# A NFXLMS ALGORITHM WITH INITIAL SUBSYSTEM ESTIMATES FOR DIGITAL PREDISTORTION OF WIENER SYSTEMS

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**ABSTRACT**

*Adaptive predistortion of nonlinear systems described as IIR Wiener models is discussed in this paper. The predistorter is modeled as an IIR Hammerstein system. The parameters of the linear and nonlinear blocks of the Hammerstein predistorter are estimated simultaneously using the Nonlinear Filtered-x Least Mean Squares (NFxLMS) algorithm. In the NFxLMS algorithm, the nonlinear system is assumed to be known or accurately identified. In this paper, a novel NFxLMS with Initial Subsystem Estimates (NFxLMS-ISE) algorithm is developed to avoid accurate identification of the nonlinear system. Simulation study shows that the NFxLMS-ISE algorithm can efficiently precompensate the nonlinear distortion similarly as the NFxLMS algorithm assuming that the nonlinear system is known.*

## 1. INTRODUCTION

In many areas, cancelling or reducing the nonlinear distortion due to the nonlinearity characteristic of the electronic or electromechanical devices are becoming more and more important. Examples can be found in communication systems, speech processing and control engineering, see [1-3]. Two kinds of adaptive compensation techniques can be used to reduce the nonlinear distortion, which are adaptive post-distortion, also named as adaptive equalization, and adaptive pre-distortion [1]. Although the post-distortion is an effective way to compensate the nonlinear distortion, the pre-distortion is more efficient and necessary in many other situations, such as compensation of the nonlinear distortion for the power amplifiers in satellite communication [4] and the active noise cancellation for loudspeakers [5].

Several time domain adaptive predistortion techniques based on Volterra model have recently been proposed [1, 3]. However, since these techniques are based on Volterra model, they are not practical for real-time implementation due to the computation complexity and slow convergence speed. The Wiener system [6-8] is a block-oriented structure to model nonlinear systems without requiring large number of parameters. A predistortion technique for the FIR Wiener system has been proposed in [9, 10]. The idea is to connect an adaptive predistorter modeled as an FIR Hammerstein system [7, 11, 12] tandemly with the nonlinear system. Then adaptively adjusting the parameters of the predistorter using the NFxLMS algorithm after accurately identifying the FIR Wiener system. The accurate estimates of the linear and nonlinear subsystems of the FIR Wiener system are used in [9, 10] in order to precompensate the nonlinear distortion.

The effect of inaccurate estimation of the system transfer function on the Filtered-x LMS (FxLMS) algorithm has been studied in [13, 14], the authors concluded that if the phase error in the estimate of the system transfer function does not exceed $\pm 90°$, stable convergence of the FxLMS algorithm can be guaranteed. This is known as "90° condition". However, the effect of the phase error between these bounds is very difficult to predict [14]. In [15], an inaccurate estimate of the system transfer function satisfying the 90° condition was used in the convergence analysis of the linear and nonlinear FxLMS algorithms.

In this paper, we describe the nonlinear system to be precompensated as an IIR Wiener system - hence the predistortion of FIR Wiener system can be regarded as a special case. Moreover, we introduce a new NFxLMS algorithm based on using an initial estimate for the linear and nonlinear subsystems as described in [16]. The method introduced in [16] is simple, fast and does not require high computation. The suggested algorithm in this paper in order to adaptively estimate the predistorter coefficients is denoted as the NFxLMS with Initial Subsystem Estimates (NFxLMS-ISE).

This paper is organized as follows. In Section 2, the structures of the IIR Wiener system and the IIR Hammerstein predistorter are defined. In Section 3, NFxLMS algorithm is derived to adaptively estimate the parameters of the predistorter. Based on NFxLMS algorithm, NFxLMS-ISE algorithm is discussed in Section 4. In Section 5, the validity of the proposed algorithm is demonstrated via computer simulation. Section 6 comes to conclusions.

