Chapter 1

Fluid-Structure Interaction by a BEM/FEM Interface Mass Matrix Approach

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Abstract. *A pure interface coupling formulation is developed for time domain analysis of 2D coupled fluid-structure systems. Finite elements are applied in standard way to model the structure as an elastic continuum, while boundary elements receive a special treatment to model the fluid region as an acoustic media. In this way, the interface variables at the fluid regions are determined by applying unit impulses at the boundary using the concept of Duhamel integrals, which are numerically approximated by means of the Convolution Quadrature Method. The proposed approach, greatly simplifies the assembly of sub-regions and the coupling to the standard finite elements. The stability and accuracy of the proposed method are verified on some selected numerical examples.*
Keywords. *Fluid-structure, boundary elements, Duhamel integral, convolution quadrature.*

1.1 Introduction

In many engineering applications, such as fluid-structure interaction analysis, the influences of both media on each other must not be neglected. Moreover, the correct representation of infinite or semi-infinite extensions is of great importance, which is usually the case of the fluid part. The Boundary Element Method (BEM) is well-suited for this task since it implicitly fulfils the radiation condition [8]. Therefore, the fluid is modelled by boundary elements, while the finite elements are used to represent the adjacent structure (with a possibly non-linear formulation).

In order to perform the coupling, the interface variables are evaluated using a direct and strong procedure. The boundary element region is modelled extending the idea of a substructure technique for elastodynamics (called Duhamel-BEM) [13] to acoustic fluids. This technique is based on the generalization of Duhamel integrals and their numerical approximation by means of the Convolution Quadrature Method (CQM), and its extension for scalar wave propagation problems. The CQM [11] is a technique which approximates convolution integrals, in this case the Duhamel integrals, by a quadrature rule whose weights are determined by Laplace transformed fundamental solutions and a multi-step method. Therefore, Laplace domain collocation BEM is used to represent the fluid.

Most of the works concerning the BEM applied to fluid-structure interaction problems are either in frequency domain or in time domain. A detailed review on different numerical models for fluid-structure interaction can be found, e.g., in [16], which shows that instability problems represent one of the major drawbacks of the pure time domain BEM formulations. To overcome these problems, a stable time domain formulation is introduced by [10] based on the linear θ method. However, a transformed domain formulation (which combines the advantages of both Laplace and time domain formulations) is used in the proposed work, which results in an interface mass matrix for the BEM allowing an easy coupling to standard finite elements.

Finally, to investigate the accuracy and the stability of the proposed coupling scheme, two selected problems are solved and the results are compared to results from the literature.

1.2 Structure Modeling by Finite Elements

The structure behaves as an elastic media, which is modeled by the dynamic equilibrium equation for undamped systems, arising from the finite element discretization [5, 4], i.e.

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f} \tag{1.1}$$

where $\mathbf{M}$ and $\mathbf{K}$ are, respectively, mass and stiffness matrices, $\ddot{\mathbf{u}}$ and $\mathbf{u}$ are, respectively, the acceleration vector and the displacement vector, and $\mathbf{f}$ is the nodal force vector.

Using the Newmark time stepping algorithm [14], one obtains the following approximations

$$\dot{\mathbf{u}}(n\Delta t) = \dot{\mathbf{u}}((n-1)\Delta t) + [(1-\gamma)\,\ddot{\mathbf{u}}((n-1)\Delta t) + \gamma\ddot{\mathbf{u}}(n\Delta t)]\,\Delta t \tag{1.2}$$

$$\mathbf{u}(n\Delta t) = \mathbf{u}((n-1)\Delta t) + \dot{\mathbf{u}}((n-1)\Delta t)\,\Delta t + [(\frac{1}{2}-\beta)\,\ddot{\mathbf{u}}((n-1)\Delta t) + \beta\,\ddot{\mathbf{u}}(n\Delta t)]\,\Delta t^2 \tag{1.3}$$

with time discretized into N time steps of equal duration Δt.

Substitution of equation (1.3), with $\beta = 1/4$ and $\gamma = 1/2$ (constant average acceleration), into equation (1.1), results in the following equation

$$\tilde{\mathbf{M}}\,\ddot{\mathbf{u}}(n\Delta t) = \tilde{\mathbf{f}}(n\Delta t) \tag{1.4}$$

with

$$\tilde{\mathbf{M}} = \mathbf{M} + \frac{\Delta t^2}{4}\mathbf{K} \tag{1.5}$$

and

$$\tilde{\mathbf{f}}(n\Delta t) = \mathbf{f}(n\Delta t) - \mathbf{K}\left\{\mathbf{u}((n-1)\Delta t) + \Delta t\,\dot{\mathbf{u}}((n-1)\Delta t) + \frac{\Delta t^2}{4}\ddot{\mathbf{u}}((n-1)\Delta t)\right\} \tag{1.6}$$

Therefore, a standard finite element formulation for elastodynamics may be used to represent the structure regions. However, the relationship between the structure and the fluid quantities (such as the force $\mathbf{f}$ and the fluid pressure p, respectively) will be only stated afterwards.

