IV International Conference on Computational Methods for Coupled Problems in Science and Engineering
COUPLED PROBLEMS 2011
M. Papadrakakis, E. Oñate and B. Schrefler (Eds)

ITERATIVE COUPLING OF BOUNDARY AND DISCRETE ELEMENT METHODS USING AN OVERLAPPING FEM ZONE

L. Malinowski*†, G.F. Karlis†, G. Beer† and J. Rojek‡

*†Institute for Structural Analysis
Technical University Graz
Lessingstrasse 25, 8010 Graz, Austria
e-mail: lmalinowski@tugraz.at

‡Institute of Fundamental Technological Research
Polish Academy of Sciences
Pawińskiego 5B, 02-106 Warsaw, Poland

Key words: Coupled Problems, Geotechnical Engineering, Tunnelling, Boundary Element Method, Discrete Element Method, Iterative Coupling

Abstract. One of the characteristics of the numerical simulation in geotechnical engineering is that non-linear/discontinuous behaviour is concentrated on small portions of the total domain. A representative example is tunnel excavation, where high stress concentration is observed only near the excavation area and not over the whole rock mass. The Discrete Element Method (DEM) is ideal for handling discontinuous behaviour. However, employing DEM over the whole domain would be prohibitive, due to the large computational effort it requires. On the other hand, the Boundary Element Method (BEM) can effectively treat infinite and semi-infinite domains. The aim of the current work is to develop a simulation methodology that can iteratively couple Boundary Element and Discrete Element Methods in order to treat discontinuous behavior that arises in small portions of the total domain. To simplify the meshing procedure, an overlapping FEM zone is introduced on the DEM side of the common boundary.

1 INTRODUCTION

The Boundary Element Method (BEM) is an established numerical method that performs particularly well in problems where infinite domains are involved. Such problems are the norm in geomechanics, where the soil and the underground rock masses can be modelled as infinite or semi-infinite domains. The BEM has been successfully appplied to a variety of problems related to underground structures [1, 2, 3, 4, 5, 6, 7].

However, the existence of non-linear behavior that can appear in problems like tunnel excavation, gives rise to additional volume integrals in BEM, that require the use of internal cells. Then the main advantage of the BEM, i.e. the discretization exclusively of the domain boundary, is partially lost, because of additional calculations that require integrations over the domain volume. In addition, discontinuous behavior can not be modelled.

In order to deal with these problems, the Discrete Element Method (DEM) is employed. DEM is a dynamic method, initially proposed by Cundall and Strack [8] and Cundall [9] and is used to investigate the behavior of geomaterials. A significant advantage of the DEM is its ability to model large displacements and discontinuous and non-linear behavior. In view of these advantages the DEM has been used to model the brittle behavior of rocks and to simulate crack initiation and propagation [16, 11, 12, 13]. However, these benefits come at a high computational cost, that prohibits the use of DEM in large domains.

The aim of the current work is to combine these two methods, by utilizing each, where they work best. It is common in geotechnical engineering problems, that discontinuous/non-linear behavior is confined to only small portions of the total domain. A representative example is tunnel excavation, where high stress concentration is observed and confined near the excavation zone and not over the whole rock mass. In the present work, an attempt is made to develop a simulation methodology that can iteratively couple the two aforementioned numerical methods in order to effectively treat non-linear/discontinuous behavior in problems related to tunnel excavation.

This paper is organized as follows: sections two and three briefly describe the BEM and DEM formulations that are used in the analysis. Section four describes the iterative coupling algorithm that is used and in section five the numerical results for a two-dimensional problem are obtained and discussed. Finally, section six contains the conclusions of this study and the plans for future work on that topic.

