ARMA/USRMS 06-1079

Kinematical analysis of rock blocks supported by 3D imaging

Pötsch, M. and Schubert, W.
Institute for Rock Mechanics and Tunnelling, Graz University of Technology, Graz, Austria
Gaich, A.
3G Software & Measurement GmbH, Graz, Austria

This paper was prepared for presentation at Golden Rocks 2006, The 41st U.S. Symposium on Rock Mechanics (USRMS): "50 Years of Rock Mechanics - Landmarks and Future Challenges.", held in Golden, Colorado, June 17-21, 2006.

ABSTRACT: Remote sensing technologies are increasingly applied in the geological and geotechnical engineering. Especially the 3D imaging technology provides important information about the geometry of rock faces and the structure of the rock mass. 3D images provide a dense grid of measurement points combined with a digital image aligned with the surface topography. This contribution presents an approach for kinematical analyses of rock blocks based on discontinuity data gained from 3D images.

In a review on block kinematics translational and rotational block displacements, the kinematical mode analysis, as well as translational and rotational failure modes are discussed. It is highlighted that a kinematical analysis comprising translational and rotational failure modes requires the determination of the block geometry. The data required for the determination of the block geometry is gained from 3D images. Two 3D imaging systems based on photogrammetry and computer vision are described. The application of 3D imaging technologies for kinematical analysis is outlined in two illustrative examples. The JointMetrix3D system provides information of discontinuities in a rock slope for a study of block kinematics. The second example shows 3D imaging in an underground mine using the ShapeMetriX3D system. Discontinuity data from face images are sampled to a trace map which serves for the identification of kinematically free blocks.

1. REVIEW ON BLOCK KINEMATICS IN JOINTED ROCK MASSES

1.1. General

Kinematics basically deals with the position and orientation of systems in space and their variation with time. It does not consider the loadings on a system which cause changes in the position and orientation. Kinematics is also called the geometry of spatial relationships and movements [1]. A system has a kinematical freedom if displacements of the entire system are possible at the same instant in at least one direction. In a kinematical analysis of rock blocks displacements relative to the block boundaries are investigated. The determination of displacements follows the assumption of rigid body dynamics, i.e. no relative displacements between two points of the block are allowed.

The established methods for block analysis also include the kinematical mode analysis [3] [4]. The mode analysis results in the determination of the failure modes of a block exposed to a load system. In contrast to the above shown definition of kinematics loadings are considered in the kinematical mode analysis. The failure mode describes how a block can fail but it says nothing if the block will fail at all. A complete block analysis finally includes a stability analysis allowing to take into account strength and deformation properties of the discontinuities involved in the failure mode. In this contribution the analysis of block kinematics and kinematical failure modes is described. Discussion of stability analysis is not part of this paper.

Failure of rock blocks is the dominating failure mode in blocky rock masses in low and intermediate stress regimes [2]. A blocky rock mass is characterised by strong rock compared to weak discontinuities, where the behaviour is dominated by the properties of the discontinuities. Failures of rock blocks occur only in the case that all of the following criteria apply:

(i) Discontinuities intersect and form finite blocks. Finite blocks are formed if discontinuities from different sets continue sufficiently wide into the rock mass and intersect. The persistence of discontinuities can be increased by rupture of rock bridges and crack coalescence.

(ii) The finite blocks have a kinematical freedom to displace.

(iii) The resultant force and moment acting on the block promote failure modes.
In accordance with the principles of rigid body dynamics the acceleration of the block must comply with the kinematical freedom to admit block displacements.

(iv) The resistance of the discontinuities is exceeded.
Discontinuities cannot support the stress imposed by the displacements. Failure of discontinuities is separation, shearing, or a combination of both.

The kinematical analysis ((ii), (iii)) is an integral part in the stability assessment of rock blocks and block assemblies. Only kinematically free blocks which exhibit a failure mode merit stability analysis. The omission of kinematical analysis can also lead to the application of wrong load systems on the blocks and misinterpretations in the stability analysis [5].

1.2. Kinematical movability

If one focuses on the determination of the kinematical freedom of a block, one has to distinguish between translational and rotational motion. Analysing translations the displacement of a block can be described by a single vector. A block is termed removable, if there are kinematically admissible displacements causing lifting from or sliding on joint planes. Theories for the analysis of the kinematical conditions of blocks have been developed by Goodman & Shi [3] and Warburton [4]. A conclusion of these works is that only the orientation of the joints and the free surface is required for the determination of the kinematical movability of a block.

In the case of rotations the displacement of a block cannot be described by one single vector but rather by displacement vectors at the block corners. Kinematically admissible displacements at the corners must not cause interpenetrations of the block with the surrounding rock mass as mentioned by Mauldon [6]. This is assured if and only if the initial displacement vectors at each corner simultaneously form part of the corner's joint pyramid. Corner displacements are in general related to a rotation axis and rotation sense. In order to assess the corners' displacement vectors it is necessary to know the spatial relationships between the rotation axis and block corners. In other words, a complete kinematical analysis of block rotations requires determination of the block geometry. Several authors addressed the analysis of block rotations [7] [8]; however, a general solution of rotational kinematics of tetrahedral blocks was originally formulated by Mauldon & Goodman [9] [10]. In their formulation they assumed that the rotation axis passes through a block edge or corner at the free surface. Remote positions of the rotation axis from the block are not considered. Mauldon & and Goodman's method for the analysis of block rotations has been recently extended to general block geometries [11].