1.3 Fluid Modeling by Boundary Elements

Small amplitude acoustic waves propagate through an ideal homogeneous fluid of density ρ_f and speed of sound c, according to the linear wave equation

$$\nabla^2 p = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} \tag{1.7}$$

where ∇^2 is the Laplacian operator and p is the acoustic pressure in the fluid at time t. Equation (1.7) must be accompanied by appropriate boundary and initial conditions.

The time-dependent differential equation (1.7) can be transformed to a time-independent one by means of the Laplace transform. The Laplace-transform $\hat{g}(s)$ of a function $g = g(t)$ is defined by

$$\hat{g}(s) = \mathscr{L}\{g(t)\} = \int_0^\infty g(t) e^{-st} dt \tag{1.8}$$

where $s \in \mathbb{C}$ is the complex Laplace transform parameter, and hats ($\hat{}$) denote Laplace transformed quantities.

Application of the Laplace transform to equation (1.7) under the assumption of zero initial conditions, i.e., $p(t=0) = 0$ and $\dot{p}(t=0) = 0$, yields

$$\nabla^2 \hat{p} = \left(\frac{s}{c}\right)^2 \hat{p} \tag{1.9}$$

which is the so-called Helmholtz equation.

Applying the Betti's reciprocal theorem to the scalar wave equation, the Somigliana's identity for a boundary element region in Laplace domain [3], relating the pressure at a point $\boldsymbol{\xi}$ in a homogeneous domain Ω with the pressures $\hat{p}(s, \mathbf{x})$ and fluxes $\hat{q}(s, \mathbf{x}) = \partial \hat{p}(s, \mathbf{x})/\partial n$ at the boundary $\Gamma = \partial\Omega$, is given by

$$\hat{p}(s, \boldsymbol{\xi}) = \int_\Gamma \hat{P}(s, \mathbf{x}, \boldsymbol{\xi})\, \hat{q}(s, \mathbf{x})\, d\Gamma(\mathbf{x}) - \int_\Gamma \hat{Q}(s, \mathbf{x}, \boldsymbol{\xi})\, \hat{p}(s, \mathbf{x})\, d\Gamma(\mathbf{x}) \tag{1.10}$$

with $\mathbf{x} \in \Gamma$. $\hat{P}(s, \mathbf{x}, \boldsymbol{\xi})$ and $\hat{Q}(s, \mathbf{x}, \boldsymbol{\xi})$ are the pressure and flux fundamental solutions of the Laplace domain wave equation. They are given, e.g., in [3, 8].

In order to solve equation (1.10), the boundary of Ω has to be discretized into elements. Introducing isoparametric boundary elements into Equation (1.10), and by collocating this equation at each boundary node, a system of equation

$$\hat{\mathbf{Q}}\, \hat{\mathbf{p}} = \hat{\mathbf{P}}\, \hat{\mathbf{q}} \tag{1.11}$$

is obtained. $\hat{\mathbf{Q}}$ and $\hat{\mathbf{P}}$ are the coefficient matrices assembled from the element contributions, as explained in detail, e.g., in [6] and $\hat{\mathbf{p}}$ and $\hat{\mathbf{q}}$ are the pressures and fluxes at all nodes.

Therefore, the fluid regions are represented by a Laplace domain boundary element formulation for wave propagation in acoustic media, where the fluid quantities are transformed back to the time domain by means of a special transformation technique, as explained subsequently.

1.4 Coupling Procedure

Consider now a linear isotropic body represented by a domain Ω, consisting of two homogeneous regions (sub-domains), such as in Figure 1.1(a), with a fluid sub-domain Ω^f coupled to a structure sub-domain Ω^s.

In our notation, the first of the two subscripts denote the location of a quantity, whereas the second one denotes its cause. The letters D, N and I refer, respectively, to the quantities at the Dirichlet boundary, Neumann boundary and coupled interface. When necessary, superscripts f and s denote quantities belonging, respectively, to the fluid and structure sub-domains. Also, overbars are used to denote prescribed quantities.

Assuming linear behaviour, enabling the superposition of effects, the time-dependent flux component at the interface of the fluid region $\Gamma_{\mathtt{I}}^f$ can be split up into three parts, i.e.,

$$\mathbf{q}_{\mathtt{I}}(t) = \mathbf{q}_{\mathtt{II}}(t) + \mathbf{q}_{\mathtt{IN}}(t) + \mathbf{q}_{\mathtt{ID}}(t) \tag{1.12}$$

