Tyre Dynamics: Model Validation and Parameter Identification

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Abstract The present paper deals with the experimental validation of tyre dynamics approaches as it is widely applied in tyre models for vehicle dynamics and handling. Firstly it gives a brief derivation of two modelling principles regarding the deflection velocity in the considered direction of the tyre's deformation. This is than followed by a brief description of the performed measurement procedure. From the measurements, a set of model parameters of the considered tyre, depending on different manoeuvre speeds and frequencies, is identified, where no particular fitting parameters for the tyre dynamics are needed. Based on these model parameters, the related dynamic simulations are carried out. The comparisons show that the applied first-order model describes the behaviour quite well within a certain operation range, whereas the second-order approach cannot deliver better results in spite of the longer computational time. However, for investigations within an enlarged frequency range of the steer input and at high slip angles, a more detailed model is recommended.

Keywords Tyre dynamics modelling · Semi-physical model · Tyre testing · Vehicle dynamics · Handling

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Introduction

Modelling and simulation of safety relevant Driver Assistance Systems (DAS) and Vehicle Dynamics Controllers (VDC) which act in standard and limit situations lead to increasing accuracy demands in the description of dynamic reactions of tyre contact forces, e.g. (Hirschberg et al. 2000; Lex and Eichberger 2011). For that purpose, first-order approaches are widely applied in this field of vehicle dynamics and handling, which originate from (Schlippe and Dietrich 1942), were modified by (Pacejka 2006) and later on refined by Rill (2012). This approach is typically characterised by the first-order differential equation

$$\tau_{x,y}\dot{F}^D_{x,y} + F^D_{x,y} = F^S_{x,y}, \tag{1}$$

where the superscripts D and S distinguish between dynamic and static tyre forces and the subscripts x and y indicate the longitudinal and lateral directions of the tyre forces F. However, the coefficient τ which corresponds to a relaxation length is not constant, but depends on the wheel load F_z and the tyre slips s_x and s_y respectively.

The line of modelling of the visco-elastic mechanism of tyre deformation is the key of a proper description of τ. Based on previous researches, e.g. (Hackl et al. 2015), the scope of the present paper is to investigate two different approaches for the above mentioned modelling method. One is the modelling principle regarding to the semi-physical tyre model TMeasy (Hirschberg et al. 2007; Rill 2012) and the other one, named here TMmass, refers to (Pacejka 2006). This is done by comparison of selected simulation results with corresponding measurement data from an extensive laboratory testing programme. With the aim to run vehicle dynamics models on uneven, but not rough roadways, a frequency range of at least 4 Hz was considered. Due to the research project's current focus on lateral vehicle dynamics, the evaluation of the correspondent relations in longitudinal direction will be dealt with on a later occasion.

Modelling Aspects

A common model approach to dynamic tyre forces and torques takes the compliance of the tyre in lateral, longitudinal, and circumferential directions into account, Fig. 1. According to (Pacejka 2006), a mass representing an appropriate part of the tyre belt in contact is considered too.

Then, the dynamic lateral force F^D_y is modelled by

$$F^D_y = c_y y_e + d_y \dot{y}_e, \tag{2}$$

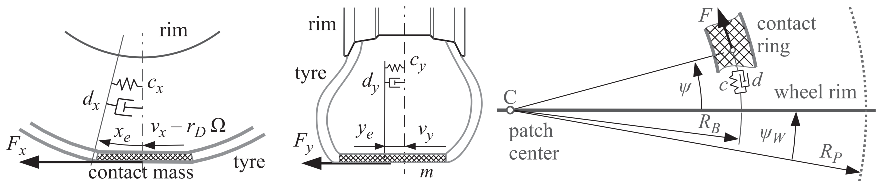

Fig. 1 Tyre deflection in the longitudinal, lateral and circumferential directions

where $y_e, \dot{y}_e$ follow from

$$m\ddot{y}_e + d_y\dot{y}_e + c_y y_e = F_y^S\left(v_y + \dot{y}_e\right). \tag{3}$$

Within this approach m denotes the corresponding belt mass, c_y and d_y are the stiffness and damping properties of the tyre, and F_y^S describe the lateral tyre characteristics evaluated at the dynamic sliding velocity $v_y + \dot{y}_e$. This model is named tyre model with mass TMmass for further investigations in the present paper under numerical solution of Eq. (3).