1.3. Kinematical mode analysis

For the kinematical mode analysis the unconstrained initial displacements of the block exposed to the load system have to be determined. In the initial stage of block movements the orientations of the displacement vectors coincide with those of the acceleration vectors. Unconstrained accelerations are calculated for the block completely separated from the surrounding rock mass and with the load system transferred to the block's centre of gravity. The transformation of the load system results in general in the active wrench resultant consisting of a resultant force and moment. For a resultant force alone the unconstrained accelerations are parallel to the force direction resulting in translational displacements. In this case the force direction can be used for the determination of the failure mode. According to block theory three failure modes are distinguished. The realisation of the analysis is outlined in [3] [4]:

- Falling/Lifting: The resultant force tends to open all joints simultaneously.
- Single face sliding: The resultant force causes a normal compressive stress on only one joint while the remaining joints tend to open.
- Double face sliding: The resultant force causes a normal compressive stress on two and only two joints while the remaining joints tend to open.

Any other force direction leads to stable conditions.

General loading conditions (wrench resultant) can additionally result in a rotational failure mode and require an extended approach. Tonon [12] derived expressions for the kinematical mode analysis of tetrahedral blocks under general loading conditions. These expressions include the law of the conservation of angular momentum. As consequence, the angular acceleration due to the moment is not parallel to the moment, but both are related by the mass moment of inertia of the block. The directions of the initial displacements of the block therefore depend on the geometry of the block. For loading conditions resulting in a rotational failure mode they have to cause a reaction at the corner or edge where the rotation axis passes through. The following rotational failure modes are distinguished:

- Free corner rotation: In the unconstrained motion the block has to tend against the joint planes forming the corner where the rotation axis passes through, while at the remaining corners at the free surface the block has to tend into the free space.
- Free edge rotation: In the unconstrained motion the block has to tend against the joint plane containing the rotation edge, while at the remaining corners at the free surface the block has to tend into the free space.

These two failure modes address motions of blocks where all joint planes simultaneously open (comparable to the lifting mode for translations). Sliding rotation modes also can occur if certain conditions apply. They are referred to as

- Torsional sliding [13]: Initial displacements cause a compressive reaction on two joint planes, comparable with the free corner rotation. In contrast one joint plane remains entirely in contact. The rotation axis is forced to be parallel to the blockside normal vector of the contact joint plane, thus, if it complies with the conditions on rotatability, this failure mode takes place.

1.4. *Input parameters for kinematical analyses*

As pointed out above, the analysis of block kinematics requires input parameters which provide information about the spatial relationships between joints, as well as joints and free surfaces. For general block motions it is not sufficient to consider only orientations of joints and free surfaces, but the entire geometry of the block is required. The knowledge of the block geometry allows determining

- directions of initial displacements at block corners related to a rotation axis,
- the mass and the mass moment of inertia of the block,
- the component of horizontal loading in case of seismic accelerations, and
- the forces which do not act through the centre of gravity of the block.

The determination of the block geometry requires information about the location, orientation, curvature, and size of the surfaces delimiting the block. Since a block is an intersection of halfspaces, it has to be defined which halfspaces of the surfaces form part of the block.

2. 3D IMAGING TECHNOLOGY

2.1. *The contribution of metric 3D images to rock engineering*

If a detailed analysis of failure mechanisms in jointed rock is required, the properties of the joints, especially location, orientation and size, have to be determined. Conventional geological field mapping for acquiring these data has several drawbacks:

- Position and orientation of discontinuities can only be determined for joints which can be accessed by the geologist.
- Commonly, the only measured geometrical property is the orientation.
- Orientation is only measured at single locations and does not necessarily represent the mean discontinuity orientation.
- It is difficult to focus on intensive data acquisition while keeping an eye on the global rock mass structure (Overview problem).
- In order to obtain a statistically sufficiently large number of samples considerable time and effort has to be spent in the field mapping.
- Visual estimation and the classification of either qualitative as well as quantitative data highly depend on the competency and experience of the geologist.

Since traditional field mapping relies on evaluation of visual information, the application of imaging technologies is evident to support the discontinuity parameter identification. 3D imaging technology is a powerful tool for geological and geotechnical engineering to overcome drawbacks of traditional field mapping in context with the determination of discontinuity parameters. 3D imaging is a remote sensing technology providing a metric three-dimensional topography of a surface (triangulated point cloud) combined with image information. A digital image is aligned with the 3D surface. The result is referred to as a 3D image.

2.2. Applied 3D imaging systems

In rock engineering currently two 3D imaging methods are used to obtain 3D models of rock faces. The one is based on images only using methods from photogrammetry, image processing and computer vision (for instance Gaich *et al.* [14]), while the other combines laser scanning and imaging (for instance Donovan *et al.* [15]). In this study systems with an image-based approach have been applied. The measurement systems are JointMetriX3D and ShapeMetriX3D. Both are 3D imaging systems using high-resolution cameras and conventional SLR cameras, respectively. According to the specifications of the systems the fields of application differ as described below.

Figure 1: Panoramic line scanner with camera unit, motor, control notebook, power supply and tripod. Note that the scanner is several hundreds of metres apart from the imaging area. Images were taken during scanning a landslide near Ashcroft, BC, Canada.

JointMetriX3D uses high-resolution digital panoramic images from which referenced, metric 3D images are generated. A high-resolution panoramic image scanner (Figure 1) is used to take two images sequentially, and the so-called Shape-from-Stereo principle applies for reconstruction of the 3D surface of an object [14]. In order to get a relationship between the resulting 3D image and the environment at least three reference points are used which must be visible in the panoramic image.

The system includes all advantages of an image-based documentation approach, but has additionally a wide operational area, since the base length between the images can be chosen freely allowing working distances from below one metre up to more than 1,500 metres. The image height and width can be individually adjusted to the shape of the investigated rock face because the vertical field of view is decoupled from the horizontal field of view. Image sizes up to 100 Megapixels are possible allowing for an identification of fine details even in a large rock face. A prototype was developed during a joint research initiative at the Graz University of Technology [16]. JointMetriX3D was finally developed by 3G Software & Measurement.