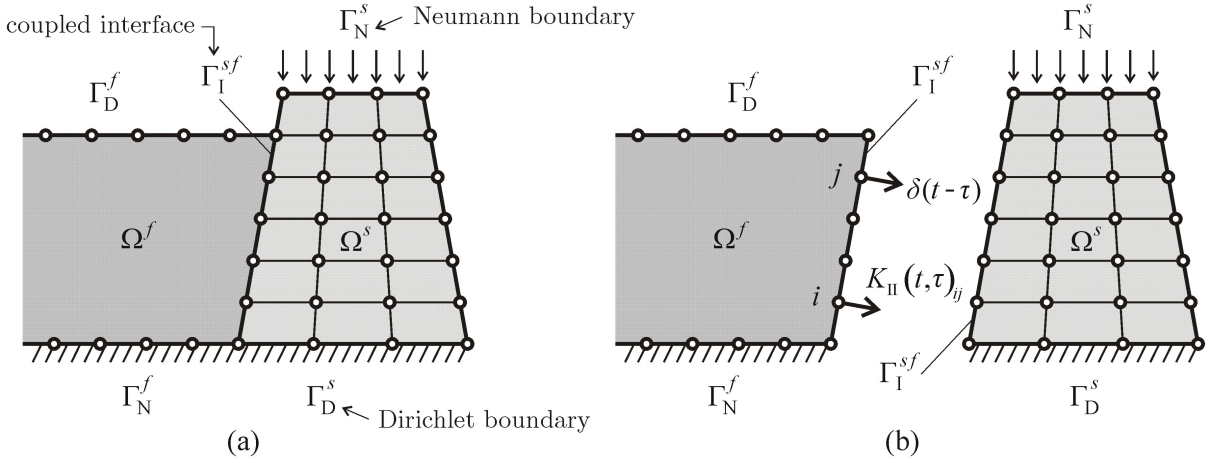

Figure 1.1: Coupled fluid-structure system.

where $\mathbf{q}_{\mathtt{II}}(t)$ is the flux vector at the interface caused by the time-dependent interface pressures $\mathbf{p}_{\mathtt{I}}(t)$; $\mathbf{q}_{\mathtt{IN}}(t)$ is the flux vector at the interface due to the transient flux $\bar{\mathbf{q}}_{\mathtt{N}}(t)$ on $\Gamma_{\mathtt{N}}^f$; and $\mathbf{q}_{\mathtt{ID}}(t)$ is the flux vector at the interface due to the transient pressure $\bar{\mathbf{p}}_{\mathtt{D}}(t)$ on $\Gamma_{\mathtt{D}}^f$.

Since the superposition law holds, $\mathbf{q}_{\mathtt{II}}(t)$ can be represented by a summation of convolution integrals, using the concept of the Duhamel integral equation [7]. Consequently, each term of equation (1.12) man be expressed in the respective form

$$\mathbf{q}_{\mathtt{I}}(t) = \int_0^t \mathbf{K}_{\mathtt{II}}(t,\tau)\,\mathbf{p}_{\mathtt{I}}(\tau)\,d\tau + \int_0^t \mathbf{K}_{\mathtt{IN}}(t,\tau)\,\bar{\mathbf{q}}_{\mathtt{N}}(\tau)\,d\tau + \int_0^t \mathbf{K}_{\mathtt{ID}}(t,\tau)\,\bar{\mathbf{p}}_{\mathtt{D}}(\tau)\,d\tau \tag{1.13}$$

In equation (1.13), the matrix $\mathbf{K}_{\mathtt{II}}(t,\tau)$ is a unit pressure impulse response matrix, where its coefficients $K_{\mathtt{II}}(t,\tau)_{ij}$ may be interpreted as the time dependent flux at the interface node i due to an impulsive unit pressure $\delta(t-\tau)$ applied at time τ at the interface node j, with zero pressure at all other interface nodes and zero boundary conditions at $\Gamma_{\mathtt{N}}^f$ and $\Gamma_{\mathtt{D}}^f$ in the same boundary element region Ω^f, as illutrated in Figure 1.1(b). Here, $\delta(t-\tau)$ denotes the Dirac delta function and $i, j = 1, ..., N_{DoF}$.

In analogous manner, the matrix $\mathbf{K}_{\mathtt{IN}}(t,\tau)$ of equation (1.13) represents the time dependent fluxes at the interface due to the impulsive unit flux $\delta(t-\tau)$ applied at time τ along the element degree of freedoms at $\Gamma_{\mathtt{N}}^f$ of region Ω^f, with zero pressure along the interface; and the matrix $\mathbf{K}_{\mathtt{ID}}(t,\tau)$ represents the time dependent fluxes at the interface due to the impulsive unit pressure $\delta(t-\tau)$ applied at time τ along the nodes at $\Gamma_{\mathtt{D}}^f$ of region Ω^f, with zero pressure along the interface. Therefore, the integrals in equation (1.13) can be understood as a generalization of the concept of Duhamel integrals [7].

There is no closed form solution for $\mathbf{K}_{\mathtt{II}}(t,\tau)$, $\mathbf{K}_{\mathtt{IN}}(t,\tau)$ and $\mathbf{K}_{\mathtt{ID}}(t,\tau)$ available in the time domain. However, they can be numerically computed in the Laplace domain without any numerical difficulties. For this reason, the fluid is modeled in Laplace domain, section 3. On the other hand, one can be benefited by all advantages of the Laplace domain boundary element formulation.