2 BEM FORMULATION

The boundary element method used in the present work is a displacement-based, collocation method that is typically derived from Betti's reciprocal theorem [14]. Based on that and Green's third identity, the following integral equation, for a linear elastic domain Ω can be derived.

$$\mathbf{c}\left(\mathbf{P}\right)\cdot\mathbf{u}\left(\mathbf{P}\right)=\int\limits_{\partial\Omega}\mathbf{U}\left(\mathbf{P};\mathbf{Q}\right)\cdot\mathbf{t}\left(\mathbf{Q}\right)\,dS-\int\limits_{\partial\Omega}\mathbf{T}\left(\mathbf{P};\mathbf{Q}\right)\cdot\mathbf{u}\left(\mathbf{Q}\right)\,dS \tag{1}$$

with $\mathbf{u}\left(\mathbf{P}\right)$, $\mathbf{t}\left(\mathbf{P}\right)$ being the displacement and traction vectors at point $\mathbf{P}$, point $\mathbf{Q}$ running over the domain boundary ($\partial\Omega$) and $\mathbf{U}\left(\mathbf{P};\mathbf{Q}\right)$, $\mathbf{T}\left(\mathbf{P};\mathbf{Q}\right)$ being the fundamental solutions for the displacements and tractions respectively. Furthermore, the tensor $\mathbf{c}\left(\mathbf{P}\right)$ is equal to the unit tensor $\mathbf{I}$, if $\mathbf{P}$ lies inside the domain, $^1/_2\mathbf{I}$ if $\mathbf{P}$ lies on the domain's smooth boundary and $\mathbf{0}$ if $\mathbf{P}$ lies outside of the domain.

Discretizing the domain boundary into nodes and elements and writing the above equation for every node of the boundary of the domain, the following integral representation for node $\mathbf{x}^k$ is obtained.

$$\mathbf{c}\left(\mathbf{x}^k\right) \cdot \mathbf{u}^k = \sum_{e=1}^{E} \sum_{i=1}^{A(e)} \mathbf{\Delta U}_{ei}^k \cdot \mathbf{t}^k - \sum_{e=1}^{E} \sum_{i=1}^{A(e)} \mathbf{\Delta T}_{ei}^k \cdot \mathbf{u}^k \tag{2}$$

with $\mathbf{u}^k$, $\mathbf{t}^k$ being the displacements and tractions of node $\mathbf{x}^k$, E being the total number of elements and $A\left(e\right)$ the total number of nodes for element e. Moreover, $\mathbf{\Delta U}_{ei}^k$ and $\mathbf{\Delta T}_{ei}^k$ are matrices that contain integrals over the element e, of the fundamental solutions U and T respectively, multiplied by the i-th interpolation function and Jacobian of element e, that are typically calculated using Gauss quadrature.

Collocating eq (2) for all boundary nodes and utilizing the boundary conditions of the problem, results to the final linear system for the BEM, which is of the form

$$\mathbf{A} \cdot \mathbf{x} = \mathbf{b} \tag{3}$$

3 DEM FORMULATION

The Discrete Element Method is a dynamic method, where the material is respresented as a collection of particles interacting with each other. The formulation utilized in the present work is the one proposed by Oñate and Rojek [15] and Labra et al.[17].

Assuming N particles in total and writing the equations of motion for translations and rotations for all of them results in

$$\mathbf{M}_D \cdot \{\ddot{\mathbf{u}}\} = \{\mathbf{F}\} \tag{4}$$

$$\mathbf{J}_D \cdot \{\dot{\boldsymbol{\omega}}\} = \{\mathbf{T}\} \tag{5}$$

with $\{\ddot{\mathbf{u}}\}$ and $\{\dot{\boldsymbol{\omega}}\}$ containing the components of acceleration and angular acceleration respectively. Furthermore, matrices $\mathbf{M}_D$ and $\mathbf{T}_D$ are block diagonal matrices that involve the particle masses m_i and moments of inertial J_i and are of the form

$$\mathbf{M}_D = \begin{bmatrix} m_1 \cdot \mathbf{I} & \dots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \dots & m_N \cdot \mathbf{I} \end{bmatrix} \quad \text{and} \quad \mathbf{J}_D = \begin{bmatrix} J_1 \cdot \mathbf{I} & \dots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \dots & J_N \cdot \mathbf{I} \end{bmatrix} \tag{6}$$

with $\mathbf{I}$ being the 2×2 and 3×3 identity matrix for 2D and 3D respectively. Finally vectors $\{\mathbf{F}\}$ and $\{\mathbf{T}\}$ contain the total force and total moment for each degree of freedom of each particle, respectively. It is worth noting that the total force applied to a particle is described as the sum of the external forces, the contact forces from the adjacent particles and the forces resulting from the imposed external damping. Similarly, the total moment is given by the sum of the external moment due to the external load, the rotational

component of the contact forces between adjacent particles and the moment resulting from the imposed external damping.