In contrast to the high-resolution scanner, the ShapeMetriX3D system uses conventional calibrated SLR cameras for image recording [17]. It is an easy-to-use measurement system without the need of surveying skills. A metric 3D image can be obtained only by installing a range pole within the imaging area. The range pole provides information about scale, verticality and reference to north. It should be noted that such a 3D image is embedded in a local coordinate system. If a referenced 3D image is required, surveying of at least three non-collinear reference points or the range pole is necessary.

2.3. Geological and geotechnical information obtained from metric 3D images

The irregular topography of the rock surface is used for geometric measurements of the rock mass structure. The accurate alignment of the topography with one of the images of the stereoscopic image pair allows for the visual identification of discontinuities. It facilitates especially the geological survey of discontinuity properties. The structures are directly marked on the 3D image using the purpose-built software component JMX Analyst. Using this software it is possible to define discontinuity traces (Figure 2) and areas (Figure 3) including their orientation, as well as measuring single orientations, distances, and coordinates [14] [17]. It promotes the determination of the completely visible discontinuity system at the rock face.

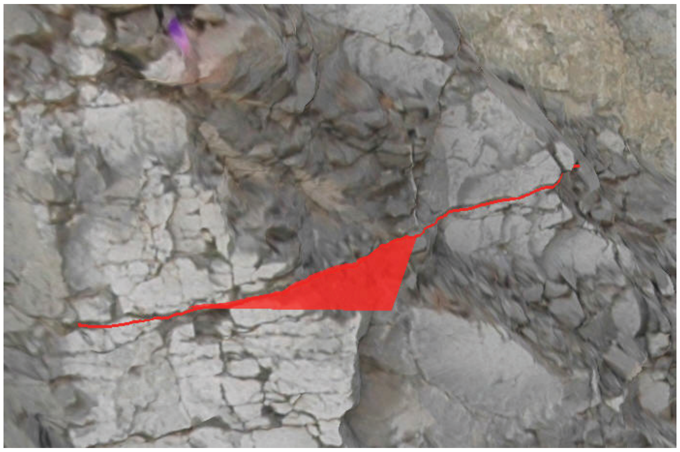

Figure 2: Joint trace at an outcrop. The joint orientation is determined by fitting a plane through the spatial joint polygon. It is represented by a triangle.

Figure 3: Joint plane as an area. The joint orientation is determined as the mean orientation of the surface elements enclosed by the polygon. It is represented by the upward normal vector.

The determination of the discontinuity system based on one or more 3D images at a certain instant in time represents the conditions of the currently visible surface. In order to improve the knowledge about the discontinuity system the rock mass structure is tracked during excavation by frequent application of 3D imaging. This is usually done during tunnel excavation by means of face mapping, either traditionally or by 3D imaging. The size of discontinuities, as well as their shape, orientation and condition can be objectively traced. For slope design based on data from natural rock slopes it is usually impossible to obtain more detailed data during the design stage. Nevertheless, the development of the discontinuity system can be observed during construction. In contrast, at mining pits the frequent and systematic monitoring of discontinuities allows establishing a detailed model of the discontinuity system for the walls. This allows for fast and rigorous stability assessments.

3. DETERMINATION OF BLOCK GEOMETRY

The discontinuity data derived from 3D images of rock faces is used to determine

- the discontinuity system at the rock face,
- the geometry of the excavation surface if the investigated excavation is represented by the 3D image,
- the trace map at the excavation surface, and
- the geometries of finite blocks adjacent to the excavation surface.

Pötsch *et al.* [18] proposed to derive a geometrical model from 3D images. The geometrical model contains all relevant information about the geometry of the rock face, as well as the rock mass structure. Basically, the geometrical model is an approximation of the irregular rock mass geometry with planes. If the investigated excavation geometry is represented by the 3D image, the establishment of the geometrical model is evident. The model geometry is derived from the 3D image topography. In design practice it is common that the excavation surface does not yet exist. The model geometry has to be separately introduced. In this case also the discontinuity system for the geometrical model has to be predicted. Depending to project-specific requirements the prediction is supported by outcrop data (3D imaging) and core drillings [19]. Statistical methods for modelling a spatial fracture network can be useful if these methods properly describe the properties of the discontinuity system.

The geometrical model is a trace network on the design excavation and is used for the determination of finite blocks at the excavation surface. The finite blocks are identified by means of a search algorithm along the trace network. Fundamental works in rock engineering have been presented by Chan [20] and Lu [21]. The algorithms find closed polygons in the trace network. These polygons represent the free surface of finite and infinite blocks. The algorithms are currently extended to meet the requirements for identification of maximum finite blocks at general excavation geometries.

From trace maps infinite and finite blocks can be distinguished. According to block theory there are two classes of finite blocks, tapered and removable blocks. Finite blocks identified from trace maps alone are simultaneously removable. Tapered blocks are formed from identified infinite blocks if a plane cuts through the joint pyramid of the infinite blocks at the rock mass side of the excavation, and it does not intersect the excavation surface within the polygon at the excavation surface. Once the polygons corresponding to finite blocks have been identified, the planes belonging to each trace are intersected to calculate the block geometries. Following assumptions apply:

- Discontinuities have no curvature.
- Although discontinuities have a limited size at the excavation surface, for calculation of the block geometry they continue sufficiently wide into the rock mass to avoid rock bridges.

4. KINEMATICAL STUDY OF A SUPERFICIAL ROCK BLOCK

The presented outline for kinematical analysis has been applied for a study of a superficial rock block in a quarry slope. Data acquisition has been performed using the JointMetriX3D system including the high-resolution panoramic line scanner. The quarry has a maximum height of approximately 150 metres and a width of around 400 metres. The average dip angle of the slope is about 67°. The quarry was recorded two times from different locations, each time in one scan. Resolution was maximised by adjusting the applied focal length to the slope height. The quarry has been recorded from an average distance of about 650 metres from the face. Figure 4 shows the stereo geometry during imaging.