The corresponding belt mass is supposed to be quite small, and may therefore be neglected in favour of computational efficiency and reduced model parameterisation effort. In case of neglecting the belt mass, $y_e, \dot{y}_e$ then follow from

$$d_y\dot{y}_e + c_y y_e = F_y^S\left(v_y + \dot{y}_e\right), \tag{4}$$

representing a first-order tyre dynamics model, which is used within the TMeasy tyre model. As shown in (Rill 2006, 2012; Hackl et al. 2015), the implicit first order differential equation is analytically expanded from $F_y^S\left(v_y + \dot{y}_e\right)$ into a Taylor Series. In contrast to Eq. (3), the formulation in Eq. (4) finally leads to a closed description of τ to be used in Eq. (1). In this article, the difference between TMmass as described in Eq. (3) and the approach within TMeasy as described in Eq. (4) will be compared.

Test Bench Setup and Tyre Measurement Programme

Measurements have been carried out on a brake and suspension test rig, which was developed for investigation of durability and fatigue of components of quarter vehicle suspensions, (Harrich et al. 2006). For the present experiments, the wheel assembly consisting of tyre, rim and wheel carrier were mounted to the test bench using a rigid suspension. No spring and damper elements were attached between wheel and test bench.

The test bench has a drum with an outer diameter of 1.219 m. It can be pivoted around the vertical axis to generate a slip angle α between ±15 deg with a maximum

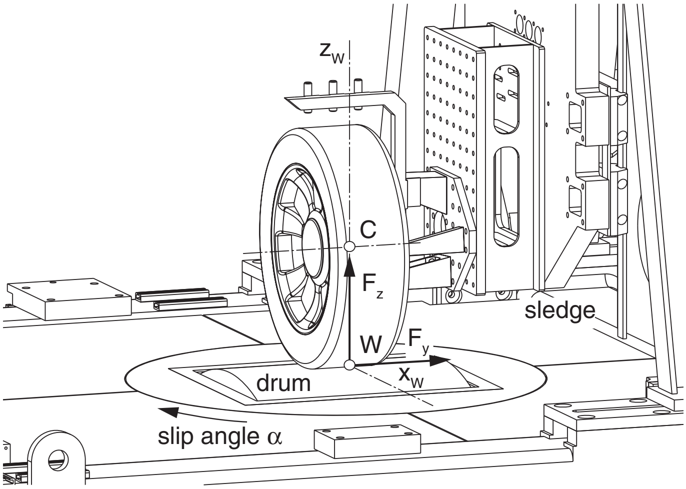

Fig. 2 Test bench setups to parametrise and validate the non-linear first-order tyre dynamics approach

rotational velocity of 25 °/s, see Fig. 2. The drum speed can be varied between 0 and 1,300 rpm. The vertical tyre load is set using a vertical hydraulic cylinder with a maximum cylinder force of 25 kN. Depending on the weight of the test assembly and the required travel range, a maximum pulse frequency of 35 Hz can be achieved with this cylinder. The maximum actuator speed is 1.1 m/s, the maximum actuator acceleration is 10 m/s^2.

The resulting forces and torques in the wheel hub are measured using a high-precision Kistler (2015) wheel force transducer (WFT), applied on a radial tyre size of 205/55 R16. Three non-contact temperature sensors were used to measure the tyre surface temperature directly after the outlet of the tyre contact patch. The internal tyre pressure was set at operational temperature of 20°. In addition, the drum steer angle α, the drum roll speed Ω_{drum}, the vertical travel z_C of the wheel carrier and the ambient temperature during the tests are measured. All signals discussed in the following were recorded at a sample rate of 1 kHz.

