## PREDICTION OF COMBUSTION NOISE OF A SWIRL-STABILIZED FLAME USING LASER INTERFEROMETRIC VIBROMETRY VALIDATED BY ACOUSTIC **MEASUREMENTS**

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## ABSTRACT

Modern gas turbines rely more and more on premixed combustion systems. While they produce less emission, they are more prone to combustion instabilities. The combustion noise emitted by turbulent swirl-stabilized flames can be calculated directly if density fluctuations in the flame are known as a function of time and space. Recently it was shown that laser interferometry records density fluctuations in the flame quantitatively. In this work a swirl-stabilized, rotationallysymmetric unconfined methane flame at lean operation conditions and low air mass flow rate was scanned by laser interferometric vibrometry (LIV) in order to calculate the overall sound power emitted by the flame. To validate the outcome calculated from the LIV data, sound power was also measured in a half-hemisphere by microphones, using pressurepressure-probes. These probes record the total sound power of the combustion noise emitted by the flame. To improve signal to noise ratio for this measurement, a siren was used to generate a reproducible excitation of the flame at 212Hz. Both measurement methods were in good agreement. With the LIV data detailed information about the local density fluctuations in the flame causing the sound emission was obtained. Also a preferred acoustic propagation direction between 40° and 80° to the burner axis in downstream direction was observed. This deviation from a uniform distribution is likely to be caused by temperature gradients in the flame. A discussion of systematic errors inherent to the LIV technique and data reduction concludes this publication.

#### NOMENCLATURE

Ameas	[m <sup>2</sup> ]	cross-section area of the laser		
		vibrometer beam		
$A_{\text{surf}}$	[m <sup>2</sup> ]	surface area of microphone		
		hemisphere		
co	[m/s]	speed of sound in the far field		

f	[Hz]	frequency			
FFT		fast Fourier transform			
G	[m³/kg]	Gladstone-Dale constant			
Ir	$[W/m^2]$	radial component of sound intensity			
Im		imaginary part of a complex number			
k	[mm/s/V]	vibrometer calibration constant			
L	[m]	optical path			
LIV		laser interferometric vibrometry			
lat		latitudinal coordinate			
lng		longitudinal coordinate			
'n	[g/s]	mass flow			
n	[]	refractive index			
Р	[W]	sound power			
pp-probe		pressure-pressure-probe			
P <sub>th</sub>	[kW]	thermal power			
p'	[Pa]	sound pressure			
PPM		perfectly premixed			
r	[m]	radial distance of observer			
$\mathbf{r}_0$	[m]	radius of flame			
SNR	[-]	signal to noise ratio			
t	[s]	time			
U	[V]	voltage			
Vr	[m/s]	radial component of particle velocity			
$V_{Fl}$	[m <sup>3</sup> ]	volume of the flame			
$\Delta r$	[m]	distance between microphones of pp-			
		probe			
v		radial direction			
Z		jet axis			
Øglobal	[-]	global equivalence ratio			
λ	[m]	wavelength			
$\rho_0$	[kg/m <sup>3</sup> ]	mean density outside of the flame			
ρ' <sub>T</sub>	[kg/m <sup>3</sup> ]	density fluctuation within the flame			
$\rho'(r)$	[kg/m <sup>3</sup> ]	density fluctuation at radius of			
		observer			
ζ	[m]	laser vibrometer beam-axis			