From the two acquired high-resolution images a 3D image was reconstructed. This 3D image has a topography with more than 1,000,000 measurement points and an aligned image with an image size of 92 Megapixels (Figure 5).

By visual inspection of the 3D image and evaluation of major discontinuities an isolated rock block with a potential for rotation is encountered. All the necessary data for the determination of the geometry have been derived from the 3D image using the evaluation software JMX Analyst. Figure 7 shows the discontinuity survey of the block on the 3D image. The obtained block geometry consists of seven planes, three joint planes and four free surface planes. It was assumed that the bottom boundary of the block is parallel to the bedding planes. Although open joints could not be observed at the bottom end of the block, the bedding planes represent planes of weakness and contribute to the kinematical freedom of the block. Table 1 shows the orientations of the block planes. Note that the irregular geometry of the free surfaces is modelled by planes defined by the orientations and locations derived from the 3D image.

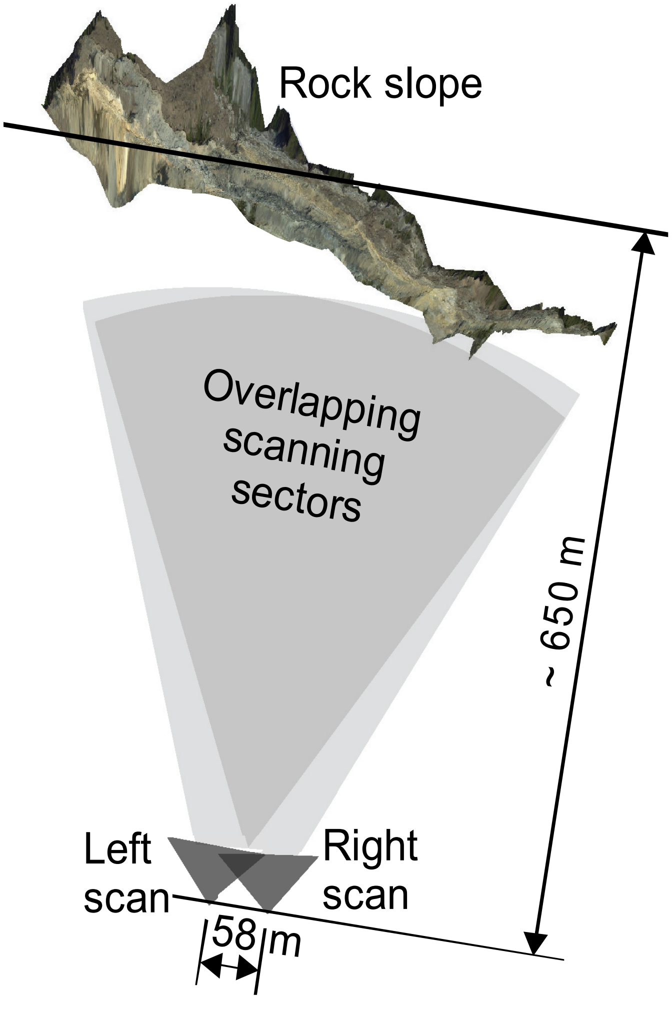

Figure 4: Stereo geometry of 3D imaging showing the rock slope and the two locations of the panoramic line scanner. The scanner locations can be freely chosen.

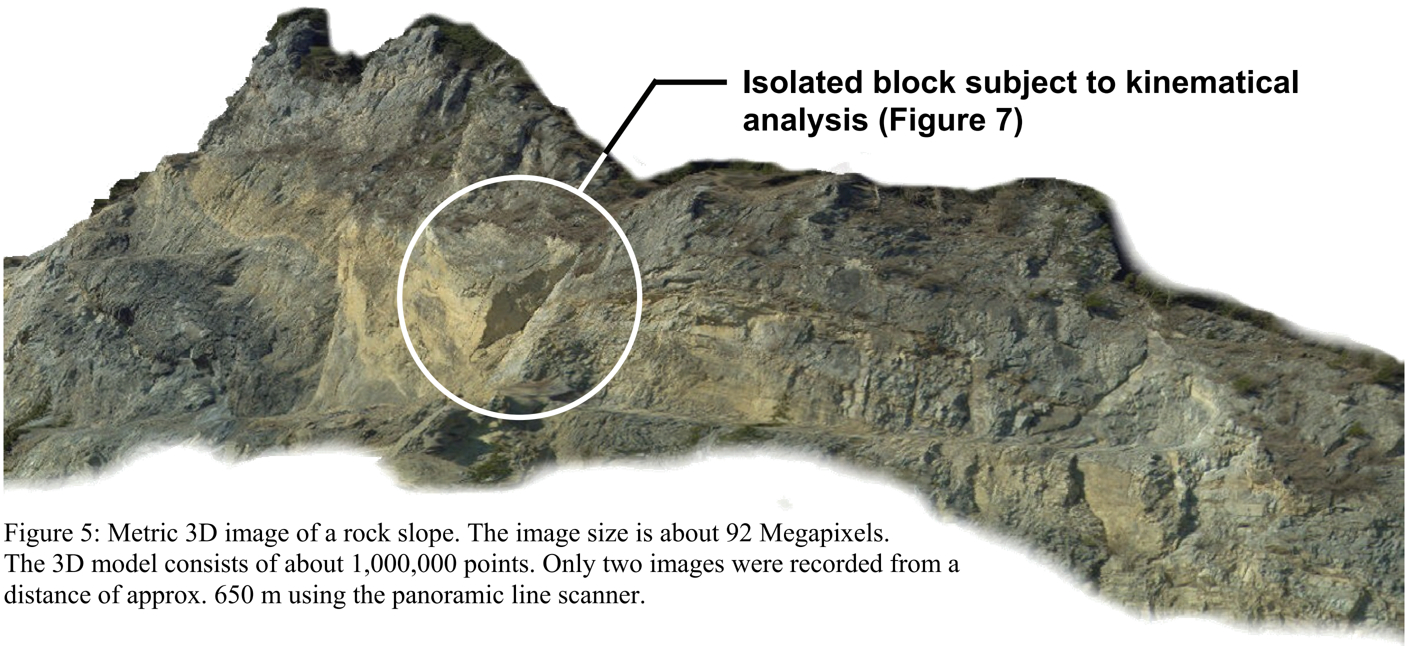

Figure 5: Metric 3D image of a rock slope. The image size is about 92 Megapixels. The 3D model consists of about 1,000,000 points. Only two images were recorded from a distance of approx. 650 m using the panoramic line scanner.