## 2. PREDISTORTION OF IIR WIENER SYSTEMS

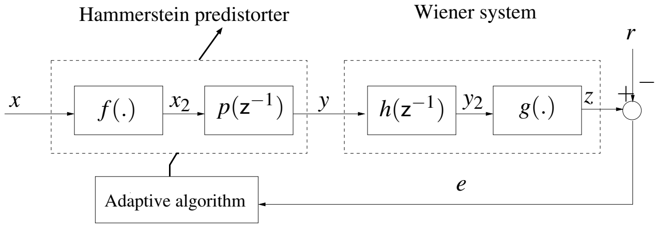


Figure 1: Predistortion of IIR Wiener system.

The IIR Wiener system to be precompensated is shown in Fig. 1. The output of this system is given by

$$\begin{aligned} z(n) &= g(y_2(n)) \\ &= g_1 y_2(n) + g_2 y_2^2(n) + \cdots + g_{m_g} y_2^{m_g}(n) \\ &= G^T Y_2(n) \end{aligned} \tag{1}$$

where

$$G = \begin{pmatrix} g_1 & g_2 & \cdots & g_{m_g} \end{pmatrix}^T \tag{2}$$

and

$$Y_2(n) = \begin{pmatrix} y_2(n) & y_2^2(n) & \cdots & y_2^{m_g}(n) \end{pmatrix}^T. \tag{3}$$

The intermediate signal $y_2(n)$ is defined as

$$\begin{aligned} y_2(n) &= h(\mathrm{z}^{-1})y(n) = \frac{B(\mathrm{z}^{-1})}{1-A(\mathrm{z}^{-1})}y(n) \\ &= \sum_{m=0}^{m_b} b_m y(n-m) + \sum_{m=1}^{m_a} a_m y_2(n-m) \end{aligned} \tag{4}$$

where $h(\mathsf{z}^{-1}) = \frac{B(\mathsf{z}^{-1})}{1-A(\mathsf{z}^{-1})}$ and the polynomials $A(\mathsf{z}^{-1})$ and $B(\mathsf{z}^{-1})$ are defined as

$$\begin{aligned} A(\mathsf{z}^{-1}) &= \sum_{m=1}^{m_a} a_m \mathsf{z}^{-m} \\ B(\mathsf{z}^{-1}) &= \sum_{m=0}^{m_b} b_m \mathsf{z}^{-m}. \end{aligned} \tag{5}$$

Here $\mathsf{z}^{-1}$ is the delay operator such that $\mathsf{z}^{-m}x(n) = x(n-m)$.

The predistorter of the IIR Wiener system is considered as an IIR Hammerstein system. The output of the predistorter is given by

$$\begin{aligned} y(n) &= p(n,\mathsf{z}^{-1})x_2(n) = \frac{D(n,\mathsf{z}^{-1})}{1-C(n,\mathsf{z}^{-1})}x_2(n) \\ &= \sum_{m=0}^{m_d} d_m(n)x_2(n-m) + \sum_{m=1}^{m_c} c_m(n)y(n-m) \end{aligned} \tag{6}$$

where $p(n,\mathsf{z}^{-1}) = \frac{D(n,\mathsf{z}^{-1})}{1-C(n,\mathsf{z}^{-1})}$ and the polynomials $C(n,\mathsf{z}^{-1})$ and $D(n,\mathsf{z}^{-1})$ are defined as

$$\begin{aligned} C(n,\mathsf{z}^{-1}) &= \sum_{m=1}^{m_c} c_m(n)\mathsf{z}^{-m} \\ D(n,\mathsf{z}^{-1}) &= \sum_{m=0}^{m_d} d_m(n)\mathsf{z}^{-m}. \end{aligned} \tag{7}$$