Tyre Measurement Manoeuvres

To investigate the non-linear tyre dynamics and validate the characteristics of the lateral spring c_y and damper d_y properties, two different measurement programmes are defined. The first is used to parametrise the steady state lateral tyre characteristics which are implemented in the tyre models and also needed for the optimisation, see Sect. "Steady State Tyre Characteristics". Secondly, high dynamic manoeuvres are performed to validate the non-linear tyre dynamics and examine the

Table 1 Overview of the measured conditions to parametrise and validate the non-linear tyre model behaviour

Tyre load F_z	3,600	N
Tyre pressure p	2.75	bar
Sine slip angle amplitude α	1, 2, 4 and 6	°
Sine slip angle frequency f	0.125, 0.25, 0.5, 1, 2 and 4	Hz
Drum circumferential speed v_x	20, 60, 100 and 130	km/h

influences on different drum speeds and frequencies on the lateral spring and damper properties. To cover vehicle dynamics on uneven roadways, a frequency range up to 8 Hz is supposed to be covered by the model.

To fulfill all requirements and include the physical limits of the test bench, a sine slip angle input with different frequencies and amplitudes and a constant normal force F_z was used for both measurement programmes. In detail, a frequency range in steps from 0.125 to 4 Hz and an amplitude range of the slip angle α from 1 to 6° were carried out. With the target of keeping a constant tyre load, a hydraulic pump with a maximum force of $F_{z,\max} = 10{,}000$ N is installed. A side slip-frequency-drum speed-matrix, with respect to the manoeuvre ranges, was set up for the parametrisation.

Table 1 gives a summary of the measured conditions used in present paper. For a detailed list of performed measurements see (Hackl et al. 2015).

Steady State Tyre Characteristics

To validate the dynamic behaviour depending on the frequency and wheel speed of the tyre, a steady state tyre model is needed. This basic tyre model is parametrised with step steer inputs during constant drum speed of 60 km/h, described in (Hackl et al. 2015). Starting from this basic tyre model, the steady state characteristics have been adapted using quasi-stationary sine manoeuvres with a frequency of 0.125 Hz, speed of 60 km/h and different slip angle amplitudes listed in Table 1. This was necessary because the tyre showed a changed behaviour due to tyre wear during these tests.

As mentioned in the literature, cf. (Rill 2012), two main characteristics are changing during the lifetime of a tyre. The main change of characteristics is an increase of the lateral stiffness because of the reduced tyre tread depth. Secondly, the force maximum is moving to a smaller slip angle value. These two factors are especially important for manoeuvres performed with a slip angle α less or equal to 6°, which have been used for these investigations.

The values of the lateral spring c_y, lateral damper d_y and the model mass m, all described in Sect. “Modelling Aspects”, are used from (Hackl et al. 2015) and (Pacejka 2006). Together, with the parameters of the steady state tyre characteristics adapted to the tire wear, these parameters are summarised in Table 2. As seen in

Table 2 Parameters to describe the lateral steady state tyre characteristics with $F_{z,Nom}$ = 3,600 N, for a parameter definition see (Rill 2012), and the dynamic model parameters as described in Sect. "Modelling Aspects"

Initial slope	*dfy0*	75,600	N/-
Maximum force	*fym*	4,000	N
Slip s where f(s) = fm	*sym*	0.1125	–
Sliding force	*fys*	3,700	N
Slip s where f(s) = fs	*sys*	0.5	N/-
Frictitious velocity	v_N	0.01	m/s
Lateral spring init	c_{y0} (Hackl et al. 2015)	126640.6	N/m
Lateral damper init	d_{y0} (Hackl et al. 2015)	1770.7	Ns/m
Contact mass init	m_0 (Pacejka 2006)	1	kg