The Laplace transform of a Dirac impulse is given by $\mathscr{L}\{\delta(t-\tau 0)\} = e^{-s\tau}$. However, for wave propagation problems, the translation principle holds which states that the response of an acoustic fluid due to an impulse does not depend on absolute time, but it is rather a function of the time difference between the instant in which the impulse is applied and the instant in which the response is observed, see equation (1.14). Without loss of generality one can therefore apply the impulses at absolute time $\tau = 0$, yielding $\mathscr{L}\{\delta(t-0)\} = 1$. Thus, the Laplace transform of

$\mathbf{K}_{\mathtt{II}}(t,\tau)$, $\mathbf{K}_{\mathtt{IN}}(t,\tau)$ and $\mathbf{K}_{\mathtt{ID}}(t,\tau)$, i.e.,

$$\begin{aligned}
\mathbf{K}_{\mathtt{II}}(t,\tau) = \mathbf{K}_{\mathtt{II}}(t+\Delta t,\tau+\Delta t) = \mathbf{K}_{\mathtt{II}}(t-\tau) \quad &\Rightarrow \quad \mathscr{L}\{\mathbf{K}_{\mathtt{II}}(t-\tau)\} = \hat{\mathbf{K}}_{\mathtt{II}}(s) \\
\mathbf{K}_{\mathtt{IN}}(t,\tau) = \mathbf{K}_{\mathtt{IN}}(t+\Delta t,\tau+\Delta t) = \mathbf{K}_{\mathtt{IN}}(t-\tau) \quad &\Rightarrow \quad \mathscr{L}\{\mathbf{K}_{\mathtt{IN}}(t-\tau)\} = \hat{\mathbf{K}}_{\mathtt{IN}}(s) \\
\mathbf{K}_{\mathtt{ID}}(t,\tau) = \mathbf{K}_{\mathtt{ID}}(t+\Delta t,\tau+\Delta t) = \mathbf{K}_{\mathtt{ID}}(t-\tau) \quad &\Rightarrow \quad \mathscr{L}\{\mathbf{K}_{\mathtt{ID}}(t-\tau)\} = \hat{\mathbf{K}}_{\mathtt{ID}}(s)
\end{aligned} \tag{1.14}$$

may be obtained by simply applying unit pressure and unit flux boundary conditions in the Laplace domain. From a more mathematical point of view, the new technique can be seen as a special transformed domain formulation or indirect time-domain boundary integral method [1].

To allow for a numerical evaluation of these Duhamel or convolution integrals, occurring in equation (1.13), the Convolution Quadrature Method (CQM) [11] is adopted. The convolution integrals are approximated by a quadrature rule (similar to ordinary Gauss quadrature), whose weights can be determined exclusively by the Laplace transformed function and a linear multi-step method.

The time t is divided into n equal time steps Δt, so that $t = n\Delta t$. The CQM approximates convolution integrals, as those integrals presented in equation (1.13), as follows

$$\mathbf{v}(n\Delta t) = \int_{\tau=0}^{t} \mathbf{h}(t-\tau)\mathbf{g}(\tau)d\tau \approx \sum_{k=0} \boldsymbol{\omega}_{n-k}(\hat{\mathbf{h}}(s),\Delta t)\mathbf{g}(k\Delta t) \tag{1.15}$$

where $\boldsymbol{\omega}_{n-k}$ are the convolution weights, and $\hat{\mathbf{h}}(s)$ are the Laplace transforms of the function $\mathbf{h}(s)$. To compute the convolution weights, it is sufficient to know $\hat{\mathbf{h}}(s)$ at least approximately for a set of discrete Laplace parameters.

In the case of the proposed methodology, the CQM requires only the knowledge of $\hat{\mathbf{K}}_{\mathtt{II}}(s)$, $\hat{\mathbf{K}}_{\mathtt{IN}}(s)$ and $\hat{\mathbf{K}}_{\mathtt{ID}}(s)$, equation (1.14), for a discrete set of Laplace parameters. Thus, the convolution integrals of equation (1.13) take the following discretized form ($t = n\Delta t$ and $\tau = k\Delta t$), with their respectives integration weight matrices,

$$\begin{aligned}
\int_0^t \mathbf{K}_{\mathtt{II}}(t-\tau)\mathbf{p}_{\mathtt{I}}(\tau)d\tau &\approx \sum_{k=0}^{n} \left\{\boldsymbol{\omega}_{n-k}^{\mathbf{K}_{\mathtt{II}}}\mathbf{p}_{\mathtt{I}}(k\Delta t)\right\}, \qquad \boldsymbol{\omega}_n^{\mathbf{K}_{\mathtt{II}}} = \frac{\mathscr{R}^{-n}}{L}\sum_{l=0}^{L-1}\hat{\mathbf{K}}_{\mathtt{II}}\left(\frac{\gamma}{\Delta t}\right)e^{-inl\frac{2\pi}{L}} \\
\int_0^t \mathbf{K}_{\mathtt{IN}}(t-\tau)\bar{\mathbf{q}}_{\mathtt{N}}(\tau)d\tau &\approx \sum_{k=0}^{n} \left\{\boldsymbol{\omega}_{n-k}^{\mathbf{K}_{\mathtt{IN}}}\bar{\mathbf{q}}_{\mathtt{N}}(k\Delta t)\right\}, \qquad \boldsymbol{\omega}_n^{\mathbf{K}_{\mathtt{IN}}} = \frac{\mathscr{R}^{-n}}{L}\sum_{l=0}^{L-1}\hat{\mathbf{K}}_{\mathtt{IN}}\left(\frac{\gamma}{\Delta t}\right)e^{-inl\frac{2\pi}{L}} \\
\int_0^t \mathbf{K}_{\mathtt{ID}}(t-\tau)\bar{\mathbf{p}}_{\mathtt{D}}(\tau)d\tau &\approx \sum_{k=0}^{n} \left\{\boldsymbol{\omega}_{n-k}^{\mathbf{K}_{\mathtt{ID}}}\bar{\mathbf{p}}_{\mathtt{D}}(k\Delta t)\right\}, \qquad \boldsymbol{\omega}_n^{\mathbf{K}_{\mathtt{ID}}} = \frac{\mathscr{R}^{-n}}{L}\sum_{l=0}^{L-1}\hat{\mathbf{K}}_{\mathtt{ID}}\left(\frac{\gamma}{\Delta t}\right)e^{-inl\frac{2\pi}{L}}
\end{aligned} \tag{1.16}$$

