{t0}}, \int_{\zeta=0}^{\infty} \vec{E_{t0}} e^{-j\beta\zeta} d\zeta = \frac{1}{j\beta}\vec{E_{t0}}$$

$$\vec{n} \times \vec{E_{t0}} = \frac{-\omega\mu\vec{H_{t0}}}{\sqrt{k^2 - {k_t}^2}} + \frac{\nabla_t \times (\nabla_t \times \vec{H_{t0}})}{\omega\epsilon\sqrt{k^2 - {k_t}^2}}$$

$$\vec{n} \times \vec{H_{t0}} = \frac{\omega\epsilon\vec{E_{t0}}}{\sqrt{k^2 - {k_t}^2}} - \frac{\nabla_t \times (\nabla_t \times \vec{E_{t0}})}{\omega\mu\sqrt{k^2 - {k_t}^2}}$$

$$\vec{n} \times \vec{E_{t0}} = \frac{-\omega\mu\vec{H_{t0}}}{\sqrt{k^2 - {k_t}^2}} - \frac{\vec{k_t} \times (\vec{k_t} \times \vec{H_{t0}})}{\omega\epsilon\sqrt{k^2 - {k_t}^2}}$$

$$\vec{n} \times \vec{H_{t0}} = \frac{\omega\epsilon\vec{E_{t0}}}{\sqrt{k^2 - {k_t}^2}} + \frac{\vec{k_t} \times (\vec{k_t} \times \vec{E_{t0}})}{\omega\mu\sqrt{k^2 - {k_t}^2}}$$

Galerkin Method for the A,v-formulation

$$-\int_{\Omega} \nabla \times \vec{N}_i \cdot \frac{1}{\mu}\nabla \times \vec{A} d\Omega + \int_{\Gamma_H} \vec{N}_i \cdot \underbrace{(\vec{n} \times (\frac{1}{\mu}\nabla \times \vec{A}))}_{\vec{n}\times\vec{H}} d\Gamma$$

$$+\int_{\Omega} \vec{N}_i \cdot (\sigma + j\omega\epsilon) j\omega(\vec{A} - \nabla v) d\Omega = 0$$

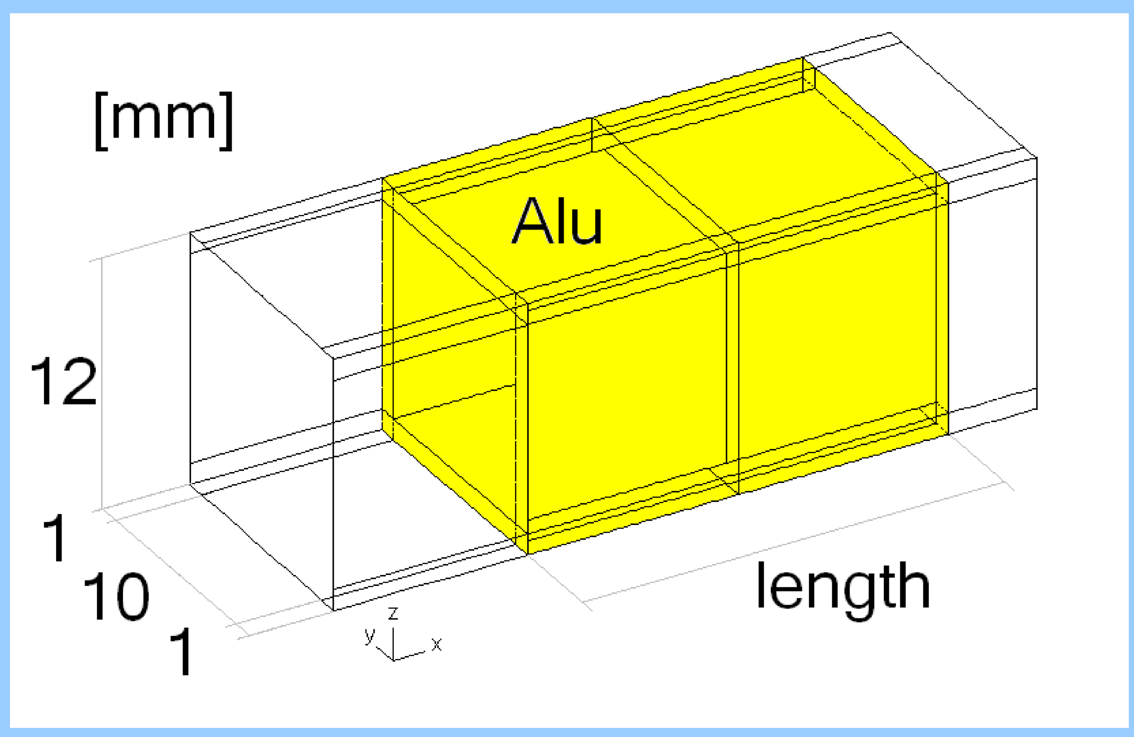
On the Neumann boundary the underbraced term is substituted by the Fourier transformed integral which prescribes the truncation of the FE-mesh directly.