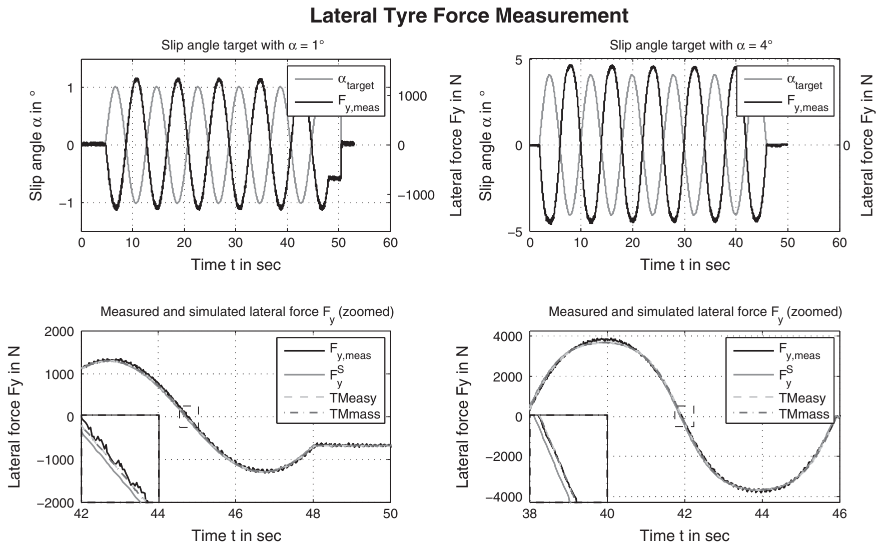

Fig. 3 Validation of the steady state tyre characteristics by comparing measurements $F_{y,meas}$ to simulations with a steady state model, the dynamic TMeasy model and the enhanced dynamic model TMmass. A quasi-stationary sine manoeuvre with a sine frequency f = 0.125 Hz, tyre load F_z = 3,600 N, drum speed v_x = 60 km/h and two different slip angle amplitudes α = 1° (*left*) and α = 4° (*right*) were used

Fig. 3, a validation with two different slip angle amplitudes was carried out, an amplitude target of α = 1° shown in the left and with α = 4° shown in the right column.

On the upper two figures, the slip angle target is shown with a constant tyre load F_z = 3,600 N, drum speed v_x = 60 km/h, and the measured lateral Force $F_{y,meas}$. In the figures below, the comparison between measured and simulated force is presented. It can be seen that there is nearly no difference between the simulation with the tyre model TMeasy and the enhanced mass model TMmass, described in

Sect. "Modelling Aspects". There is just a small delay between the steady state tyre characteristics F_y^S and the two dynamic models, which are traceable to the dynamic behaviour also on just a small frequency.

Comparison and Validation of the Simulation with Measurement Data

In this main part of the paper, the simulation with the two tyre models, both described in Sect. "Modelling Aspects", is presented and compared with measurement data. The principal focus was to investigate the influences of different frequencies and drum speeds on lateral spring c_y and damper d_y properties. Therefore, an optimiser was used to minimize the least squares error between the two models and the manoeuvres, with respect to the two parameters. In addition, these two models were compared with respect to optimisation/calculation time and accuracy.

To find the minimum between the measurement and the simulation deviation, a stochastic method called *Particle Swarm Optimisation* based on (Eberhart and Kennedy 1995) and (Kennedy and Eberhart 1995) with an extension from (Liu et al. 2005) was used. In addition, for all optimisations a constant number of iteration steps and swarm particles were used.

Variation of the Frequency

The first part of the validation describes the influences of the frequency f. Therefore, manoeuvres with a constant speed v_x = 60 km/h, slip angle amplitude of $\alpha = 2°$ and a frequency range from f = 0.125–4 Hz were applied.

Figure 4 shows an example of the optimisation with a frequency f = 1 Hz. On the upper left subfigure, the slip angle target α_{target} and the measured lateral force $F_{y,meas}$ is shown. On the two right subfigures, the good agreement between the simulated and measured lateral force is presented with different zoom factors. Also the behaviour between the steady state and the dynamic characteristics can be seen in the below right subfigure. A second representation, with the lateral deflection versus the lateral force, is presented in subfigure four, as can be seen left below, to show the small deviation between the two models.