Table 1: Orientation of joints and free surfaces of the block

No.		Dip Direction [°]	Dip angle [°]
1	Free surface	351	75
2	Joint	346	67
3	Joint	232	66
4	Free surface	221	85
5	Free surface	194	67
6	Free surface	81	33
7	Joint (Bedding plane)	120	33

According to the kinematical constraints the block was identified to be removable and rotatable (Figure 6). Removable means that there are directions of translational motion possible without interpenetration of the rock mass. For a block to be removable its joint pyramid must entirely plot outside the excavation pyramid [3]. Rotatable means that there are rotation axes located at the intersection polygon joint – free surface which allow for rotational motion without interpenetration of the rock mass. Directions of rotation axes passing

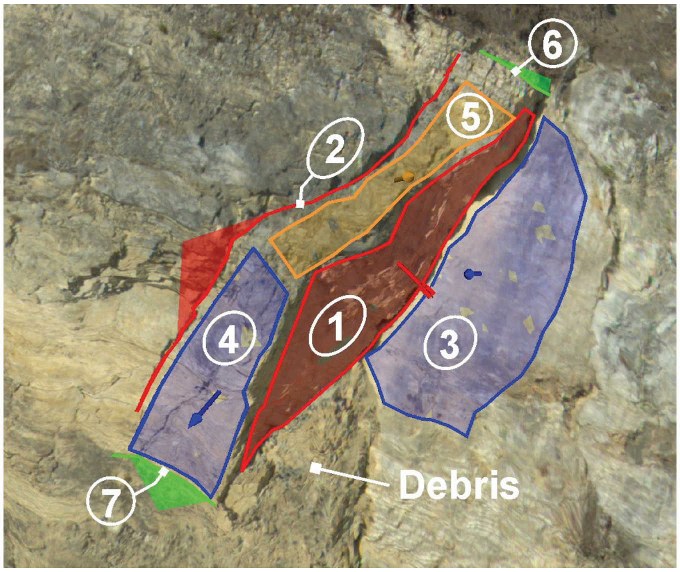

Figure 7: Survey of joints and free surfaces of a rock block on a 3D image. The numbers refer to planes described in Table 1.

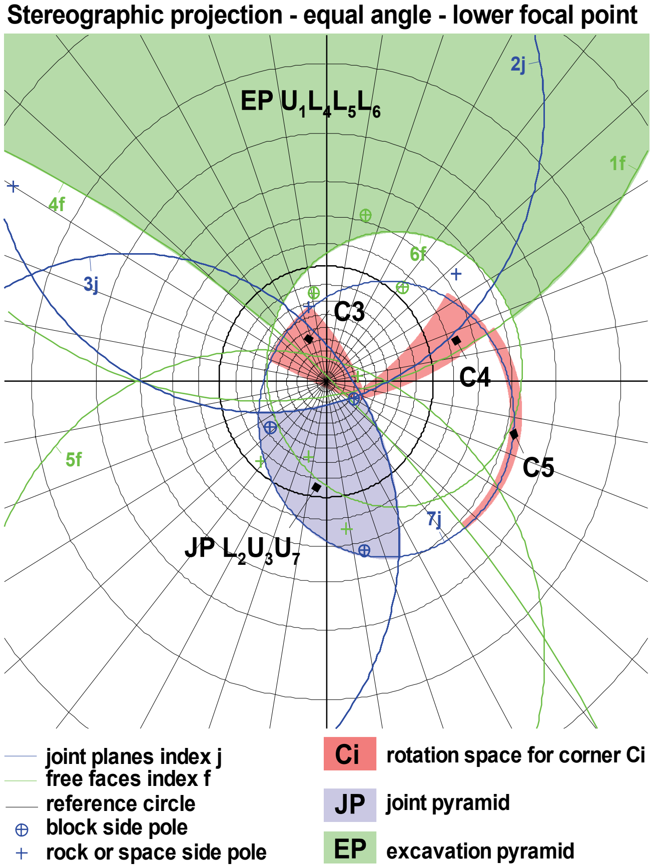

Figure 6: Stereographic projection of joints and free faces of the block in Figure 7. It shows also the joint pyramid, excavation pyramid and rotation spaces for corners defined in Figure 8. Analysis after [3] and [11].