Surface Impedance Boundary Conditions (SIBC)

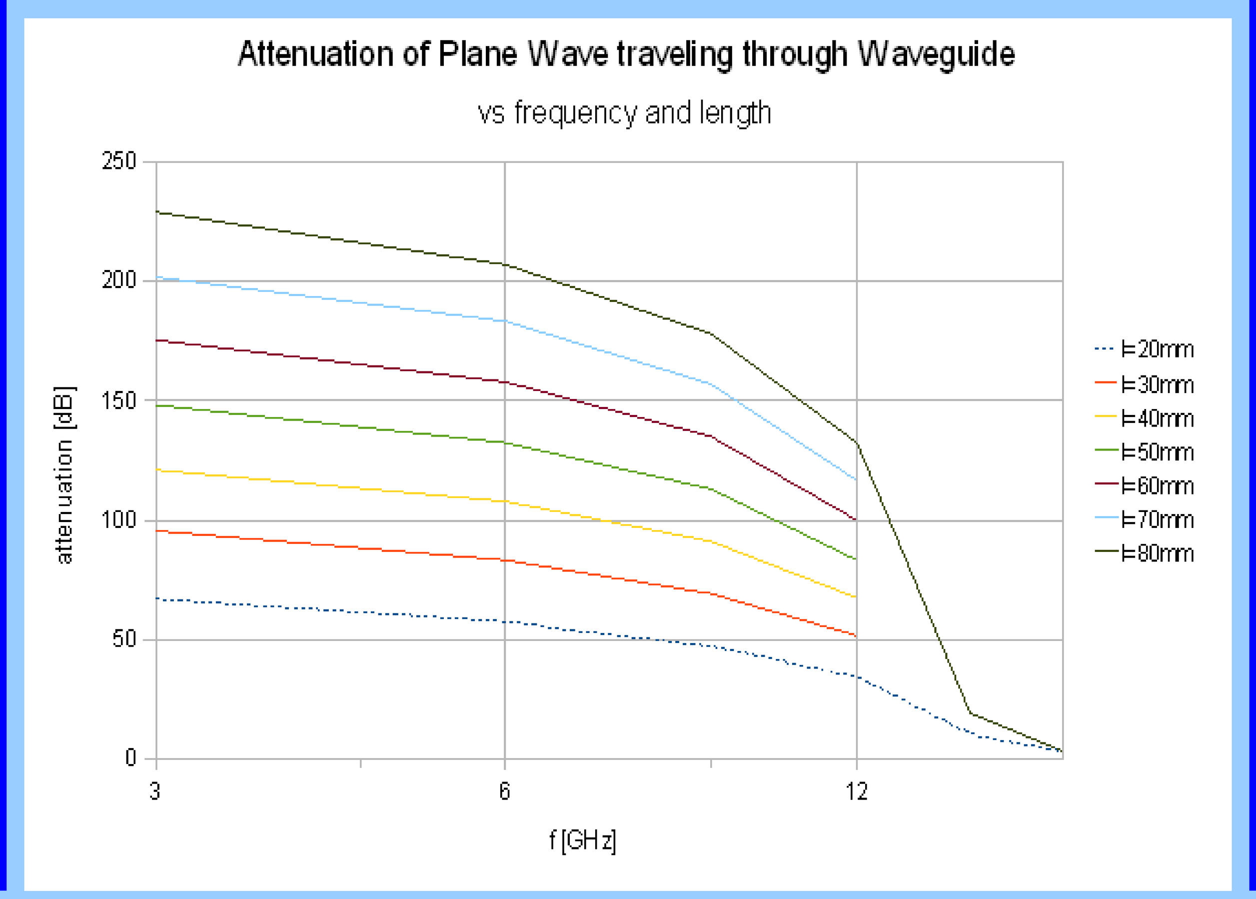
$$\vec{n} \times \vec{E_{t0}} = \frac{-\omega\mu\vec{H_{t0}}}{k} = -\sqrt{\frac{\mu}{\epsilon}}\vec{H_{t0}} = -Z_0\vec{H_{t0}}$$

$$\vec{n} \times \vec{H_{t0}} = \frac{\omega\epsilon\vec{E_{t0}}}{k} = \sqrt{\frac{\epsilon}{\mu}}\vec{E_{t0}} = \frac{1}{Z_0}\vec{E_{t0}}$$

Waveguide with rectangular cross-section penetrated by plane wave



Calculating the attenuation of a plane wave by a waveguide of constant cross-section but variable length at selected frequencies clearly shows the waveguide beyond cutoff and the non-linear behaviour of the shielding attenuation.



