NOVEL ANALYTICAL SOLUTIONS OF THE REACTION FIELDS OF ARBITRARY ORIENTATED SPHEROIDS IN A HOMOGENEOUS MAGNETIC FIELD EXPRESSED IN CARTESIAN COORDINATES

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Target audience: In the current work novel analytical solutions capable to compute field inhomogeneities induced by introducing spheroids with arbitrary symmetry axis into a homogenous field are presented. Thus, facilitating the modelling of MR signal formation and spectral line broadening in the vicinity of biological structures such as spongy bone and capillary networks. The authors believe that these findings are of major interest to the scientific community which are exploring and applying susceptibility phenomena.

Purpose: In the field of MRI the use of spheroids has attracted considerable interest as a block to model and predict static magnetic fields around biological structures [1]. MR-Osteodensitometry utilizes inhomogeneous magnetic fields within trabecular bone to gain information about its mechanical competence [2]. Generally, infinite cylinder models are applied to study the field properties of such osseous networks. Due to their spatial constraint of infinite extent this approach is not capable of assessing the fields in the vicinity of micro cracks. Alternatively, spongy bone can be modelled as arrays of spheroids, in conjunction with their analytical solutions derived in spheroidal coordinates, e.g. see [3]. Thus, structures of interest have to be necessarily modelled in the spheroidal system or it is required to subsequently transfer the computed field values into the desired coordinate system.

To overcome these restrictions we derived novel analytical solutions, where the magnetic fields for prolate and oblate spheroids are entirely expressed in Cartesian coordinates and applicable for arbitrary directions of both the symmetry axis and of the field axis. Additionally, we demonstrate their applicability through computing the impact of field inhomogeneities onto T2' along trabecular rarefaction within a simplified 3D vertebra model.

Methods: The solutions were derived from the potential and field of the spheroid in a given uniform magnetic field [4]. Expressing the resulting Legendre polynomials by elementary functions, and applying transformation equations we obtained the solutions in Cartesian coordinates. The general case with arbitrary symmetry axis **n** was derived by decomposing the field vectors into components parallel and normal to **n**. This description is rather concise; details may be found in [5-6]. To evaluate the field distribution within the spongy bone a two-compartment model, consisting of marrow and bone, was utilized. Prolate ellipsoids were appropriately arranged within a 3D unit cell to mimic the trabecular micro structure (Fig 1a.). The impact of the disturbances on the signal course of the free induction decay was analyzed by computing the magnitude signal according to [7] and approximating the resulting transversal decay time T2' by means of a mono-exponential function.

Results: The resulting reaction fields H_{r1} pre- and post bone rarefaction are depicted in Fig.1b. Within the unit cell the initial field distribution ranged around ±1 A/m, whereby micro fractions induced inhomogeneities of ±1.5 A/m. When evaluating the variations in terms of T2', the modelled cracks gave rise to a decrease of the initial T2' of 26.1 ms to approximately 14.4 ms, see Fig. 1c.

Conclusion: The novel analytical expressions were successfully applied to the field of MR-Osteodensitometry, proving its advantage that it is very easy to model and investigate structures built from spheroids with different axes and positions. There is no need of complicated coordinate transformations. The novel expressions make it possible to study bone rarefaction along osteoporosis, whereby either cracks of the horizontal, the vertical or arbitrary structures are accessible for modelling. Possible further applications are the computations of induced reaction fields in the surrounding of vascular networks, or of magnetic distortions for techniques using the proton resonance frequency shift method.

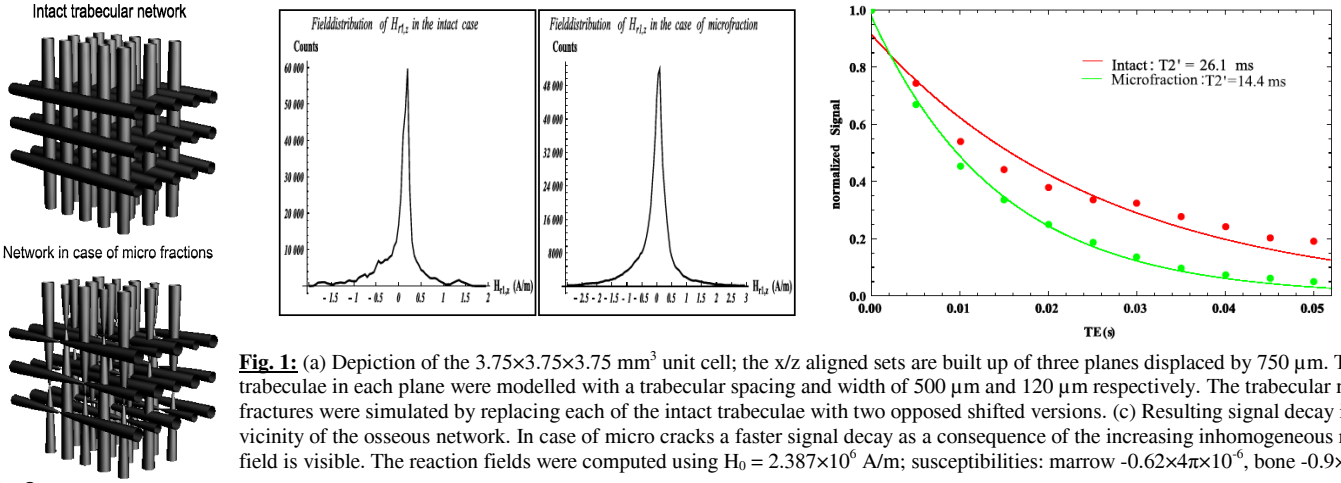

Fig. 1: (a) Depiction of the 3.75×3.75×3.75 mm^3 unit cell; the x/z aligned sets are built up of three planes displaced by 750 µm. The trabeculae in each plane were modelled with a trabecular spacing and width of 500 µm and 120 µm respectively. The trabecular micro fractures were simulated by replacing each of the intact trabeculae with two opposed shifted versions. (c) Resulting signal decay in the vicinity of the osseous network. In case of micro cracks a faster signal decay as a consequence of the increasing inhomogeneous reaction field is visible. The reaction fields were computed using $H_0 = 2.387\times10^6$ A/m; susceptibilities: marrow $-0.62\times4\pi\times10^{-6}$, bone $-0.9\times4\pi\times10^{-6}$.

References: [1] Sukstanskii and Yablonskiy, JMR 151: 107-117 (2001), [2] Wehrli et al.: NMR in Biomed 19: 731-764 (2006), [3] Landau and Lifshitz: Pergamon Press (1960), [4] Kuchel and Bulliman.: NMR in Biomed 2:151-160 (1989), [5] Kraiger and Schnizer to appear in COMPEL (2012), [6] Kraiger and Schnizer: http://itp.tugraz.at/~schnizer/MedicalPhysics/, [7] Selby et al., JMRI 6:549-559 (1996)