In Table 3, the average, the maximum and minimum results from ten independently executed optimisations for a manoeuvre frequency of f = 1 Hz are written. The biggest difference between the tyre model TMeasy and TMmass is seen in the calculation time by an increase of up to about 350 % for the model TMmass. Therefore, the model with the enhanced mass may be theoretically more accurate, but at a cost of higher calculation time. In both cases the simulation was

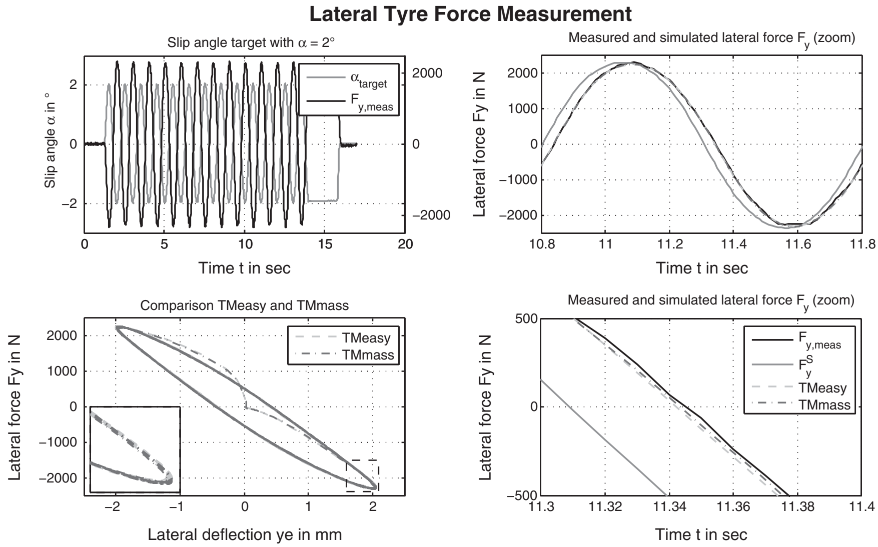

Fig. 4 Comparison of the two optimised tyre models TMeasy and TMmass using measurements of a sine manoeuvre with a sine frequency f = 1 Hz, tyre load F_z = 3,600 N, drum speed v_x = 60 km/h and a slip angle amplitude α = 2°

Table 3 Parameter results of the optimisation from the manoeuvres with a sine frequency f = 1 Hz, tyre load F_z = 3,600 N, drum speed v_x = 60 km/h and a slip angle amplitude α = 2°

	c_y in N/m	d_y in Ns/m	m in kg	t_{sim} in % (approx.)
Tyre model TMeasy				
Minimum value	131.454	1.072	–	–
Average value	132.166	1.316	–	100
Maximum value	133.204	1.489	–	–
Tyre model TMmass				
Minimum value	123.707	1.230	1	–
Average value	124.348	1.327	1	350
Maximum value	125.499	1.442	1	–

obtained by using the Matlab-Solver ode15 s that applies implicit multi-step formulas with step-size and order control.

Regarding the spring characteristics c_y, it can be see that in both tyre models, the three values are in a quite small range and it can be assumed that a good minimum was found. But the damper characteristics, especially in the tyre model TMeasy, show a higher deviation. Therefore, the influences of the two parameters with respect to the optimisation result were investigated separately.

Using the initial parameters from Table 3, and changing the spring characteristics with ±25 % and the damper characteristics by multiply and dividing by 5, a parameter study with the tyre model TMeasy was done. The results of this parameter study, presented in Fig. 5, shows that the influence of the damper value is quite smaller compared to that of the spring, which mirrors the results from Table 3. Comparing the deviations of the different simulations shows that a small change of the spring causes a much higher change in the lateral force characteristics than changing the damper properties. The tyre model with the enhanced mass showed the same behaviour, and thus is not presented. To minimize the influence of the damper on optimisations and focus on the spring properties, a constant damper value during the sine frequency variation was further assumed.

The results presented in Fig. 6 show the optimised spring parameter c_y for different frequencies. The solid line represents the average and the grey hatching the whole area of results of the carried out optimisations for the two different tyre models.