Since DEM is a dynamic method, eqs (4) and (5) have to be integrated in time. This is done be employing a central difference scheme. The calculation of the displacements and the incremental rotations for step $n+1$ is done in terms of the displacement and rotation and their time derivatives over the previous time step, utilizing also an intermediate, auxiliary time step, indicated as $n + 1/2$.

$$\{\ddot{\mathbf{u}}\}^n = \mathbf{M}_D^{-1} \{\mathbf{F}\}^n \tag{7}$$

$$\{\dot{\mathbf{u}}\}^{n+1/2} = \{\dot{\mathbf{u}}\}^{n-1/2} + \{\ddot{\mathbf{u}}\}^n \Delta t \tag{8}$$

$$\{\mathbf{u}\}^{n+1} = \{\mathbf{u}\}^n + \{\dot{\mathbf{u}}\}^{n+1/2} \Delta t \tag{9}$$

$$\{\dot{\boldsymbol{\omega}}\}^n = \mathbf{J}_D^{-1} \{\mathbf{T}\}^n \tag{10}$$

$$\{\boldsymbol{\omega}\}^{n+1/2} = \{\boldsymbol{\omega}\}^{n-1/2} + \{\dot{\boldsymbol{\omega}}\}^n \Delta t \tag{11}$$

$$\Delta \{\boldsymbol{\theta}\}^{n+1} = \{\boldsymbol{\omega}\}^{n+1/2} \Delta t \tag{12}$$

with $\Delta \{\boldsymbol{\theta}\}$ being the incremental rotations.

4 ITERATIVE COUPLING

In order to couple the BEM and DEM subdomains, an iterative coupling scheme has been adopted. An important advantage of iterative coupling is that it allows solving each region separately, without having to assemble a global linear system. The interaction between the adjacent regions during the iteration steps is achieved with the exchange of boundary conditions on their common boundary. From now on, the term interface will be used to describe the common boundary of the two subdomains. The proposed scheme is a strong coupling scheme, i.e. there must be a "1-1" correspondence of the nodes of the two adjacent subdomains on their common boundary.

A basic requirement for the coupling scheme to work, is that the same fields must be calculated on all interfaces, regardless the method that is implemented in each subdomain. Otherwise, the exchange of boundary conditions will not be possible. To that end, the BEM formulation described in Section 2 was modified to calculate or accept as boundary conditions the nodal point forces instead of the element-based tractions on the interface.

4.1 BEM - Nodal Force Calculation on the Interface

As explained by Beer et al.[14], the nodal tractions on the interface of a BEM subdomain can be expressed as

$$\{\mathbf{t}\}_c = \{\mathbf{t}\}_{c0} + \mathbf{K}_{BE} \{\mathbf{u}\}_c \tag{13}$$

where the vector $\{\mathbf{t}\}_c$ contains the tractions on the interface nodes, $\{\mathbf{t}\}_{c0}$ contains the tractions on the interface nodes due to the applied loads on the rest of the boundary, $\{\mathbf{u}\}_c$

contains the displacements on the interface nodes and $\mathbf{K}_{BE}$ is the so called pseudo-stiffness matrix, calculated for the interface nodes.

To calculate $\{\mathbf{t}\}_{c0}$, one has to solve the BEM subdomain using the linear system (3) with a constrained interface ($\{\mathbf{u}\}_c = \mathbf{0}$) and the boundary conditions on the rest of the domain boundary as specified by the problem.