The intermediate signal $x_2(n)$ is defined as

$$\begin{aligned} x_2(n) &= f(x(n)) \\ &= f_1(n)x(n) + f_2(n)x^2(n) + \cdots + f_{m_f}(n)x^{m_f}(n) \\ &= F^T(n)X(n) \end{aligned} \tag{8}$$

where

$$F(n) = \begin{pmatrix} f_1(n) & f_2(n) & \cdots & f_{m_f}(n) \end{pmatrix}^T \tag{9}$$

and

$$X(n) = \begin{pmatrix} x(n) & x^2(n) & \cdots & x^{m_f}(n) \end{pmatrix}^T. \tag{10}$$

Let us define the parameter vector $\theta$ of the predistorter as

$$\begin{aligned} \theta &= \begin{pmatrix} \theta_d^T & \theta_c^T & \theta_f^T \end{pmatrix}^T \\ \theta_d &= \begin{pmatrix} d_0 & d_1 & \cdots & d_{m_d} \end{pmatrix}^T \\ \theta_c &= \begin{pmatrix} c_1 & c_2 & \cdots & c_{m_c} \end{pmatrix}^T \\ \theta_f &= \begin{pmatrix} f_1 & f_2 & \cdots & f_{m_f} \end{pmatrix}^T. \end{aligned} \tag{11}$$

The goal of this paper is to estimate the parameter vector $\theta$ by minimizing the Mean Square Error (MSE) which is defined as

$$E\{e^2(n)\} = E\{(r(n) - z(n))^2\} \tag{12}$$

where $E\{.\}$ denotes Expectation. Here $r(n)$ is the reference signal which is defined as [3]

$$r(n) = x(n-\tau) + \upsilon(n) \tag{13}$$

where $\tau$ is the time delay and $\upsilon(n)$ is zero-mean additive white Gaussian noise (AWGN). The NFxLMS algorithm can be developed to estimate the parameter vector $\theta$. This is the topic of the next section.

**Remark 1:** The delay time $\tau$ equals zero in case the system to be compensated is minimum phase [3].

## 3. THE NFXLMS ALGORITHM

The adaptation of the NFxLMS algorithm is obtained by applying the stochastic gradient algorithm [17, 18]:

$$\theta(n+1) = \theta(n) - \frac{\mu}{2}\Delta^T(n) \tag{14}$$

where $\mu$ is a positive constant with value less than 1 and usually defined as the *step-size* parameter. The gradient vector $\Delta(n)$ is defined as

$$\Delta(n) = \frac{\partial e^2(n)}{\partial \theta(n)} = -2e(n)\frac{\partial z(n)}{\partial \theta(n)}. \tag{15}$$

Using Eqs. (1)-(3), $\frac{\partial z(n)}{\partial \theta(n)}$ can be derived as

$$\begin{aligned} \frac{\partial z(n)}{\partial \theta(n)} &= G^T \frac{\partial Y_2(n)}{\partial \theta(n)} = G^T \begin{pmatrix} \frac{\partial y_2(n)}{\partial \theta(n)} \\ \frac{\partial y_2^2(n)}{\partial \theta(n)} \\ \vdots \\ \frac{\partial y_2^{m_g}(n)}{\partial \theta(n)} \end{pmatrix} \\ &= G^T \begin{pmatrix} 1 \\ 2y_2(n) \\ \vdots \\ m_g y_2^{m_g-1}(n) \end{pmatrix} \frac{\partial y_2(n)}{\partial \theta(n)}. \end{aligned} \tag{16}$$