The main aspect shown in this figure is the increasing behaviour of the spring characteristics which reflects the results from (Hackl et al. 2015) that there is an influence of the frequency on the dynamic tyre behaviour. It seems there is a degree of dynamic hardening of the lateral stiffness with decreasing frequency. To investigate this effect, a more detailed model is needed.

Summarized, in this subsection it was shown that the spring parameter influences the dynamic behaviour to a higher degree than the damper. In addition, it was presented that the spring properties of the used models are not constant with respect to the frequency. Both models can describe the behaviour of the lateral force for a fixed frequency with optimised parameters.

It was shown that the inclusion of the mass may theoretically bring a more accurate calculation of the dynamic lateral force, but, like within the tyre model TMeasy, it does not solve the measured influences of different frequencies on the

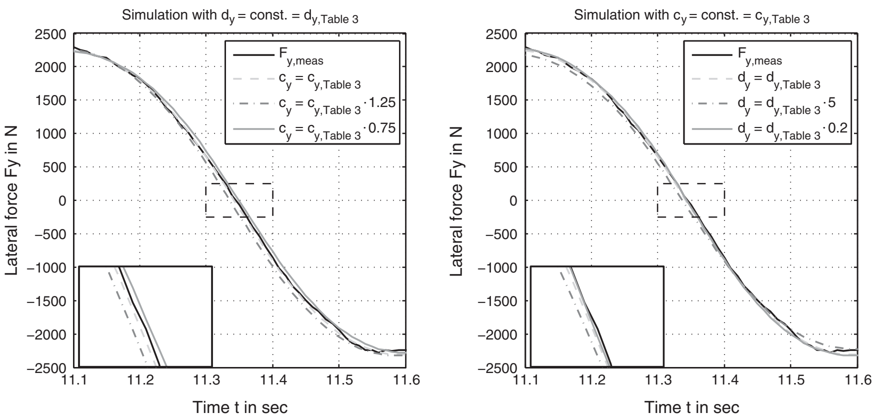

Fig. 5 Comparison of measurement and simulation results by using different values for spring parameter c_y (*left subfigure*) and damper parameter d_y (*right subfigure*) values on the sine manoeuvres with a sine frequency $f = 1$ Hz, tyre load $F_z = 3{,}600$ N, drum speed $v_x = 60$ km/h and a slip angle amplitude $\alpha = 2°$

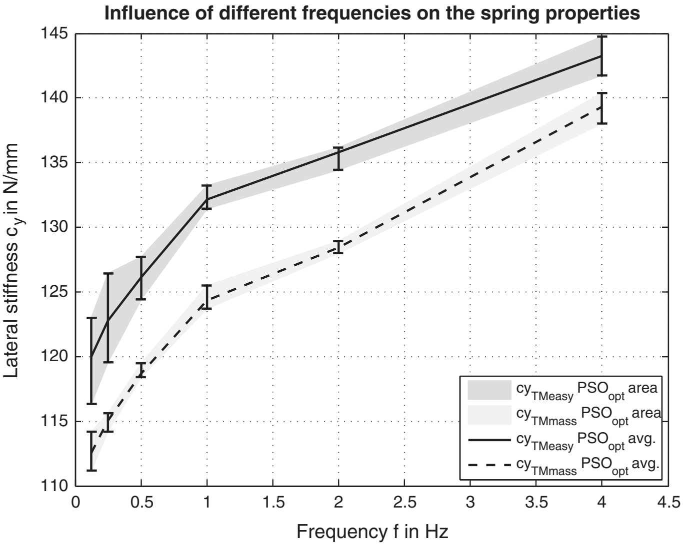

Fig. 6 Results of the influence of different sine-slip angle frequencies on the spring characteristics c_y with a manoeuvre with constant tyre load $F_z = 3{,}600$ N, drum speed $v_x = 60$ km/h and a slip angle amplitude of $\alpha = 2°$

spring characteristics. To describe these effects with constant parameters a more detailed model is needed. Another aspect between the two models is the different calculation time, which should not be neglected, especially with regard to real time applications.