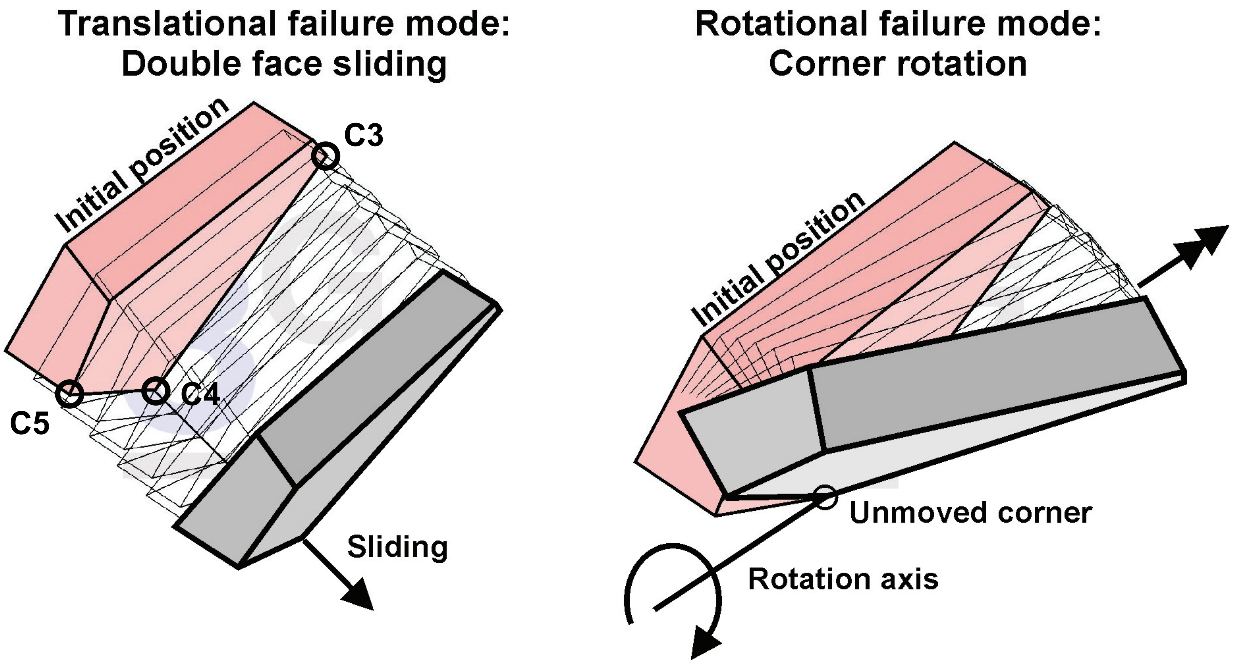

Figure 8: Failure modes of the rock block from Figure 7 under gravity loading. The translational failure mode is double face sliding. The rotational failure mode is corner rotation.

through a block corner, and allowing for kinematically feasible displacements define the rotation space. A block is called rotatable if its rotations space is not empty. Figure 6 shows the rotation spaces for three corners defined in Figure 8.

Two failure modes could be identified for gravity loading. The translational failure mode is sliding on two joint planes (Figure 8). This failure mode obviously is not critical, however, without further stability considerations no objective conclusion could be made about the most critical failure mode. Apart of the translational failure mode a corner rotation mode (Figure 8) could also be identified. From the kinematical point of view, it can be stated that there is a rotational failure mode leading to failure of the block.

5. APPLICATION FOR UNDERGROUND EXCAVATIONS

The application of the ShapeMetriX3D system for acquisition of information about the rock mass structure is shown for a drift excavation in an underground mine. The data was gained during an interdisciplinary study including remote sensing in underground excavations, blast engineering and rock mechanics [22]. The following paragraphs focus on the discontinuity characterisation and sampling using 3D images.

The basic data acquisition included the sequential recording of drift faces using the ShapeMetriX3D system. The 3D images of the faces typically have around 100,000 measurement points. This results in a point density of about 2.0 – 2.5 points / centimetre which is considered to be sufficient for rock engineering tasks. The general layout of the imaging locations is shown in Figure 9 for a drift width of 8 m. The distance of the imaging locations to the face depends on the size of the cross section and the available focal lengths of the camera. Note that the imaging locations not necessarily need to be

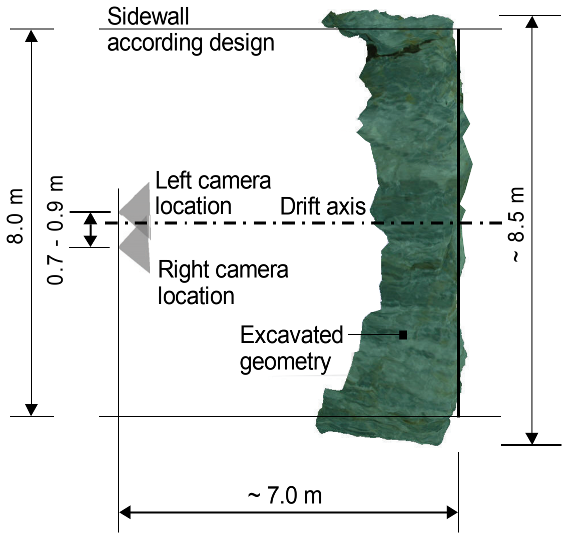

Figure 9: Camera locations during 3D imaging at the drift face (plan view). The camera locations can be freely chosen. It is not necessary to put the camera on a tripod nor to survey the camera position.

in the drift axis but can be freely chosen. This significantly facilitates the application at the site, since possible sight shadows due to obstacles (muck piles, equipment, etc.) can be easily avoided.

Although for the generation of a metric 3D image it would not have been necessary, several reference points have been surveyed by means of a total station. The use of reference points was exclusively required for the establishment of an accurate relationship between different 3D face images. Twelve faces have been recorded during the time period of the study; however, seven only are shown in this paper. The corresponding drift length is approximately 33 metres. The intended round length is 4.5 metres. For the discontinuity evaluation, the faces were successively arranged according to the drift advance (Figure 11). The arrangement eases tracking of discontinuities and the identification of persistent joints. Three categories of discontinuities can be distinguished.

- Discontinuities with high persistence: Discontinuities can be tracked along several faces.
- Discontinuities with low persistence: Discontinuities are only visible in one face.
- Non-visible discontinuities: Discontinuities do not intersect the face. This can typically take place in the case of steeply dipping joints striking perpendicularly to the drift axis.