Since it is not possible to measure the intermediate signal $y_2(n)$, using Eq. (4) and defining

$$\begin{aligned} s_1(n) &= G^T \begin{pmatrix} 1 \\ 2y_2(n) \\ \vdots \\ m_g y_2^{m_g-1}(n) \end{pmatrix} \\ &= G^T \begin{pmatrix} 1 \\ 2\left[h(\mathsf{z}^{-1})y(n)\right] \\ \vdots \\ m_g \left[h(\mathsf{z}^{-1})y(n)\right]^{m_g-1} \end{pmatrix} \end{aligned} \tag{17}$$

we have

$$\frac{\partial z(n)}{\partial \theta(n)} = s_1(n)\frac{\partial y_2(n)}{\partial \theta(n)} \tag{18}$$

where

$$\frac{\partial y_2(n)}{\partial \theta(n)} = \sum_{m=0}^{m_b} b_m \frac{\partial y(n-m)}{\partial \theta(n)} + \sum_{m=1}^{m_a} a_m \frac{\partial y_2(n-m)}{\partial \theta(n)}. \tag{19}$$

Assuming that $\theta(n)$ changes slowly [3, 19, 20], we have

$$\begin{aligned} \frac{\partial y(n-m)}{\partial \theta(n)} &\approx \frac{\partial y(n-m)}{\partial \theta(n-m)}, \quad m = 0,1,\cdots,m_b \\ \frac{\partial y_2(n-m)}{\partial \theta(n)} &\approx \frac{\partial y_2(n-m)}{\partial \theta(n-m)}, \quad m = 1,\cdots,m_a. \end{aligned} \tag{20}$$

Thus Eq. (19) becomes

$$\begin{aligned} \frac{\partial y_2(n)}{\partial \theta(n)} &= \sum_{m=0}^{m_b} b_m \frac{\partial y(n-m)}{\partial \theta(n)} + \sum_{m=1}^{m_a} a_m \frac{\partial y_2(n-m)}{\partial \theta(n)} \\ &\approx \sum_{m=0}^{m_b} b_m \frac{\partial y(n-m)}{\partial \theta(n-m)} + \sum_{m=1}^{m_a} a_m \frac{\partial y_2(n-m)}{\partial \theta(n-m)} \\ &= \frac{B(\mathsf{z}^{-1})}{1-A(\mathsf{z}^{-1})}\frac{\partial y(n)}{\partial \theta(n)}. \end{aligned} \tag{21}$$

Again using Eq. (4), Eq. (21) can be written as

$$\frac{\partial y_2(n)}{\partial \theta(n)} = h(\mathsf{z}^{-1})\left( \begin{array}{ccc} \frac{\partial y(n)}{\partial \theta_d(n)} & \frac{\partial y(n)}{\partial \theta_c(n)} & \frac{\partial y(n)}{\partial \theta_f(n)} \end{array} \right). \tag{22}$$

Differentiating both sides of Eq. (6) with respect to $d_k(n)$ and $c_k(n)$ give

$$\begin{aligned} \frac{\partial y(n)}{\partial d_k(n)} &= x_2(n-k) + \sum_{m=1}^{m_c} c_m(n) \frac{\partial y(n-m)}{\partial d_k(n)}, \\ & k = 0,1,\cdots,m_d \\ \frac{\partial y(n)}{\partial c_k(n)} &= y(n-k) + \sum_{m=1}^{m_c} c_m(n) \frac{\partial y(n-m)}{\partial c_k(n)}, \\ & k = 1,\cdots,m_c. \end{aligned} \tag{23}$$

Since the parameter vector $\theta$ is assumed to be changing slowly, we can write

$$\begin{aligned} \frac{\partial y(n-m)}{\partial d_k(n)} &\approx \frac{\partial y(n-m)}{\partial d_k(n-m)}, \quad m = 1,2,\cdots,m_c \\ \frac{\partial y(n-m)}{\partial c_k(n)} &\approx \frac{\partial y(n-m)}{\partial c_k(n-m)}, \quad m = 1,2,\cdots,m_c. \end{aligned} \tag{24}$$