Variation of the Speed

To validate and with the aim to enhance a tyre model to describe the dynamic behaviour in more detail, the influence of the speed should be additionally investigated. Therefore, the influence of different drum speeds from $v_x = 20$–130 km/h on the lateral spring and damper properties are investigated and presented in this chapter. Starting from the results in Table 3, the drum speed was varied. In Fig. 7, the results of the drum speed variation is presented. Just three drum speeds are presented because of a measurement error during the 100 km/h measurements.

It can be seen that both the lateral stiffness and the lateral damper properties increase with a higher drum speed. It has to be mentioned that the damper characteristics of the tyre model TMeasy for a drum speed of 130 km/h seems slightly too high and should be checked with further measurements. It is also shown that the results of the optimisations are quite equal to that of the investigations on the

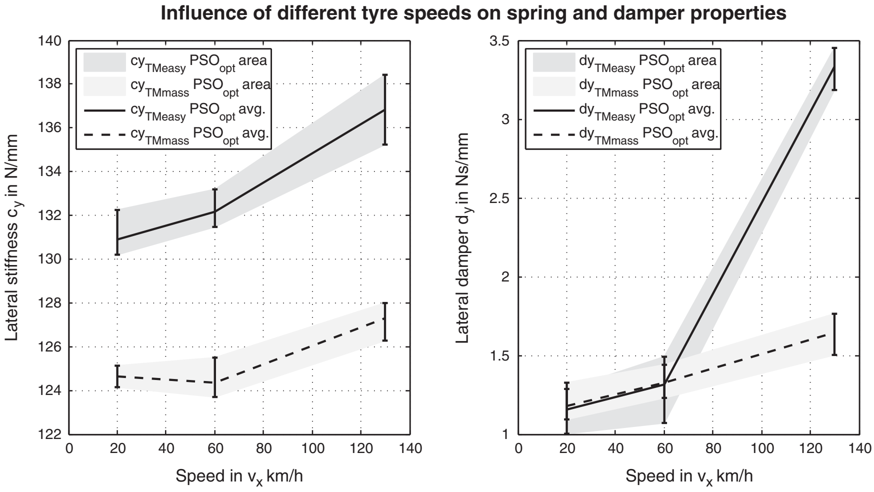

Fig. 7 Results of the influence of different drum speeds v_x on spring c_y and damper d_y characteristics with a sine manoeuvre under constant tyre load $F_z = 3{,}600$ N, a slip angle amplitude of $\alpha = 2°$ and a slip angle frequency $f = 1$ Hz

frequency dependency. The deviations of the spring properties are in a smaller range in comparison to the damper.

In summary, it is presented that there is an influence of the speed on the parameters that characterise the dynamic tyre behaviour. Both models are not able to handle the influence without adapting the model parameters to the speed. In the near future, new measurement manoeuvres with a higher range of speed values and smaller steps between the values are planned to investigate the influence in more detail.

Mass Variation in the Enhanced Tyre Model TMmass

In Sects. "Variation of the Frequency" and "Variation of the Speed" it was shown that an enhanced model is needed to describe the influence of the frequency and the speed. In the last part of this article, the mass variation of the enhanced model is considered. The goal was to investigate the influences of the mass parameter on the spring and damper properties and validate this approach for further research in the area of tyre dynamics. Therefore, the influence of varied mass $m = 0.1$–10 kg on the lateral tyre force behaviour is done with respect to the optimised spring and damper characteristics for a fixed manoeuvre. For this manoeuvre, a constant speed $v_x = 60$ km/h, tyre load of $F_z = 3{,}600$ N, slip angle amplitude of $\alpha = 2°$ and a frequency $f = 1$ Hz was used.