Engineering guidelines use rules-of-thumb formulae which give a rough estimate up to some GHz.

cutoff frequency

$$f_c = \frac{150}{b} \qquad [GHz], [mm]$$

attenuation

$$a = \frac{27.3}{b} \cdot l \qquad [dB], [mm]$$

ratio

$$l \geq 3b$$

usable frequency

$$f \leq \frac{f_c}{10} \qquad [GHz]$$

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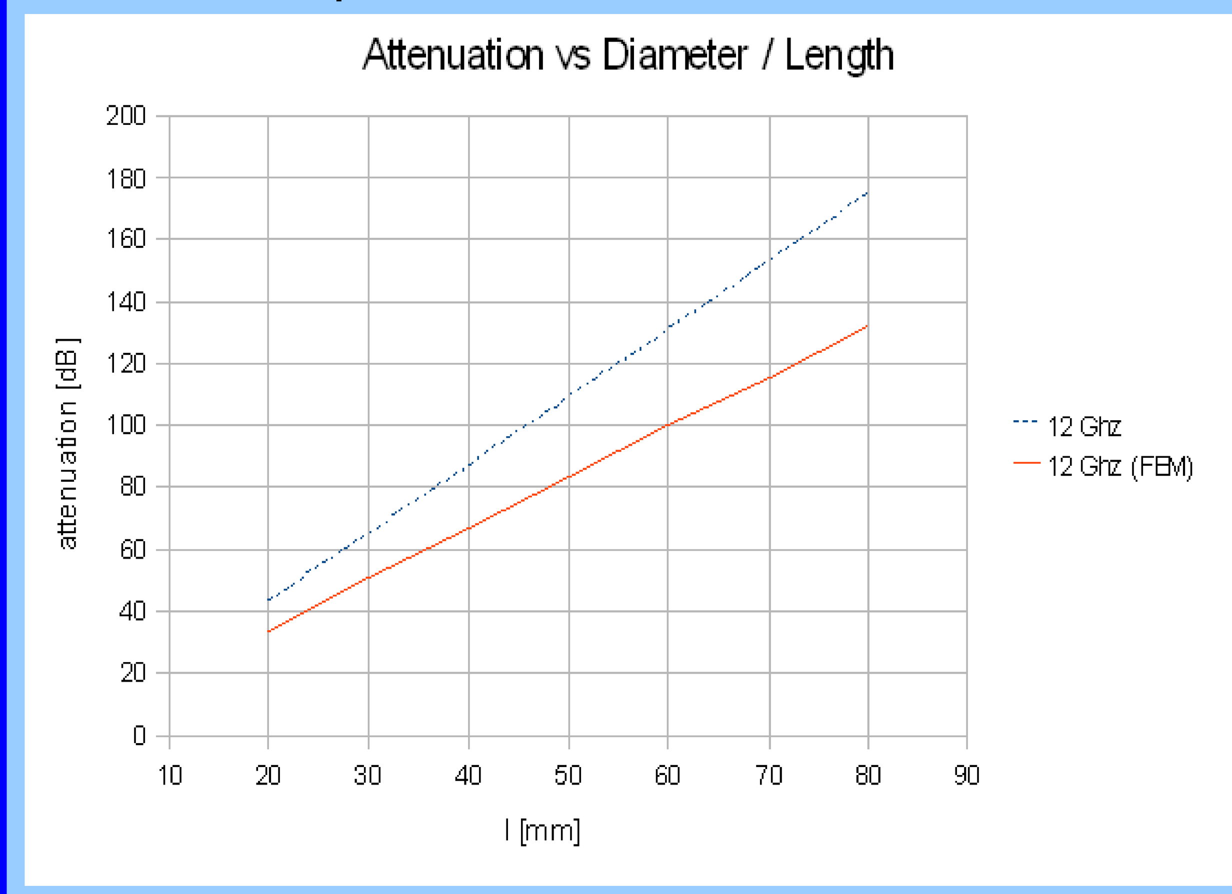
The waveguide under consideration provides a square aperture of 10mm. With this diagonal the cutoff-frequency reads ~12GHz.
Applying the formula given for the shielding attenuation provides:



Comparing these results to the FEM solution suggests that linear extrapolation is valid in a limited frquency range only. Requiring the length of the waveguide being at least three times its diameter provides trustworthy results for frquencies much lower than the cutoff frequency.
Since supposing that plane waves launch fields according to a TE_{10} mode pattern, the cutoff frequency can be determined precisely for waveguides with square apertures according to:

$$f_g = \frac{1}{2\sqrt{\epsilon\mu}}\frac{1}{a}$$

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which gives a cutoff frequency of 14.99GHz for the geometry given – matching numerical results.

Conclusion – Based on the results given here, it can be shown that conventional engineering rules are reliably applicable below 1 GHz. At frequencies above approximately 1GHz careful consideration of the waveguide-beyond-cutoff condition is required.
The behaviour of such a structure under different angles of incidence of the plane wave on the honeycomb is due to be investigated in depth.