For calculating the pseudo-stiffness matrix $\mathbf{K}_{BE}$, the linear system (3) is utilized again. The collocation procedure is conducted for all nodes of the domain, in order to assemble the system matrix and all nodes, including those of the interface, are considered to have unknown tractions. Then the linear system is solved (number of interface nodes) $\times$ (number of dofs per node) times. Each time, a unit displacement is applied to one degree of freedom of one node of the interface, whereas all other displacements are set to zero. This results to the following traction vector:

$$\{\bar{\mathbf{t}}\}_i = \begin{Bmatrix} \{\mathbf{t}\}_i^b \\ \{\mathbf{t}\}_i^c \end{Bmatrix} \tag{14}$$

with i ranging from 1 to (number of interface nodes) $\times$ (number of dofs per node), $\{\mathbf{t}\}_i^c$ indicating the inteface tractions and $\{\mathbf{t}\}_i^b$ indicating the tractions on the rest of the boundary of the subdomain. Using vectors $\{\bar{\mathbf{t}}_i\}$ as columns, a matrix B can be constructed, with dimensions [(number of nodes) $\times$ (number of dofs per node)]$\times$[(number of interface nodes) $\times$ (number of dofs per node)].

$$\mathbf{B} = \begin{bmatrix} \{\mathbf{t}\}_1^b & \{\mathbf{t}\}_2^b & \dots \\ \{\mathbf{t}\}_1^c & \{\mathbf{t}\}_2^c & \dots \end{bmatrix} \tag{15}$$

Isolating the part of $\mathbf{B}$ that corresponds to the tractions on the interface, yields the pseudo-stiffness matrix $\mathbf{K}_{BE}$.

$$\mathbf{K}_{BE} = \begin{bmatrix} \{\mathbf{t}\}_1^c & \{\mathbf{t}\}_2^c & \dots \end{bmatrix} \tag{16}$$

After $\mathbf{K}_{BE}$ and the applied load contribution $\{\mathbf{t}\}_{c0}$ have been calculated, eq (13) can be solved either for displacements or for tractions on the interface nodes. However, it requires further processing in order to utilize nodal forces instead of tractions.

To obtain the x-component of the equivalent nodal point force for a node k, we apply a unit virtual displacement to that node in the x direction. Then, the work done by the tractions at that point must be equal to the work done by the equivalent nodal point force.

$$F_x^k \cdot 1 = \int_S t_x \delta u_x \, dS \tag{17}$$

with S being the boundary of the domain (including the interface). By discretizing eq (17)

and introducting the displacement and traction interpolation functions

$$\begin{aligned} t_x &= \sum_{n=1}^{N_e} N_n t_{xe}^n \\ \delta u_x &= N_j \delta u_{xj} = N_j \cdot 1, j = \text{local node numbering of node } k \end{aligned}$$

we end up with the following expression:

$$F_x^k = \sum_{e=1}^{E} \int_{S_e} \sum_{n=1}^{N_e} N_n t_{xn}^e \tag{18}$$

with E being the total number of elements that node k belongs to and N_e being the number of nodes of element e. The same procedure can be followed for the y-component of the nodal force and the total equivalent nodal force at node k can be expressed using vector notation as follows:

$$\mathbf{F}_k = \sum_{e=1}^{E} \sum_{n=1}^{N_e} \mathbf{N}_{jn}^e t_n^e \tag{19}$$

with E, N_e being the same as in eq (18) and $\mathbf{N}_{jn}^e$ being a matrix of the form

$$\mathbf{N}_{jn}^e = N_{jn}^e \cdot \mathbf{I}, \text{ with } N_{jn}^e = \int_{S_e} N_j N_n \, dS_e \tag{20}$$

where N_j, N_n are the displacement and traction interpolation functions respectively.

Repeating the same procedure for every node of the interface (i.e. where we need the equivalent nodal point forces) we obtain

$$\{\mathbf{F}\}_c = \mathbf{N} \{\mathbf{t}\}_c = \mathbf{N} \{\mathbf{t}\}_{c0} + \mathbf{N}\mathbf{K}_{BE} \{\mathbf{u}\}_c \tag{21}$$

with matrix $\mathbf{N}$ containing the contributions of $\mathbf{N}_{jn}^e$. The above procedure is explained in further detail in [14] and results to

$$\{\mathbf{F}\}_c = \{\mathbf{F}\}_{c0} + \mathbf{K} \{\mathbf{u}\}_c \tag{22}$$

where $\{\mathbf{F}\}_c$ and $\{\mathbf{u}\}_c$ are the nodal point forces and the displacements on the interface nodes respectively, $\{\mathbf{F}\}_{c0}$ is the contribution of the external load on the interface nodes and $\mathbf{K}$ is the stiffness matrix of the interface.