Figure 11: 3D images of a face successively arranged according to drift advance. The advance direction is from bottom-right to top-left. At the face images the rock mass structure, reference points and the bore hole pattern for the blasts can be observed. Note that the face of the third round has not been recorded.

For the identification of discontinuities from images by visual inspection they must have some peculiarities, such as an observable separation or a discolouration due to weathering. If those open or weathered joints are highly persistent, they significantly affect the kinematics around the drift. Joints with low persistence are important, as they decrease the rock mass strength and can cause local kinematical freedom. In engineering practice it is impossible to consider all joints having a low persistence. Nevertheless, they should be statistically introduced in order to assess their effects. Non-visible joints have to be considered in the discontinuity model, especially if they are highly persistent. Anyhow, other sources of information have to be used (for instance, mapping of the wall, evaluation of core drillings).

Table 2: Orientation of joints and drift advance

No.		Dip Direction [°]	Dip angle [°]
1	Joint01	169	58
2	Joint02	172	81
3	Joint03	167	43
4	Normal strike joint	289	68
5	Bedding planes	017	44
	Drift advance	088	-02

The sampling of discontinuities from face documentation and its peculiarities has been addressed by Pötsch *et al.* [23]. Figure 10 shows selected high-persistent discontinuities evaluated from the face images. The discontinuities belong to

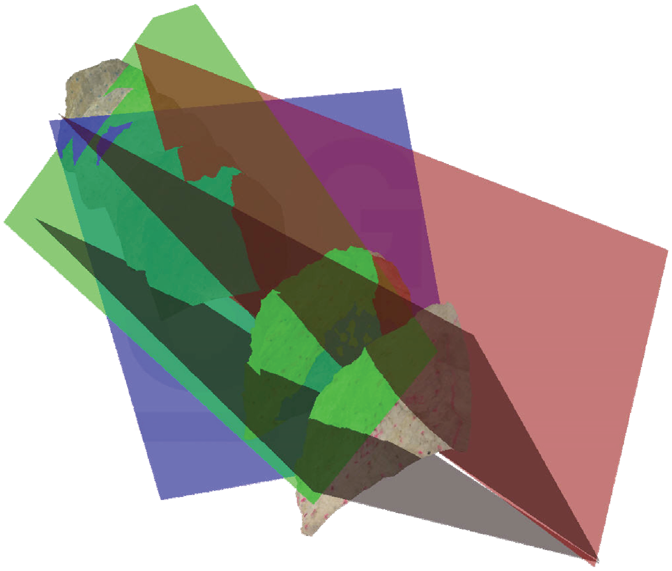

Figure 10: Faces from Figure 11 with high-persistent joints from different sets. Only selected discontinuities are displayed for reasons of visibility.

different sets. Table 2 shows the orientation of the selected discontinuity planes and drift advance.

The exemplary discontinuity system shown in Figure 10 has been used to determine a trace map on the drift wall. Discontinuities named as joints have been considered to be specifically located while the bedding planes due to their nature are considered to be closely spaced. Figure 12 displays the trace map on the design drift geometry including the second and fourth face image. The traces caused by intersection of the discontinuities and the drift geometry are drawn as polygons. Two removable blocks could be identified from trace map analysis. The block in the crown exhibits a lifting mode under gravity loading while the block in the sidewall has a sliding mode on one plane. Rotational failure modes have not been considered.

6. CONCLUSION

3D imaging technologies are increasingly used for data acquisition in rock engineering. They provide valuable information on the geometry of rock faces and the rock mass structure. It has been shown how these data are valuable input for kinematical analysis.

Basic concepts of kinematical analyses have been reviewed, especially discussing the kinematics of rock blocks. Kinematical analyses include the investigation of the movability and the kinematical mode analysis. The importance of the determination of the block geometry for the analysis of block kinematics has been pointed out. The block geometry dominates initial displacements as well as controls the eccentricity of loads. According to rigid body dynamics initial block displacements under general loading conditions can only be determined by knowing the properties of the rock block. Initial block displacements form the basis for determination of the failure mode.

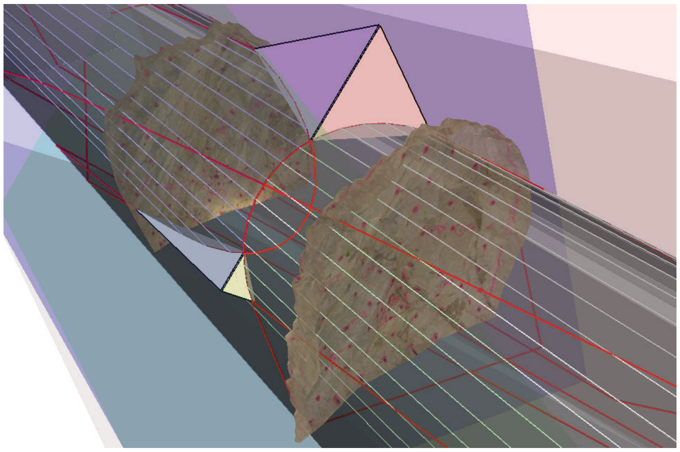

Figure 12: Identified kinematically removable blocks based on discontinuity sampling and 3D imaging. The blocks are located between the 2nd and 4th round. The discontinuity planes as well as the design drift geometry are shaded. The traces at the drift wall are displayed as polygons.

The determination of the block geometries is facilitated by data gained from 3D images. Two 3D imaging systems have been presented. For large rock faces (highwalls, valley rock slopes) the JointMetriX3D system with its high-resolution capabilities is typically applied. For benches or underground faces the ShapeMetriX3D system has been used. The evaluated data from 3D images is processed into a discontinuity system. The discontinuity system is intersected with the excavation geometry forming a trace map. This trace map serves for identification of finite blocks adjacent to the free surfaces.

With two illustrative examples the application of the different 3D imaging systems has been presented. It has been shown how the data have been used for kinematical analyses of blocks.