## INTRODUCTION

In thermal turbomachinery applications, combustion noise is not only a pollutant, but also causes thermo-acoustic oscillations in the combustion chamber. Low-NOx strategies include lean combustion in order to reduce the flame temperature. Together with high-power densities and reduced damping capabilities, lean combustion increases the vulnerability of these systems to thermo-acoustic oscillations. These are instabilities arising from the positive coupling between pressure and heat release. A good overview on the dynamics of swirl-stabilized flames - the type of flame commonly used in turbomachinery - is given by Candel et al. [1]. This research is part of an Austrian-German joint research project entitled "Full-field laser vibrometry for combustion diagnostics" to quantitatively validate the applicability of Laser Interferometric Vibrometry (LIV) for combustion research. Laser vibrometers are commercially available interferometers, used to detect surface vibrations without mechanical contact to the surface. They are also capable to record density fluctuations along the line-of-sight direction of the laser beam. Assuming rotational symmetry of the flame, local density fluctuations can be obtained by tomographic algorithms. A number of authors discuss the application of LIV in confined and unconfined flames (a first application of LIV in combustion research can be found in [2], [3] describes the recording of flame transfer functions in confined flames via LIV, [4] and [5] give further details on LIV in combustion diagnostics). Other authors are using microphones to detect pressure fluctuations [6], [7]. The underlying hypothesis to this paper is that LIV is capable to quantitatively predict the sound power emitted by a flame. According to combustion theory ([8] [9]) the second time derivative of density fluctuation integrated over the flame volume should result in a value which is proportional to the acoustic pressure fluctuations in the far field. Since LIV directly records the first time derivative of density fluctuations, a comparison of sound power calculated from LIV measurements to microphone measurements is a plausible method to validate the reliability of LIV data in a quantitative way. A former attempt by the authors to verify LIV recordings by microphone measurements [10] resulted in some discrepancies between the precise values of sound power calculated from LIV recordings, compared to the data obtained by a microphone array. In the meantime, these discrepancies were resolved mainly by scanning the flame with higher spatial resolution, enhanced signal processing and a new digital LIV system with higher sensitivity and enhanced accuracy in order to derive more accurate data from LIV recordings. Additionally the theory now corresponds to recent works concerning combustion noise [9], [11]. This publication will first present the theoretical background and relates density fluctuations integrated over the flame with the sound power in the far field. After presenting the test object - an unconfined swirl-stabilized methane-fired burner - the LIV and its signal processing are presented in detail. At the end a comparison between LIV and microphone recordings are discussed.

## THEORETICAL BACKGROUND

As suggested by Strahle [8] density fluctuations in the far field of a combustion process can be related to density fluctuations in the flame. This approximation holds true for the assumption of long acoustic wavelengths compared to the size of the combustion field, no preferred directionality in the far field and low aerodynamic noise. LIV directly records the time derivate of density fluctuations as integral values along the laser beam (line-of-sight measurement). By summing up all line of sight data of the measurement grid with respect to their phase, LIV should be capable to predict the sound power of a flame, since sound power is a function of the integral value of density fluctuations within the flame [8].

The relation between density fluctuations in the flame and the far field is given by [12]

$$\rho'(r,t) = \frac{1}{4\pi c_0^2 r} \frac{\partial^2}{\partial t^2} \int_{V_{Fl}} \rho'_T(r_0, t - \frac{r}{c_0}) dV(r_0)$$
(1)

with *r* radial distance of the observer,  $\rho'$  density fluctuation in the far field,  $c_{\theta}$  speed of sound at ambient conditions and  $\rho'_{T}$  the density fluctuations in the flame.

In the far field where particle velocity and sound pressure p' are in phase, sound power P can be calculated the following way:

$$P = \int I \, dA = 4\pi r^2 \frac{p'(r)^2}{\rho_0 c_0} = 4\pi r^2 \frac{(c_0^2 \rho'(r))^2}{\rho_0 c_0} \tag{2}$$

The factor  $4\pi r^2$  denotes a spherical control surface at a specified radius r, p' the sound pressure in the far field,  $\rho'$  the density fluctuation in the far field and  $\rho_0$  the density at ambient conditions. Combining (1) and (2) results in an equation relating sound power to the second time derivative of density fluctuations:

$$P = \frac{1}{4\pi\rho_0 c_0} \frac{\partial^2}{\partial t^2} \left( \int_{V_{Fl}} \rho'_T(r_0, t - \frac{r}{c_0}) dV(r_0) \right)^2$$
(3)

Using a Fast Fourier Transform (FFT), data is transferred into the frequency domain. The Fourier transform of a time derivative of a function corresponds to the transform of the function itself multiplied by the angular frequency and considering a phase lag of  $-\pi/2$ .

Summing up the sound power spectra from all line-of-sight measurement points recorded by LIV over all positions in the measurement grid (the flame area) leads to eq. (4). The following equation describes the sound power in the far field predicted by LIV.

$$P = \sum_{f} P(f) = \sum_{f} \frac{1}{4\pi\rho_{0}c_{0}} 2\pi f \left( \sum_{MP} \frac{4}{\pi} A_{meas} \int_{\zeta} \frac{\partial \rho'}{\partial t} (r_{0}, t - \frac{r}{c_{0}}, phase - \frac{\pi}{2}) dV(r_{0}) d\zeta \right)^{2}$$

$$(4)$$

with  $A_{meas}$  the measurement area of the laser beam and  $\zeta$  the integrated line-of-sight length of the laser beam. The factor  $4/\pi$  corrects the round shaped laser beam in a rectangular grid with scanning positions spaced by a distance equal to the laser beam diameter.