Hence, Eq. (23) can be rewritten as

$$\begin{aligned} \frac{\partial y(n)}{\partial d_k(n)} &\approx x_2(n-k) + \sum_{m=1}^{m_c} c_m(n) \frac{\partial y(n-m)}{\partial d_k(n-m)}, \\ & k = 0,1,\cdots,m_d \\ \frac{\partial y(n)}{\partial c_k(n)} &\approx y(n-k) + \sum_{m=1}^{m_c} c_m(n) \frac{\partial y(n-m)}{\partial c_k(n-m)}, \\ & k = 1,\cdots,m_c \end{aligned} \tag{25}$$

or

$$\begin{aligned} \frac{\partial y(n)}{\partial d_k(n)} &\approx \frac{\mathsf{z}^{-k}}{1-C(n,\mathsf{z}^{-1})} x_2(n) \\ &= \frac{\mathsf{z}^{-k}}{1-C(n,\mathsf{z}^{-1})} \left(\theta_f^T(n) X(n)\right), \\ & k = 0,1,\cdots,m_d \\ \frac{\partial y(n)}{\partial c_k(n)} &\approx \frac{\mathsf{z}^{-k}}{1-C(n,\mathsf{z}^{-1})} y(n), \quad k = 1,\cdots,m_c. \end{aligned} \tag{26}$$

Similarly, differentiating both sides of Eq. (6) with respect to $f_k(n)$ gives

$$\begin{aligned} \frac{\partial y(n)}{\partial f_k(n)} &= \sum_{m=0}^{m_d} d_m(n) \frac{\partial x_2(n-m)}{\partial f_k(n)} + \\ & \sum_{m=1}^{m_c} c_m(n) \frac{\partial y(n-m)}{\partial f_k(n)}, \quad k = 1,\cdots,m_f. \end{aligned} \tag{27}$$

Again because the parameter vector $\theta$ is assumed to be changing slowly, we can write

$$\begin{aligned} \frac{\partial x_2(n-m)}{\partial f_k(n)} &\approx \frac{\partial x_2(n-m)}{\partial f_k(n-m)}, \quad m = 0,\cdots,m_d \\ \frac{\partial y(n-m)}{\partial f_k(n)} &\approx \frac{\partial y(n-m)}{\partial f_k(n-m)}, \quad m = 1,\cdots,m_c. \end{aligned} \tag{28}$$

Hence, Eq. (27) can be rewritten as

$$\begin{aligned} \frac{\partial y(n)}{\partial f_k(n)} &= \sum_{m=0}^{m_d} d_m(n) \frac{\partial x_2(n-m)}{\partial f_k(n-m)} + \sum_{m=1}^{m_c} c_m(n) \frac{\partial y(n-m)}{\partial f_k(n-m)} \\ &= \sum_{m=0}^{m_d} d_m(n) x^k(n-m) + \sum_{m=1}^{m_c} c_m(n) \frac{\partial y(n-m)}{\partial f_k(n-m)} \\ &= \frac{D(n,\mathsf{z}^{-1})}{1-C(n,\mathsf{z}^{-1})} x^k(n) \\ &= p(n,\mathsf{z}^{-1}) x^k(n), \quad k = 1,\cdots,m_f. \end{aligned} \tag{29}$$

In summary, the gradient vector $\Delta(n)$ can be written as

$$\Delta(n) = -2e(n)s_1(n)h(\mathsf{z}^{-1})\left( \begin{array}{ccc} \frac{\partial y(n)}{\partial \theta_d(n)} & \frac{\partial y(n)}{\partial \theta_c(n)} & \frac{\partial y(n)}{\partial \theta_f(n)} \end{array} \right) \tag{30}$$

where $s_1(n)$ is calculated in Eq. (17), and $\frac{\partial y(n)}{\partial \theta_d(n)}$, $\frac{\partial y(n)}{\partial \theta_c(n)}$ and $\frac{\partial y(n)}{\partial \theta_f(n)}$ are given by Eqs. (26) and (29), respectively. Predistortion of FIR Wiener system using the NFxLMS algorithm in [9, 10] is a special case when $A(\mathsf{z}^{-1}) = 0$ in $h(\mathsf{z}^{-1})$ and $C(n,\mathsf{z}^{-1}) = 0$ in $p(n,\mathsf{z}^{-1})$.