which are the quotients of characteristic polynomials of the underlying multi-step method. More details on the CQM can be found in [11, 15].

Therefore, the convolution weights are computed using equation (1.16) combined with the boundary element formulation shown in section 3, by applying unit impulses on the boundary and obtaining the responses for the interface fluxes due to each unit impulse. This methodology is explained in details in [6].

Substitution of equation (1.16) into equation (1.13) and extracting $\mathbf{p}_{\mathtt{I}}(n\Delta t)$ yields for each fluid region

$$\mathbf{q}_{\mathtt{I}}(n\Delta t) \approx \boldsymbol{\omega}_0^{\mathbf{K}_{\mathtt{II}}}\mathbf{p}_{\mathtt{I}}(n\Delta t) + \bar{\mathbf{q}}_{\mathtt{I}}(n\Delta t) \tag{1.17}$$

with

$$\bar{\mathbf{q}}_{\mathtt{I}}(n\Delta t) = \sum_{k=0}^{n-1}\boldsymbol{\omega}_{n-k}^{\mathbf{K}_{\mathtt{II}}}\mathbf{p}_{\mathtt{I}}(k\Delta t) + \sum_{k=0}^{n}\boldsymbol{\omega}_{n-k}^{\mathbf{K}_{\mathtt{IN}}}\bar{\mathbf{q}}_{\mathtt{N}}(k\Delta t) + \sum_{k=0}^{n}\boldsymbol{\omega}_{n-k}^{\mathbf{K}_{\mathtt{ID}}}\bar{\mathbf{p}}_{\mathtt{D}}(k\Delta t) \tag{1.18}$$

which contains only known values.

Now, splitting the nodal accelerations of the structure sub-domain Ω^s into coupled accelerations $\ddot{\mathbf{u}}_{\mathtt{I}}$ and uncoupled ones $\ddot{\mathbf{u}}_{\mathtt{F}}$, the equation (1.4) is rewritten as

$$\begin{bmatrix} \tilde{\mathbf{M}}_{\mathtt{II}} & \tilde{\mathbf{M}}_{\mathtt{IF}} \\ \tilde{\mathbf{M}}_{\mathtt{FI}} & \tilde{\mathbf{M}}_{\mathtt{FF}} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}}^s_{\mathtt{I}}(n\Delta t) \\ \ddot{\mathbf{u}}_{\mathtt{F}}(n\Delta t) \end{Bmatrix} - \begin{Bmatrix} \tilde{\mathbf{f}}^s_{\mathtt{I}}(n\Delta t) \\ \tilde{\mathbf{f}}_{\mathtt{F}}(n\Delta t) \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}^s_{\mathtt{I}}(n\Delta t) \\ \mathbf{0} \end{Bmatrix} \tag{1.19}$$

where subscript $\mathtt{F}$ denotes quantities at the finite element uncoupled boundary ($\Gamma_{\mathtt{F}} = \Gamma^s_{\mathtt{N}} \cup \Gamma^s_{\mathtt{D}}$). $\mathbf{f}^s_{\mathtt{I}}(n\Delta t)$ is the coupling nodal force vector, containing the interaction forces on $\Gamma^s_{\mathtt{I}}$. As in general the tangential forces exerted by the fluid are negligible the vector $\mathbf{f}^s_{\mathtt{I}}$ can be given in terms of pressure p along $\Gamma_{\mathtt{I}}$ as [17]

$$\mathbf{f}^s_{\mathtt{I}} = \int_{\Gamma_{\mathtt{I}}} \bar{\mathbf{N}}^{\mathtt{T}}\, \mathbf{n}\, p\, d\Gamma \tag{1.20}$$

where $\mathbf{n}$ is a unit normal vector to the interface, and $\bar{\mathbf{N}}$ represents the same shape functions which interpolate the nodal displacements.