The characteristic acoustic spectrum of the methane-burner used in this investigation, has a frequency range of up to about 500Hz. Due to the long acoustic wavelength (low frequencies) it was not possible to meet the requirements of the far field condition in the experiment. In the acoustic near field sound pressure p' and the radial component of the particle velocity  $v_r$  are not necessarily in phase. Therefore it is preferable to record the total sound power with a microphone array and compare this number to the result obtained by LIV and eq. (4). The sound power P is calculated via the radial component of the sound intensity  $I_r$  and the surface of a hemisphere  $A_{surf}$ .

$$P = \int_{A} I_r dA_{surf} = \int_{A} p' v_r dA_{surf}$$
(5)

Since pressure is much easier to measure than particle velocity, the Euler equation is used to estimate the particle velocity [11]:

$$\frac{\partial v_r}{\partial t} = -\frac{1}{\rho_0} \frac{\partial p}{\partial r} \tag{6}$$

By measuring the sound pressure at two closely spaced positions in the acoustic field with two microphones facing each other, the particle velocity can be calculated as follows:

$$v_r = -\frac{1}{\rho_0} \int_t \frac{p_{r+\Delta r} - p_r}{\Delta r} dt \tag{7}$$

with  $\Delta r$  the distance between the two microphones. Finally the sound intensity in radial direction can be calculated:

$$I_{r} = p'v_{r} = -\frac{p_{r+\Delta r} + p_{r}}{2} \frac{1}{\rho_{0}} \int_{t} \frac{p_{r+\Delta r} - p_{r}}{\Delta r} dt$$
(8)

In this work pressure-pressure-probes (pp-probes) recorded the pressures in eq. (8). In the frequency domain sound intensity can be calculated with the imaginary part of the cross spectral density of the two pressures from the pp-probe [13]:

$$I_r(f) = -\frac{1}{\rho_0 2\pi f \Delta r} Im \big( FFT(p_{r+\Delta r}) \times FFT(p_r) \big)$$
(9)

#### THE SWIRL-STABILIZED METHANE-FIRED BURNER

The investigations presented in this paper were performed on an unconfined swirl-stabilized methane-fired burner at ambient conditions. The working principle of this burner is documented in detail by Giuliani et al. [14]. As shown by Peterleithner et al. [15] this burner has a rotational symmetry of the flame. Therefore only half of the flame was scanned via LIV. Because of the rotational symmetry the microphone measurements recorded only one segment of the hemisphere. In FIGURE 1 the experimental setup of the burner can be seen. For Perfectly Premixed (PPM) operation, methane was injected into the axial air feedline far upstream before entering the burner, to ensure a homogenous mixture of air and fuel. By the means of a stratifier (FIGURE 1, white) the axial flow was forced into an axial direction. While the tangential air entered the plenum trough 32 cylindrical bores aligned tangentially and symmetrical around the burner z-axis. This tangential flow generated a simplified swirl number of 0.52 according to Candel et al. [1].

Fuel and air mass flow rate were measured using caloric mass flow meters of the EL-FLOW series from Bronkhorst, Netherlands, with an accuracy of 0.6% full scale. The specific mass flow rates, global equivalence ratio and the thermal power are summed up in the following table, assuming complete combustion:

$\dot{m}_{ax}$	$\dot{m}_{tan}$	$\dot{m}_{cooling}$	$\dot{m}_{fuel}$	Ø <sub>global</sub>	$\mathbf{P}_{\mathrm{th}}$
[g/s]	[g/s]	[g/s]	[g/s]	[-]	[kW]
0.424	0.378	0.969	0.068	0.66	3.44

Table 1: Investigated operation point of the methane burner

In this configuration the flame stayed detached from the center cone (FIGURE 1, blue) in a stable way.

A siren [16] was used to modulate the axial airflow shortly before entering the burner by rotating a disc with rectangular shaped teeth, blocking or releasing the full cross section area of the nozzle at a certain rate. The blocking and the releasing sections on the disc were designed with the same angular spacing. By choosing a perturbation frequency of 212Hz, both measurement techniques recorded a sharp peak of thermoacoustic oscillations at this frequency. Thus it was easier to compare the quantitative results of the LIV and the microphone measurements, since the underlying noise band was low compared to the excitation.

The whole test rig was set up in a thermoacoustic laboratory in a 3x3x2.5 m<sup>3</sup> box with a sound absorbing ceiling and two layers of low reflective curtains.



FIGURE 1: EXPERIMENTAL SETUP OF THE BURNER FOR PERFECTLY PREMIXED OPERATION OF THE SWIRL-STABILIZED FLAME; AXIAL AIR AND FUEL ARE ALREADY MIXED BEFORE ENTERING THE BURNER; AXIAL AIRFLOW IS MODULATED WITH A ROTATING DISC (SIREN) SHORTLY BEFORE ENTERING THE BURNER. THE FLAME IS INDICATED BY THE CORRESPONDING MEAN OH\* EMISSION.