## 4. THE NFXLMS-ISE ALGORITHM

As it is clear from Eqs. (17) and (30), the nonlinear system should be known or identified since the linear subsystem, $h(\mathsf{z}^{-1})$, and the nonlinear subsystem, $g(.)$, are needed in order to calculate the gradient vector. Notice that in Eq. (30), $h(\mathsf{z}^{-1})$ performs as a filter for the gradient components. Hence, it can be replaced by any other filter with a similar frequency response. On the other hand, the accuracy of estimating the nonlinear subsystem is not critical here since this will only contribute to the constant $s_1(n)$ [20]. The Initial Subsystem Estimates (ISE) method [16] can be used to approximately estimate the linear and nonlinear subsystems of the Wiener system. Therefore, the ISE method can be applied as an initial step and then make use of the estimated subsystems in the NFxLMS algorithm. This will save the effort needed to evaluate accurate estimates for the linear and nonlinear subsystems. The ISE method uses the Discrete Multitone (DMT) signals to construct the excitation inputs of the system. The ISE method follows the following lines:

(1) Generate $M$ different DMT singals $y^{[m]}, 1 < m \leq M$.

(2) For each DMT signal $y^{[m]}$, measure $N$ periods of output signal $z^{[m]}$ in steady state excitation, take the average over these periods to obtain the mean output signal $\bar{z}^{[m]}$ and similarly evaluate the mean input signal $\bar{y}^{[m]}$.

(3) Use Fast Fourier Transform (FFT) to calculate the spectrum of $\bar{z}^{[m]}$, $F_{\bar{z}^{[m]}}(j\omega)$ and the spectrum of $\bar{y}^{[m]}$, $F_{\bar{y}^{[m]}}(j\omega)$. Then, estimate the frequency response $\widehat{H}^{[m]}(j\omega)$ for each excitation, *e.g.* $\widehat{H}^{[m]}(j\omega) = F_{\bar{z}^{[m]}}(j\omega)/F_{\bar{y}^{[m]}}(j\omega)$.

(4) Take the average of all frequency response functions: $\widehat{H}(j\omega) = \frac{1}{M}\sum_{m=1}^{M} \widehat{H}^{[m]}(j\omega)$. From $\widehat{H}(j\omega)$, estimate a parametric linear model $\widehat{h}(\mathsf{z}^{-1})$.

(5) For each $y^{[m]}$, compute the intermediate signal $Y_2^{[m]}$ using $\widehat{h}(\mathsf{z}^{-1})$. From the signals $Y_2^{[m]}$ and $z^{[m]}$, $G$ can be estimated using different techniques, e.g. Least Squares (LS) fitting [18].

Therefore, the filter $h(\mathsf{z}^{-1})$ can be replaced in Eq. (30) by the initial linear subsystem estimate $\widehat{h}(\mathsf{z}^{-1})$ and the constant $s_1(n)$ is replaced with

$$\widehat{s}_1(n) = \widehat{G}^T \left( \begin{array}{c} 1 \\ 2[\widehat{h}(\mathsf{z}^{-1})y(n)] \\ \vdots \\ \widehat{m}_g[\widehat{h}(\mathsf{z}^{-1})y(n)]^{\widehat{m}_g-1} \end{array} \right) \tag{31}$$

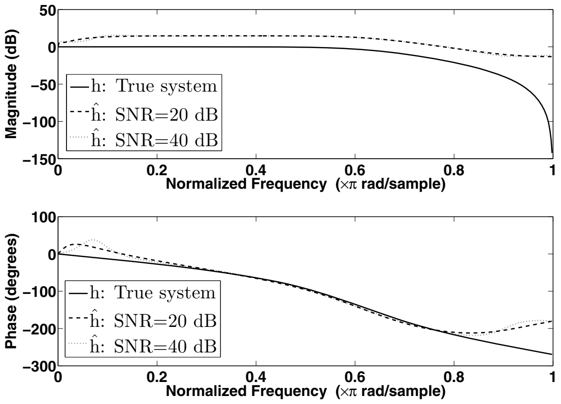


Figure 2: Frequency responses of the initial estimates of the linear subsystem.