Now using eq (22) the displacements or the forces on the interface can be calculated, when the forces or displacements are provided from the adjacent region respectively. The results of the calculation can then be transfered back to the adjacent subdomain. Note that during the iterative procedure only the interface has to be solved.

From the above, it is easy to see that in order to calculate the solution at the subdmain interface, only the calculation of the matrix $\mathbf{K}$ and of the forces on the interface nodes $\{\mathbf{F}\}_{c0}$ is required and is done only once. This means that the fields on the boundary of the subdomain do not have to be calculated in every iteration step, but only in the end, after the iterative procedure has converged.

4.2 DEM - Overlapping FEM Zone

Directly coupling DEM and BEM using a strong coupling scheme would impose additional requirements on the DEM mesh, such as gradually increasing the particle size to match that of the BEM element size on the interface. In order to avoid this problem, a FEM zone was introduced between the BEM and DEM subdomains, that partially overlaps with the DEM subdomain, as described by Rojek and Oñate [16] and Labra et al.[17].

In short, the methodology described in Section 3 is enhanced with the so called explicit dynamic FEM formulation over the FEM zone.

$$\mathbf{M}_F \cdot \{\ddot{\mathbf{u}}\}_F = \{\mathbf{F}\}_F^{ext} - \{\mathbf{F}\}_F^{int} \tag{23}$$

with $\mathbf{M}_F$ being the mass matrix and $\mathbf{u}_F$, $\mathbf{F}_F^{ext}$ and $\mathbf{F}_F^{int}$ being the displacements, external and internal forces respectively. The same central difference scheme as before has been utilized for the time integration.

Over the overlapping zone, where DEM and FEM coexist, additional kinematic constrains have to be applied to the DEM particles in order to constrain their displacement field by that of the underlying finite elements. The above constrains are imposed by means of a penalty function which forms the final linear system for the DEM-FEM subdmain along with the equations of motion derived from the DEM and FEM formulations. This formulation is described in great detail in [17].

4.3 Coupling Algorithm

A common problem in iterative coupling is avoiding floating domains. In that context, each static subdomain should either have sufficiently prescribed displacements or be of infinite extent, to avoid being subjected to rigid body motion. In the case neither of the above is true, the required displacements should be provided by an adjacent region in the form of appropriate boundary conditions. To fulfill this requirement in a general manner, an algorithm has been developed to determine the sequence in which the subdomains will be solved during each iteration step and what kind of boundary conditions should be imposed on their interfaces with the adjacent regions. This algorithm is invoked only once, before starting the iterative procedure and results in an array, reffered to as the solving sequence array, that contains all the subdomains of the problem in the reverse sequence in which they should be solved. Furthermore, the boundary conditions that will be used on the interfaces are determined for each subdomain. The algorithm consists of the following steps:

- While there are still unhandled subdomains
    - If current subdomain has sufficient displacement boundary conditions (bcs) or is infinite
        - Set Neumann type bcs on its interfaces with other subdomains

- ◇ For all adjacent subdomains set Dirichlet type bcs on their interfaces with the current subdomain
- ◇ Add current subdomain to the solving sequence array

After the solving sequence and the types of boundary conditions have been determined on all interfaces, the iterative procedure takes place:

– While convergence or maximum number of iterations have not been reached

- ◦ Run through the solving sequence array from the back to the front and back again
  - ◇ For the current subdomain, obtain boundary conditions from adjacent subdomains
  - ◇ Solve current subdomain
  - ◇ Check for convergence

The convergence criterion used, was the one proposed by Elleithy et al.[18]. Namely, the displacements of the BEM subdomain on the interface $\{\mathbf{u}_{BEM}\}_I$ are checked over two consequent iteration steps.

$$\frac{\left|\left|\{\mathbf{u}_{BEM}\}_I^{n+1} - \{\mathbf{u}_{BEM}\}_I^{n}\right|\right|}{\left|\left|\{\mathbf{u}_{BEM}\}_I^{n+1}\right|\right|} < \epsilon \tag{24}$$

with n and $n+1$ being consequent iteration steps and ϵ being the predefined tolerance.