# MEASUREMENT EQUIPMENT AND SIGNAL PROCESSING

#### Laser interferometric vibrometry

For the recording of density fluctuations a single LIV was used (interferometer head OFV-503, velocity decoder OFV-5000, calibration factor 2mm/s/V, 100kHz bandwidth, no filters, Polytec, Waldbronn, Germany). These systems are commonly used to detect surface vibrations without mechanical contact to the surface. Including acousto-optical modulators they provide sub-nanometer resolution in wide frequency ranges [17]. LIVs also record changes in optical path length L'caused by refractive index fluctuations n' when the geometrical path  $\zeta$  is kept constant

$$L' = \int n'(\zeta, t) d\zeta \tag{10}$$

with  $\zeta$  the laser beam direction. The laser beam from the LIV is first collimated by a -40mm lens and then reflected by a mirror back into the LIV passing the flame zone twice. (A factor 2 from passing the flame section twice is already considered in eq.10). The integral indicates that this interferometric technique is a line-of-sight measurement technique, integrating all index or density fluctuations along the beam path. With an acoustooptical modulator as frequency modulator these interferometers record vibration velocity rather than vibration amplitude.

$$\frac{d}{dt}L'(t) = k_{vib}U(t) \tag{11}$$



FIGURE 2: EXPERIMENTAL SETUP FOR LIV MEASUREMENT: VIBROMETER, LENS, BURNER, MIRROR; BOTTOM LEFT: PRINCIPLE FUNCTION OF A LASER VIBROMETER: BS... BEAMSPLITTER, L... LENS, D... DETECTOR

with  $k_{vib}$  the calibration factor of the LIV (here 2mm/s/V) and U(t) the output voltage. Fluctuations in the refractive index can be related to density fluctuations  $\rho'(t)$  by the Gladstone-Dale relation

$$n' = G \rho_T' \tag{12}$$

with the Gladstone-Dale constant G for the gas mixture. The concept of optical path length is extremely important to interferometry and flow visualization. The fluid flow as refractive index field is discussed by [18]. This author also discussed the concept of optical path length for interferometry and shearography, basic concepts on interferometry for transparent objects of radial symmetry or without any symmetry is provided by [19]. To relate the optical path difference to a voltage output from a laser vibrometer by a calibration constant is a basic engineering concept for laser vibrometers [17].

The LIV system measured the density fluctuations - integrated along the line-of-sight - with a signal bandwidth from 0 to 100kHz. Additionally, 2D- traversing (DANTEC lightweight traverse, DANTEC Dynamics, Roskilde, Denmark) was used to scan the flame, and the integration over those measurement points provided the global density fluctuations.Scanning positions were spaced by a distance equal to the laser beam diameter (more details on the basics this technique can be found in [2] [4] [5] [15]). Since the burner is rotational symmetric, an Abel inversion was used to convert line-of-sight data into local density data for visualization purposes. A leastsquare fit in a matrix was used for Abel inversion [20].

Data acquisition was performed with analog-to-digital input modules NI-91215 and LabView 2012 software (both National Instruments, Austin, Texas). Each position was scanned for 30 seconds and was recorded with a sample rate of 16384 S/s.

## **Pressure-Pressure-Probe**

Due to the long acoustic wavelengths (low frequencies) it was not possible to meet the requirements of the far field



FIGURE 3: LEFT: MICROPHONE ARRAY, CONSISTING OF 14 PP-PROBES (TOP LEFT) WITH AN ANGLE OF 6 DEGREE BETWEEN EACH PROBE. RIGHT:

2D-MEASUREMENT GRID OF THE LIV: LINE OF SIGHT INTEGRATED DENSITY FLUCTUATIONS REPRESENTING THE AVERAGE SHAPE OF THE FLAME; THE POSITION OF THE REFERENCE MICROPHONE IS SHOWN IN THE MIDDLE

condition in the experiment. To verify the assumption that the LIV is capable to predict the sound power of the flame, ppprobes were used. With this microphone configuration it was possible to record sound power in the near field of the flame. Therefore a custom built microphone array with 14 measurement positions, aligned in an arch of one meter radius was used (FIGURE 3). The first pp-probe was mounted at a lateral angle of three degrees. The following microphones were aligned in steps of six degrees in latitudinal direction. Due to the rotational symmetric flame no longitudinal traversing was necessary. Each pp-probe consisted of two microphones (G.R.A.S. 40BD 1/4" prepolarized pressure microphone; G.R.A.S. Sound & Vibration A/S, Holte, Denmark) facing each other [21]. This configuration makes it possible to derive the particle velocity from the directly measured acoustic pressure and furthermore sound power can be calculated avoiding the need to fulfill the far-field assumption.