Therefore, equation (1.20) can be written as

$$\mathbf{f}^s_{\mathtt{I}} = \int_{\Gamma_{\mathtt{I}}} \bar{\mathbf{N}}^{\mathtt{T}}\, \mathbf{n}\, \mathbf{N}\, \mathbf{p}^s_{\mathtt{I}}\, d\Gamma = \int_{\Gamma_{\mathtt{I}}} \bar{\mathbf{N}}^{\mathtt{T}}\, \mathbf{n}\, \mathbf{N}\, \mathbf{p}^f_{\mathtt{I}}\, d\Gamma = \mathbf{T}\, \mathbf{p}_{\mathtt{I}} \tag{1.21}$$

In analogous manner, one can write for the coupling nodal flux vector $\mathbf{q}_{\mathtt{I}}$ the following expression [17]

$$\mathbf{q}^f_{\mathtt{I}} = \int_{\Gamma_{\mathtt{I}}} \mathbf{N}^{\mathtt{T}} \rho\, \mathbf{n}^{\mathtt{T}}\, \bar{\mathbf{N}}\, \ddot{\mathbf{u}}^f_{\mathtt{I}}\, d\Gamma = \int_{\Gamma_{\mathtt{I}}} \mathbf{N}^{\mathtt{T}} \rho\, \mathbf{n}^{\mathtt{T}}\, \bar{\mathbf{N}}\, \ddot{\mathbf{u}}^s_{\mathtt{I}}\, d\Gamma = \rho\, \mathbf{T}^{\mathtt{T}}\, \ddot{\mathbf{u}}_{\mathtt{I}} \tag{1.22}$$

From now on it is possible to couple the fluid (boundary elements) to the structure (finite elements), just transforming equation (1.17) by means of equations (1.21) and (1.22). In this way, the interaction force vector $\mathbf{f}^f_{\mathtt{I}}$, containing all interaction forces generated by the boundary element region coupled to a finite element region along the interface can be expressed in terms of the interface accelerations, i.e.,

$$\tilde{\mathbf{M}}^f_{\mathtt{II}}\, \ddot{\mathbf{u}}^f_{\mathtt{I}}(n\Delta t) = \mathbf{f}^f_{\mathtt{I}}(n\Delta t) + \tilde{\mathbf{f}}^f_{\mathtt{I}}(n\Delta t) \tag{1.23}$$

where

$$\tilde{\mathbf{M}}^f_{\mathtt{II}} = \mathbf{T}\, (\boldsymbol{\omega}_0^{\mathbf{K}_{\mathtt{II}}})^{-1} \rho\, \mathbf{T}^{\mathtt{T}} \quad \texttt{and} \quad \tilde{\mathbf{f}}^f_{\mathtt{I}}(n\Delta t) = \mathbf{T}\, (\boldsymbol{\omega}_0^{\mathbf{K}_{\mathtt{II}}})^{-1}\, \bar{\mathbf{q}}_{\mathtt{I}}(n\Delta t) \tag{1.24}$$

Assuming bonded contact between coupled regions and invoking equilibrium and compatibility, the necessary coupling conditions at the interface should be considered

$$\mathbf{f}^s_{\mathtt{I}}(n\Delta t) + \mathbf{f}^f_{\mathtt{I}}(n\Delta t) = \mathbf{0} \qquad \texttt{(equilibrium)} \tag{1.25}$$

$$\ddot{\mathbf{u}}^s_{\mathtt{I}}(n\Delta t) = \ddot{\mathbf{u}}^f_{\mathtt{I}}(n\Delta t) = \ddot{\mathbf{u}}^f_{\mathtt{I}}(n\Delta t) \qquad \texttt{(compatibility)} \tag{1.26}$$

Finally, introducing equations (1.19) and (1.23) into equations (1.25) and (1.26), one obtains the following system of equations

$$\begin{bmatrix} \tilde{\mathbf{M}}^s_{\mathtt{II}} + \tilde{\mathbf{M}}^f_{\mathtt{II}} & \tilde{\mathbf{M}}_{\mathtt{IF}} \\ \tilde{\mathbf{M}}_{\mathtt{FI}} & \tilde{\mathbf{M}}_{\mathtt{FF}} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}}_{\mathtt{I}}(n\Delta t) \\ \ddot{\mathbf{u}}_{\mathtt{F}}(n\Delta t) \end{Bmatrix} = \begin{Bmatrix} \tilde{\mathbf{f}}^s_{\mathtt{I}}(n\Delta t) + \tilde{\mathbf{f}}^f_{\mathtt{I}}(n\Delta t) \\ \tilde{\mathbf{f}}_{\mathtt{F}}(n\Delta t) \end{Bmatrix} \tag{1.27}$$

Equation (1.27) must be solved for each time step but this involves a re-solution with different right hand side only. It is important to note, that the system becomes non-symmetric due to the characteristics of the employed BEM formulation. In principle, the coupling matrices $\tilde{\mathbf{M}}^f_{\mathtt{II}}$ could be symmetrized in order to save storage space. However, it was shown by [12] that this could lead to an unacceptable loss of accuracy. It might prove to be the better strategy to employ fast iterative solvers that do not need symmetric matrices (see [2]). The coupling strategy presented here may be easily implemented by means of very simple adaptations of an existing finite element code and this is the main advantage of the approach proposed in this paper.

1.5 Numerical Results

In order to investigate the accuracy and the stability of the proposed technique, two problems are analyzed and the obtained results are compared to the ones from the literature.