To illustrate the procedure described above, consider two subdomains, Ω_1 and Ω_2 that have a common boundary, as shown in Figure 1. The elements of Ω_1 that lie on the common boundary with Ω_2 form the interface I_{12}, whereas the elements of Ω_2 that lie on the common boundary with Ω_1 form the interface that is indicated as I_{21}. The displacements and forces on I_{12}, at the n-th iteration step, are denoted as $\mathbf{u}_{12}^n$ and $\mathbf{f}_{12}^n$ respectively. Similarly, $\mathbf{u}_{21}^n$ and $\mathbf{f}_{21}^n$ are used to indicate the displacements and forces on the interface I_{21}.

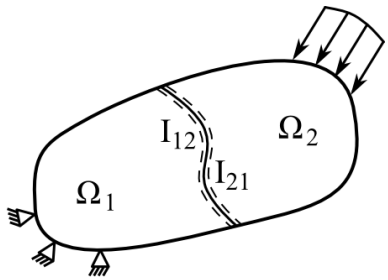

Figure 1: A problem with two subdomains sharing a common boundary

Now assume that the subdomain Ω_1 is fixed and that a constant load is applied to Ω_2. In that case, the algorithm sequence determination algorithm would check Ω_1, and since it is fixed, it would apply Neumann boundary conditions to I_{12}, Dirichlet boundary conditions on I_{21} and would store Ω_1 in the solving sequence array. Afterwards, the algorithm would check subdomain Ω_2, which now has prescribed Dirichlet boundary conditions (on I_{21}). It has no interfaces with subdomains other than Ω_1, which has already been addressed. The subdomain Ω_2 is added to the solving sequence array and the algorithm ends. At this point both subdomains have been assigned with a boundary condition on their interface.

The next step is to proceed to the iterative procedure itself. The first subdomain to be solved is the last added to the solving sequence array, i.e. Ω_2. In order to solve Ω_2, the displacements have to be specified on the interface I_{21}. For the first iteration, zero displacements are assumed, whereas fo the next iterations, the displacements are provided by Ω_1.

To ensure convergence for the aforementioned iterative procedure, a relaxation scheme had to be adopted. A wide range of relaxation algorithms exist in the literature [19]. For the present work, the same relaxation scheme was applied on all interfaces, regardless the type of the imposed boundary conditions, as shown in eqs (25) and (26).

$$\mathbf{u}_{12}^{n+1} = \alpha\, \mathbf{u}_{21}^{n+1} + (1-\alpha)\, \mathbf{u}_{12}^{n} \tag{25}$$

$$\mathbf{f}_{12}^{n+1} = \beta\, \mathbf{f}_{21}^{n+1} + (1-\beta)\, \mathbf{f}_{12}^{n} \tag{26}$$

The values of the relaxation parameters α and β are problem dependent and can affect not only the convergence speed of the iterative procedure, but also the convergence itself.

5 NUMERICAL EXAMPLE

In order to demonstrate the aforementioned coupling strategy, a two dimensional, linear elastic compression test is solved. The problem domain consists of an $\alpha \times 2\alpha$ rectangular plate, with $\alpha = 0.5$ meters.

The upper half of the plate is modelled as a linear elastic BEM region with Young's modulus $E = 100GPa$ and Poisson's ratio $\nu = 0$. The lower half of the plate was modelled as a DEM region with a partially overlapping FEM zone, as shown in Figure 2(a). Note that the DEM and the FEM regions are treated as one subdomain, as described in Section 4.2. For the FEM zone, the same material parameters are used, whereas for the DEM the particle radius is $r = 0.025\alpha$ and the normal stiffness was set to $k_n = 94.6GPa$ in order to obtain the same effective Young's modulus as in the BEM subdomain.