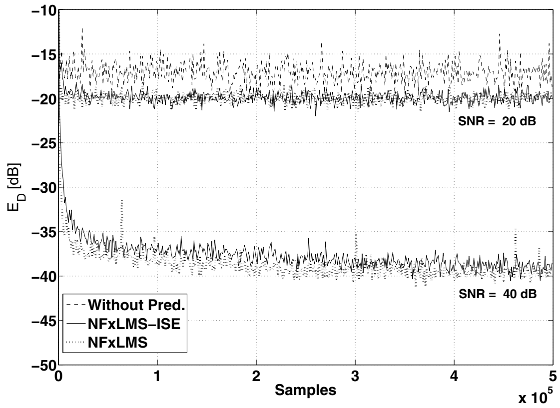


Figure 3: $E_D$ for different SNRs.

where $\widehat{G}$ represents the initial nonlinear subsystem estimate with order $\widehat{m}_g$.

Now, the coefficient adaptation of the NFxLMS with Initial Subsystem Estimates (NFxLMS-ISE) algorithm can be described as

$$\theta(n+1) = \theta(n) - \frac{\mu}{2}\Delta^T(n) \tag{32}$$

with

$$\Delta(n) = -2e(n)\widehat{s}_1(n)\widehat{h}(\mathsf{z}^{-1})\begin{pmatrix} \frac{\partial y(n)}{\partial \theta_d(n)} & \frac{\partial y(n)}{\partial \theta_c(n)} & \frac{\partial y(n)}{\partial \theta_f(n)} \end{pmatrix} \tag{33}$$

where $\widehat{h}(\mathsf{z}^{-1})$ is the initial linear subsystem estimate, $\widehat{s}_1(n)$ is calculated using the initial nonlinear subsystem estimate $\widehat{G}$ as in Eq. (31), and $\frac{\partial y(n)}{\partial \theta_d(n)}$, $\frac{\partial y(n)}{\partial \theta_c(n)}$ and $\frac{\partial y(n)}{\partial \theta_f(n)}$ are given by Eqs. (26) and (29), respectively.

## 5. SIMULATION STUDY

The comparative simulation study between the NFxLMS-ISE and NFxLMS algorithms is given in this section. The following IIR Wiener system was considered.

$$\begin{aligned} z(n) &= y_2(n) + 0.25y_2^2(n) + 0.125y_2^3(n) \\ y_2(n) &= \frac{0.72 + 1.51\mathsf{z}^{-1} + 1.04\mathsf{z}^{-2} + 0.26\mathsf{z}^{-3}}{1 + 1.46\mathsf{z}^{-1} + 0.89\mathsf{z}^{-2} + 0.18\mathsf{z}^{-3}}y(n). \end{aligned} \tag{34}$$

The orders of the linear and nonlinear blocks of the IIR Hammerstein predistorter were chosen as $m_c = 3$, $m_d = 3$ and $m_f = 9$. The input signal was chosen to be a random signal with uniform distribution over $(-1, 1)$ and data length of $5 \times 10^5$ samples. The bandwidth of the input signal was limited in order to prevent aliasing [21].

As a measurement of the performance, we define the mean square distortion of the system consisting of the predistorter and the nonlinear system as

$$E_D(n) = 10\log_{10}\left(\frac{\hat{E}\{e^2(n)\}}{\hat{E}\{r^2(n)\}}\right) \tag{35}$$

where $\hat{E}\{.\}$ is the mean over 200 independent realizations.