One dimensional fluid-structure rod. A benchmark example frequently used to validate fluid-structure interaction formulations is the wave propagation in a rod with fluid container, as shown in Figure 1.2. The purpose of this example is to investigate the results obtained by the proposed formulation against the corresponding analytical solution of the problem.

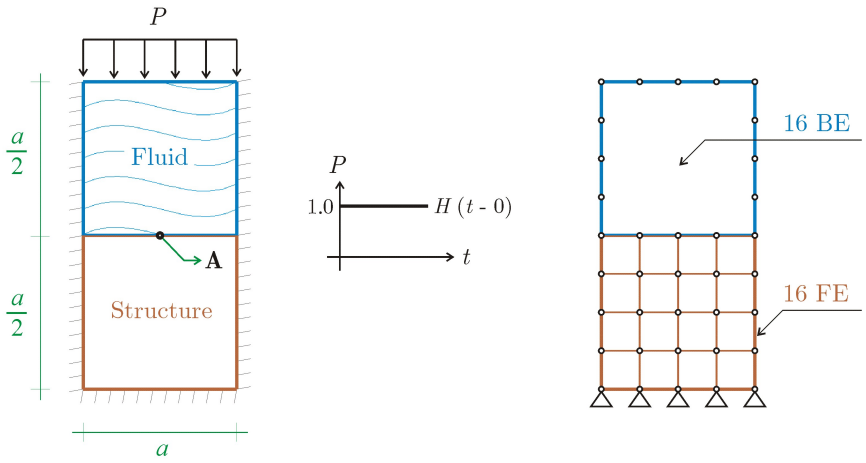

Figure 1.2: Step function excitation of a fluid-structure rod with a BEM/FEM discretization.

The rod is fixed at the bottom and the fluid surface is subjected to a Heaviside compression load. The material properties of the elastic structure are given by: Young's modulus E; Poisson's ratio $\nu = 0$; mass density ρ^s; and compression wave speed of the elastic domain $c_p = \sqrt{E/\rho^s}$. The mass density of the fluid is taken the same that of the structure $\rho^f = \rho^s$, as well as for the wave speed of the fluid $c = c_p$. The boundary conditions and the discretization used to model the fluid-structure system are illustrated in Figure 1.2. The strucuture is discretized with 16 finite elements, and the fluid with 16 boundary elements with the same length L.

The vertical displacements and the fluid-structure pressure at point **A**, midpoint of the coupled interface, are plotted in Figure 1.3 and Figure 1.4, respectively. The time step size can be estimated by $\beta = c\Delta t/L$, with β restricted to a very small range where stable and satisfactory results are achieved ($0.2 < \beta < 0.6$). The results for the rod are obtained with $\beta = 0.4$.

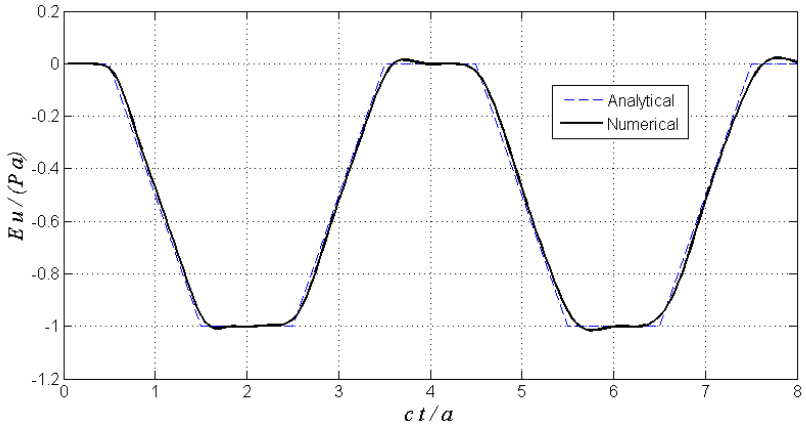

Figure 1.3: Vertical displacement at point **A** in the fluid-structure rod.

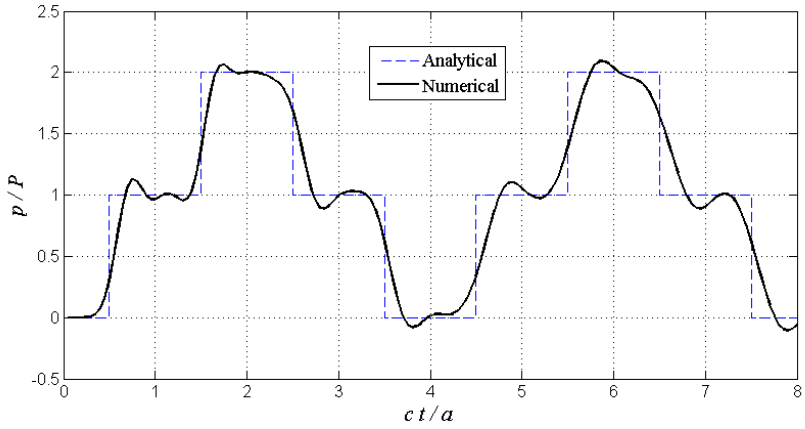

Figure 1.4: Fluid-structure pressure at point **A** in the fluid-structure rod.