A uniform load $L = 0.05GPa$ is applied to the top of the plate, whereas its base is constrained with zero displacement in all directions. During the iterative procedure, the BEM subdomain is solved first with zero displacement prescribed on its interface with the adjacent subdomain. The forces calculated on the interface are applied to the DEM as boundary condition. Since the DEM is dynamic, the boundary condition was applied incrementally starting from zero force at time $t = 0$ and increasing linearly until the

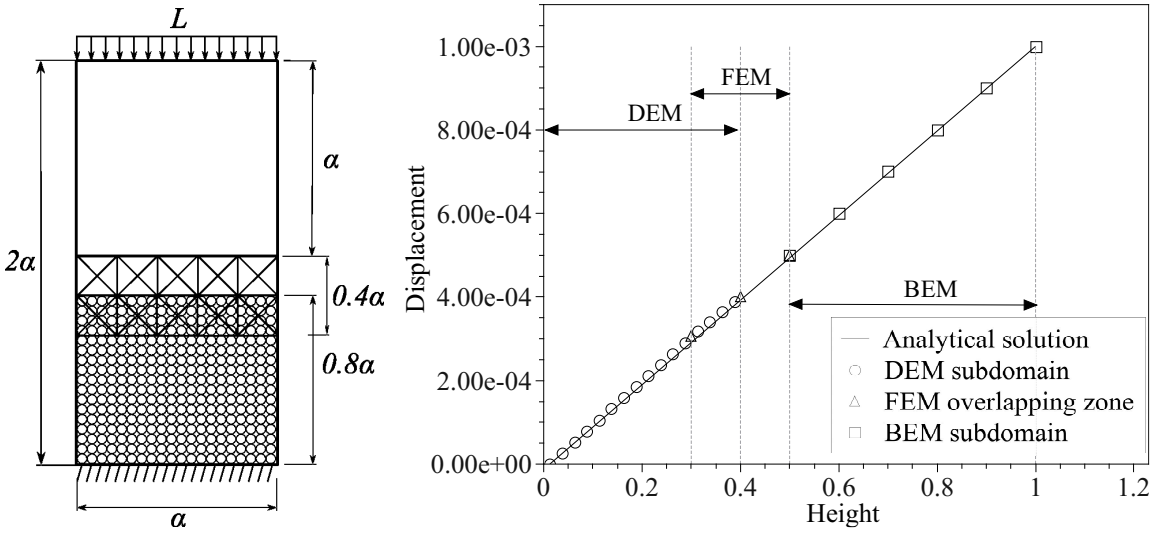

Figure 2: (a) The problem domain and (b) the displacements along the vertical axis.

value provided by the BEM is reached at time $t = 0.01$sec and remains constant until the the total time $T = 0.02$sec is reached. Then, the obtained results on the interface are transfered to the BEM subdomain allowing the iterative process to continue.

The convergence kriterion was set to $\epsilon = 1e - 4$. After the iterative procedure has converged, the displacements along the vertical direction were compared to the analytical solution of the problem (Figure 2(b)). The mean error obtained was 0.02%.

6 CONCLUSIONS AND FUTURE WORK

In the present work a DEM-BEM iterative coupling scheme has been presented. In order to simplify the coupling procedure, a FEM zone has been introduced between the two subdomains, partially overlapping with the DEM region. The DEM with the overlapping FEM zone and the BEM subdomains were solved independently from one another during the coupling procedure and the interaction between them was achieved by exchanging boundary conditions on their common boundary. The type of boundary conditions to be used on the interface was automatically determined, taking into account the existing boundary conditions of each subdomain, in order to avoid floating domains.

A numerical example has been solved to demonstrate the aforementioned algorithm. The covergence of the coupling scheme depends on the values of the relaxation parameters. The optimal values of these parameters are problem specific and to calculate them efficiently, an automatic relaxation algorithm should be used, as the ones proposed by Yan et al.[20] and Lin et al.[21].

In order to be able to model non-linear effects on the DEM subdomain with the proposed scheme, additional feedback from the BEM subdomain is required, during the

solution of the DEM region. Since DEM is a dynamic method, the strategy to follow would be to pause the DEM calculation at specific time steps, couple iteratively with the BEM region and after the iterative procedure has converged, proceed to the next time step. However, this case requires further investigation and is the subject of future work.