In Table 4 and Fig. 8, the results for different mass variations are shown. On the left side, the influences on the spring, and on the right on the damper characteristics

Table 4 Parameter average results of the optimisation with different model masses with a sine manoeuvre and sine frequency $f = 1$ Hz, tyre load $F_z = 3{,}600$ N, drum speed $v_x = 60$ km/h and a slip angle amplitude of $\alpha = 2°$

	m in kg	c_y in N/m	d_y in Ns/m	t_{sim} in % (approx)
Enhanced model with mass TMmass				
	0.1	124.3826	1.3157	385
	0.5	124.3182	1.2611	385
	1	124.3480	1.3270	350
	2	124.4516	1.3479	315
	10	124.3585	1.6930	260

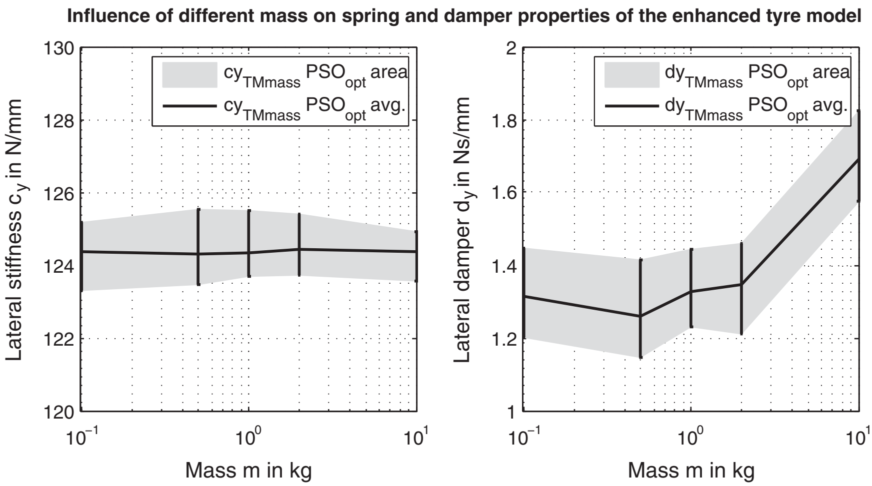

Fig. 8 Results of the influence of model mass in the enhanced tyre model TMmass on spring c_y and damper d_y characteristics using a sine manoeuvre with constant tyre load $F_z = 3{,}600$ N, a slip angle amplitude $\alpha = 2°$ and a slip angle frequency $f = 1$ Hz

is presented. As can be seen on the left, there is nearly no influence on the spring characteristics. Regarding the damper characteristics, it seems to slightly increase with a higher mass. The larger deviation of the damper in the optimisation is attributed to the smaller influence of the damper behaviour on the lateral force behaviour compared to the spring.

Conclusion

The present paper investigates the tyre dynamic approaches as they are widely applied in tyre models for vehicle dynamics and handling simulation. Firstly, two modelling principles are briefly described which consider different methods to obtain a proper description of the tyre's relaxation behaviour, particularly depending on the tyre load and the amount of the acting slip. Therein, the tyre stiffness and the viscous damping parameters are included in the relaxation functions. How these effects are implemented distinguishes the tyre model TMeasy from an alternative approach, named here TMmass, which includes an additional mass for the description of the visco-elastic deformation of the tyre in the contact area. This represents a second-order approach for the computation of the tyre deflection. However, in order not to exceed the extent, only the lateral relations were considered in this article.

An extended measurement programme on a test bench under laboratory conditions was carried out to validate both the above mentioned modelling approaches. The testing conditions are briefly documented, where particularly sine inputs on the slip angle with different amplitudes and frequencies under different tyre speeds were applied. Advantageously the results shown in the paper are restricted to a nominal tyre load of 3,600 N.

The essential model parameters in tyre dynamics are the tyre stiffness and damping with respect to the particular direction of deformation. Both these properties are carefully identified using measurement results. As already previously mentioned, their variability with respect to the excitation frequency, amplitude and rolling speed does not allow linear modelling, but requires more detailed models for stiffness and damping. Also, the hyper-elasticity seems to be worth to be taken into consideration.

In addition, the results of this investigation demonstrate that the additional model mass introduced in TMmass, which is considered in the calculation of the tyre's deformation velocity, only weakly influence the resulting model accuracy. Summing up, one may conclude that this artificial mass, which significantly extends the computational effort, may not have real relevance for the mentioned dynamic tyre modelling.

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