It can be observed, which the results are stable and in good agreement with the correspondent analytical solution. The same example is analyzed in [16], which introduces a stable pure time domain formulation for fluid-structure interaction problems. In the same work, the authors show that for the standard BEM/FEM coupling procedure a immediate instability appears for the fluid-structure pressure at point **A**. Note that in the proposed formulation, this instability problem does not occur.

Dam-water interaction. In order to demonstrate the relevance and applicability of the proposed formulation to realistic problems, a dam-reservoir system is simulated dynamically, as shown in Figure 1.5. An identical problem is analyzed by [9]. Therefore, the results obtained in [9] may be used here as reference solutions to the proposed formulation.

The considered linear elastic dam is discretized with 54 isoparametric linear finite elements while the adjacent semi-infinite water reservoir is modeled with 40 linear boundary elements. The material properties of the dam are: Young's modulus $E = 3.437 \times 10^6$ kN/m^2; Poisson's ratio $\nu = 0.25$; and mass density $\rho^s = 2.0$ t/m^3. The adjacent water is characterized by its pressure wave velocity $c = 1436$ m/s and density of $\rho^f = 1.0$ t/m^3. At the free surface of the fluid region the hydrodynamic pressure are prescribed to be zero, as well as the accelerations along the bottom of the reservoir.

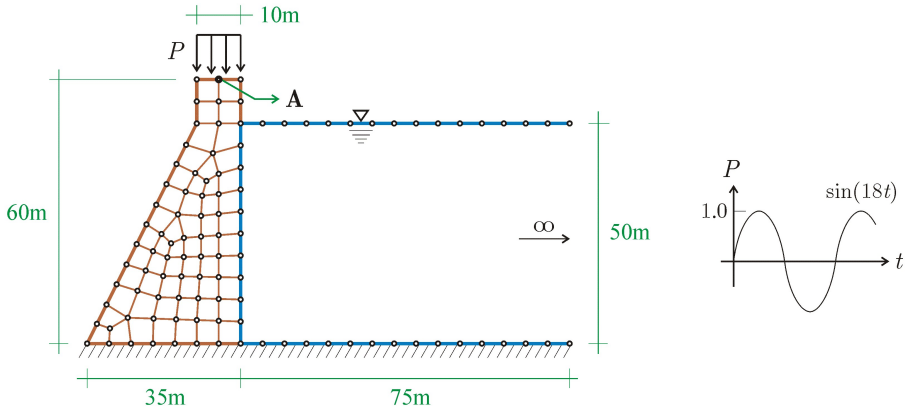

Figure 1.5: Dam-reservoir system: geometry, discretization and loading.

Figure 1.6 illustrates the transient response for the vertical displacements of point **A** at the crest of the dam due to a vertical sinusoidal load of $P = \sin(18t)$. In [9], the authors perform

a serie of numerical experiments varying the water level, to study the influence of this level on the dam. However, just the highest level (50m) was considered in the current paper. For this case, the results obtained by the proposed formulation are stable and in good agreement with the results found in the reference solution [9].

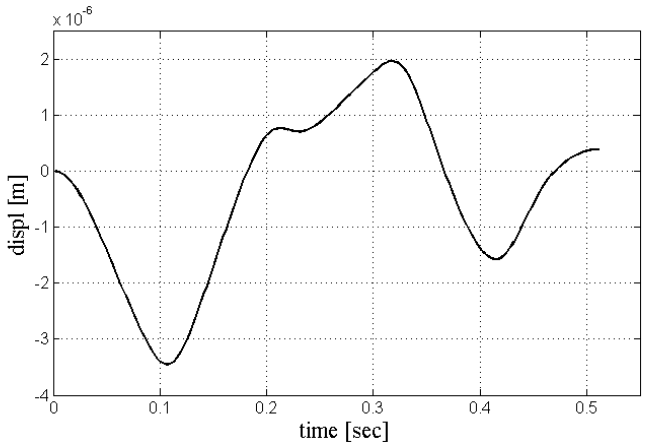

Figure 1.6: Vertical displacement at point **A** in the dam-reservoir system.

1.6 Conclusions

A novel technique for the analysis of dynamic fluid-structure interaction problems with BEM/FEM coupling in time domain has been presented in this work. It is based on simple engineering concepts, i.e., the concept of stiffness (mass) matrix that is widely used in structural engineering and the idea of Duhamel integrals. In addition, the combination of the Laplace domain BEM with a powerful mathematical tool, namely the CQM, allows the numerical modeling of the fluid regions. Whereas the structure is modeled by standard finite elements.

The main advantage of this approach is that it requires only the discretization of the boundary of the problems, since when dealing with most of the fluid-structure systems also semi-infinite media should be taken in account. Moreover, one can benefit from the advantages of the Laplace domain without worry with the time discretization, and still obtain stable and accurate results.

Satisfactory agreement is obtained and the results are very stable with respect to time step size. Therefore, the proposed formulation may be used as a powerful simulation tool for fluid-structure systems, in which the design of such systems subjected to earthquake or other non-static loads is an indispensable requirement.

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