REFERENCES

[1] Beer, G. and Watson, O.W. and Swoboda, G. Three-dimensional analysis of tunnels using infinite Boundary Elements. *Computers and Geotechnics* (1987) **3(1)**:37–58.

[2] Pande G.N. and Beer, G. and Williams, J.R. Numerical Methods in Rock Mechanics. J.Wiley, (1987).

[3] Beer, G. Numerical Simulation in Tunneling. Springer, (2003).

[4] Beer, G. Technology Innovation in Underground Construction. CRC Press, (2009).

[5] Moser, W. and Duenser, Ch. and Beer G. Mapped infinite elements for three-dimensional multi-region boundary element analysis. *International Journal for Numerical Methods in Engineering* (2004) **61(3)**:314–328.

[6] von DEstorff, O. and Stamos, A.A. and Beskos, D.E. and Antes H. Dynamic interaction effects in underground traffic systems. *Engineering Analysis with Boundary Elements* (1991) **8(4)**:167–175.

[7] Pöttler, R. and Swoboda, G.A. Coupled Beam-Boundary-element model (BE-BEM) for analysis of underground openings. *Computers and Geotechnics* (1986) **2(4)**:239–256.

[8] Cundall, P.A. and Strack, O.D.L. A discrete numerical method for granular assemblies. *Geotechnique* (1979) **29**:47–65.

[9] Cundall, P.A. Formulation of a three dimensional distinct element model-Part I. A scheme to detect and represent contacts in a system of many polyhedral blocks. *International Journal of Rock Mechanics, Mining Sciences & Geotechnics Abstracts* (1988) **25(3)**:107–116.

[10] Rojek, J. Discrete Element Modelling of Rock Cutting. *Computer Methods in Material Science* (2007) **7(2)**:224–230.

[11] Potyondy, D. and Cundall, P.A. A bonded-particle model for rock. *International Journal of Rock Mechanics and Mining Sciences* (2004) **41**:1329–1364.

[12] Shiu, W. and Donzé, F.V. and Daudeville, L. Compaction process in concrete during missile impact: a DEM analysis. *Computers and Concrete* (2008) **5(4)**:329–342.

[13] Moon, T. and Nakagawa, M. and Berger, J. Measuring of Fracture Toughness using the Distinct Element Method. *International Journal of Rock Mechanics and Mining Sciences* (2006) **44(3)**:449–456.

[14] Beer, G. and Smith, I. and Duenser, Ch. The Boundary Element Method in Programming for Engineers and Scientists. Springer, (2008).

[15] Oñate, E. and Rojek, J. Combination of discrete and finite element methods for dynamic analysis of geomechanics problems. *Computer Methods in Applied Mechanics and Engineering* (2004) **193(27-29)**:3087–3128.

[16] Rojek, J. and Oñate, E. Multiscale analysis using a coupled discrete/finite element model. *Interaction and Multiscale Mechanics* (2007) **1(1)**:1–31.

[17] Labra, C. and Rojek, J. and Oñate, E. and Zarate, F. Advances in discrete element modelling of underground excavations. *Acta Geotechnica* (2008) **3(4)**:317–322.

[18] Elleithy, W.M. and Al-Gahtani, H.J. and El-Gebeily, M. Iterative coupling of BE and FE methods in elastostatics. *Engineering Analysis with Boundary Elements* (2001) **25**:685–695.

[19] Elleithy, W.M. and Tanaka, M. Interface relaxation algorithms for BEM-BEM coupling and FEM-BEM coupling. *Computer Methods in Applied Mechanics and Engineering* (2003) **192**:2977–2992.

[20] Yan, B. and Du, J. and Hu, Ning. and Sekine, H. A domain decomposition algorithm with finite element - boundary element coupling. *Applied Mathematics and Mechanics (English Edition)* (2006) **27(4)**:519–525.

[21] Lin, C.-C. and Lawton, E.C. and Caliendo, J.A. and Anderson, L.R. An iterative finite element - boundary element algorithm. *Computers & Structures* (1996) **59(5)**:899–909.