ACKNOWLEDGEMENT

The authors would like to thank Mr Matthias Wimmer for performing face imaging at the underground marble mine of Sterzing/Italy.

REFERENCES

1. Magnus, K. and H. H. Müller. 1990. *Grundlagen der Technischen Mechanik.* Stuttgart: Teubner.
2. Kaiser, P. K., M. S. Diederichs, C. D. Martin, J. Sharp, and W. Steiner. 2000. Underground Works in Hard Rock Tunnelling and Mining. In *Proceedings of the Int. Conf. on Geotechnical & Geological Engineering – GeoEng2000, Melbourne, 19 – 24 November 2000,* ed. M. C. Ervin.
3. Goodman, R. E. and G.-H. Shi. 1985. *Block Theory and Its Application to Rock Engineering.* New Jersey: Prentice-Hall.
4. Warburton, P. M. 1981. Vector Stability Analysis of an Arbitrary Polyhedral Rock Block with any Number of Free Faces. *Int. J. Rock Mech. Min. Sci. & Geomech. Abstr.* 18, 415-427.
5. Luo, H. Y. and W. Zhou. 2005. A Three-Dimensional Method for Multiple Rock Block Stability Analysis and Its Application in Dam Abutment and Slope Stability Assessment. In *Proceedings of the 40th U.S. Rock Mechanics Symposium – Alaska Rocks 2005, Anchorage, 25 – 29 June 2005,* eds. G. Chen et al., Paper No. 05-725.

6. Mauldon, M. 1992. *Rotational failure modes in jointed rock: A generalization of block theory.* PhD dissertation, University of California, Berkeley.

7. Londe, P., G. Vigier, and R. Vormeringer. 1969. Stability of rock slopes, a three-dimensional study. *J. Soil Mech. and Found. Div., ASCE,* 95(1), 235-262.

8. Wittke, W. 1984. *Felsmechanik.* Berlin: Springer Verlag.

9. Mauldon, M. and R. E. Goodman. 1990. Rotational Kinematics and Equilibrium of Blocks in a Rock Mass. *Int. J. Rock Mech. Min. Sci. & Geomech. Abstr.* 27(4), 291-301.

10. Mauldon, M. and R. E. Goodman. 1996. Vector Analysis of Keyblock Rotations. *J. Geotech. Engineering* 122(12), 976-987.

11. Pötsch, M. and W. Schubert. 2006. Rotational kinematics of rock blocks with arbitrary geometries. *Felsbau Rock and Soil Engineering* 24(3), in press.

12. Tonon, F. 1998. Generalization of Mauldon's and Goodman's Vector Analysis of Keyblock Rotations. *J. Geotech. Geoenviron. Engineering* 124(10), 913-922.

13. Goodman, R. E. 1995. Block theory and its application. The Rankine Lecture. *Géotechnique* 45(3), 383-423.

14. Gaich, A., M. Pötsch, A. Fasching and W. Schubert. 2004. Measurement of rock mass parameters based on 3D imaging. In *Caratterizzazione degli ammassi rocciosi nella progettazione geotecnica. X Ciclo di Conferenze di Meccanica e Ingegneria delle Rocce, Turin, 24 – 25 November 2004,* eds. G. Barla and M. Barla, 21-45, Bologna: Patron editore.

15. Donovan, J., J. Kemeny and J. Handy. 2005. The application of three-dimensional imaging to rock discontinuity characterization. In *Proceedings of the 40th U.S. Rock Mechanics Symposium – Alaska Rocks 2005, Anchorage, 25 – 29 June 2005,* eds. G. Chen et al., Paper No. 05-745.

16. Gaich, A., A. Fasching and W. Schubert. 2003. Improved site investigation – Acquisition of geotechnical rock mass parameters based on 3D computer vision. In *Numerical Simulation in Tunnelling.* ed. G. Beer, 13-46, Vienna: Springer-Verlag.

17. Gaich, A., M. Pötsch and W. Schubert. 2006. Acquisition and assessment of geometric rock mass features by true 3D images. In *Proceedings of the 41st U.S. Rock Mechanics Symposium – Golden Rocks 2006, Golden, 17 – 21 June 2006,* eds. N.N., Paper No. 06-1051, under preparation.

18. Pötsch, M., W. Schubert and A. Gaich. 2004. Computer based stability analysis of rock slopes in a blocky rock mass. In *Proceedings of the ISRM Regional Symposium Eurock 2004 & 53rd Geomechanics Colloquy, Salzburg, 7-9 October 2004,* ed. W. Schubert, 431-434.

19. Pötsch, M., W. Schubert and A. Gaich. 2005. Application of metric 3D images of rock faces for the determination of the response of rock slopes to excavation. In *Proceedings of the ISRM Regional Symposium Eurock 2005, Brno, 18-20 May 2005,* ed. P. Konečný, 489-497.

20. Chan, L.-Y. 1987. *Application of block theory and simulation techniques to optimum design of rock excavations.* PhD Dissertation, University of California, Berkeley.

21. Lu, J. 2002. Systematic identification of poyhedral rock blocks with arbitrary joints and faults. *Computers and Geotechnics* 29, 49-72.

22. Wimmer, M. 2006. *Optimisation of the Drill and Blast Work in the Underground Marble Mine Sterzing of Omya S.p.A.* Diploma Thesis, Department of Mineral Resources and Petroleum Engineering, Montanistic University of Leoben, Austria.

23. Pötsch, M., W. Schubert and A. Gaich. 2005. From documentation to prediction – a computer-based method for discontinuity analysis and block stability assessment in tunnelling. In *Proceedings of the ITA-AITES 2005 World Tunnel Congress & 31st General Assembly, Istanbul, 9-11 May 2005,* eds. Y. Erdem and T. Solak, 1059-1063.