The parameter vectors were initialized as

$$\begin{aligned} \theta_d(0) &= \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix}^T \\ \theta_c(0) &= \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}^T \\ \theta_f(0) &= \begin{pmatrix} 1 & 0 & \cdots & 0 \end{pmatrix}^T. \end{aligned} \tag{36}$$

For the initial estimate of the IIR Wiener system, 10 different 64-tones DMT signals with constant crest factor 3 and Root Mean Squares (RMS) value 0.25 were used as the system inputs. The output measurement noise was considered as zero-mean AWGN with different Signal to Nose Ratio (SNR). $\widehat{h}(\mathsf{z}^{-1})$ was assumed to be an IIR filter with numerator order 5 and denominator order 5, and $\widehat{G}$ was assumed to be a 5th order static nonlinearity. Fig. 2 gives the frequency responses of the initial estimates of $h(\mathsf{z}^{-1})$. The initial estimates of $\widehat{G}$ were $(1.0001 \quad 0.2500 \quad 0.1249 \quad 0 \quad -0.0008)^T$ and $(1.0000 \quad 0.2500 \quad 0.1249 \quad -0.0001 \quad 0.0001)^T$, for SNR=20 dB and SNR=40 dB, respectively.

The distortions of the nonlinear system without predistorter were $-16.98$ dB in noise-free scenario. The $E_D$ comparison between the NFxLMS-ISE and NFxLMS algorithms is given in Fig. 3. The step size $\mu$ for the NFxLMS algorithm was chosen as 0.05 and 0.08 for SNR=20 dB and SNR=40 dB, respectively. For the NFxLMS-ISE algorithm, $\mu$ was chosen differently as 0.005 and 0.02 for SNR=20 dB and SNR=40 dB, respectively. The difference in the selected values of the step size $\mu$ for the two algorithms is due to the fact that the estimate of the linear subsystem in Eq. (33) is scaled with a constant as compared to the true subsystem [16]. On average, the NFxLMS-ISE algorithm achieved about $-19.99$ dB and $-39.01$ dB, and the NFxLMS algorithm achieved about $-20.00$ and $-39.64$ dB, for SNR=20 dB and SNR=40 dB, respectively.

Figure 4 shows power spectral densities (PSDs) of the output signals of the IIR Wiener system with and without predistorter. From this figure, we can see that the IIR Hammerstein predistorter using either the NFxLMS or the NFxLMS-ISE algorithm can effectively reduce spectral regrowth.

## 6. CONCLUSIONS

Adaptive predistortion of nonlinear systems described using Wiener models is considered in this paper. The NFxLMS algorithm for estimating the parameters of the Hammerstein predistorter has been derived. Accurate estimates of the linear and nonlinear subsystems are required in the NFxLMS algorithm for the gradient calculations. A new NFxLMS-ISE algorithm is proposed in this paper to avoid using the accurate estimates. The new algorithm is based on using the initial subsystem estimates (ISE) method. A numerical simulation study has been introduced to compare between the NFxLMS-ISE and NFxLMS algorithms. The simulation results show that the nonlinear distortion can be efficiently precompensated using the NFxLMS-ISE algorithm as well as the NFxLMS algorithm.


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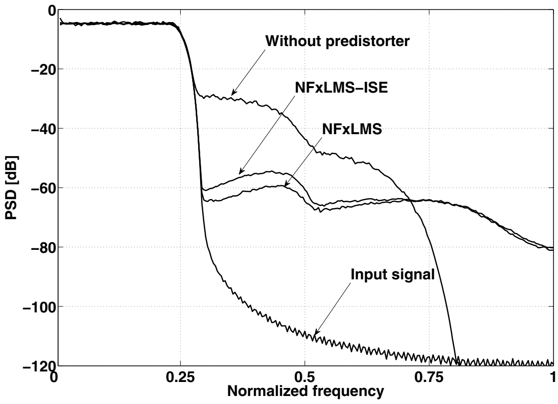


Figure 4: Power spectral density for SNR=40 dB.

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