The distance between the two microphones of each pp-probe was set to 52mm. This provided an accurate measurement of pressure fluctuations of interest from 50 to 1000Hz, considering that the wavelength of the fluctuations is much longer than the distance between the microphones. For the recording of the acoustic data a sample rate of 100kS/s was chosen with a recording duration of 60 seconds, using a PXI-module and LabView 2013 (National Instruments, Austin, Texas USA).

## Signal processing

In order to calculate the amplitude of the siren-excited thermo-acoustic oscillation, a fast Fourier transform was performed by a MATLAB routine (MATLAB 2015a, The



FIGURE 4: DISTRIBUTION OF SOUND INTENSITY AT 212Hz ON A HEMISPHERE AROUND THE FLAME, MEASURED WITH PP-PROBES

MathWorks GmbH, Ismaning, Germany) using a sample length of 16384 for the LIV and 100000 for the microphone measurements. In order to calculate the entire sound power at 212Hz from the LIV measurements as given by eq. (4), the signals from all positions had to be summed up with respect to the signal phase. This phase relation resulted from the crosscorrelation of LIV and siren data. All cross-correlations and Fourier transforms were calculated using Welch's periodogram [22] and the analogues cross-correlation function. For all single frequency calculations the MATLAB built in flat-top window with an overlap of 50% was used, in order to determine the exact amplitude [23]. The phase from the cross power spectrum was acquired using a rectangular window. Flat-top window functions are generally used for calibration purposes, where the frequency is known beforehand. They provide maximum accuracy in amplitude but low frequency resolution. This is the case here, since fluctuations are caused by a siren with a defined excitation frequency. All operations were performed in the power spectrum only.

#### **RESULTS AND DISCUSSION**

The results obtained in this publication allowed a quantitative comparison between LIV measurements and more common microphone measurements. As a reference, sound power was recorded with pp-probes according to eqs. (9) and (5). The sound intensity (sound power per area) was calculated from the pp-probes and is shown in FIGURE 4 for the excited frequency of 212Hz of thermo-acoustic oscillation, plotted on a corresponding hemisphere. FIGURE 5 shows the distribution of sound intensity and sound power in latitudinal direction as recorded by the pp-probes. Since sound intensity is sound power per unit area, high values of intensity at high latitudinal angles have only small contribution to the sound power emitted in these directions. For the swirl-stabilized flame used here the



FIGURE 5: LATITUDINAL DISTRIBUTION OF SOUND INTENSITY AND SOUND POWER AT 212Hz, MEASURED WITH PP-PROBES

maximum sound power was emitted downstream at a latitudinal angle from 30° to 60°. Strahle's work and early experimental work already point towards a weak downstream directionality of sound emitted by the flame [12] [24]. According to Smith and Kilham [24] the preferred acoustic propagation lies between 40° and 80° to the burner axis in downstream direction. As possible causes the convection of the sound source, refractions in the temperature field, reflections at base plate or casing and velocity gradients are discussed by Strahle. These effects may alter the ideal circular sound radiation of a monopole to a more downstream oriented sound propagation. Herein a plausible explanation for this non-uniformity in the angular distribution is acoustic wave refraction in the temperature field around the flame, leading to a spatial redistribution of sound intensity without changing the integral value. Such an effect would be similar to the refraction of light in temperature or refractive index fields commonly used in shadowgraphy. This technique 'shadows' the refractive index field since light refraction at the field gradients redistribute the light intensity on a background screen. The shadows on the screen visualize the second spatial derivative of the refractive index (or temperature, or density) distribution, its curvature. According to Strahle's assumption in eq. (3), the measurement of density fluctuations via LIV should predict the same sound power as measured with the pp-probes if the flame can be reduced to a monopole. Therefore a high signal-to-noise-ratio (SNR) is necessary to ensure comparability between the two measurement techniques. In particular the calculation of the second time derivative of density fluctuations in the frequency domain in eq. (4) enhances noise. By multiplying the LIVsignal with the angular frequency, noise at higher frequencies lead to high SNR-amplitudes in these spectra. In a former work by Peterleithner et al. [10] it was shown that a lower SNR lead to problems measuring the natural spectra of the flame. Using LIV, the two dimensional distribution of the first time



FIGURE 6: PHASE RESOLVED LOCAL DENSITY FLUCTUATIONS AT 212Hz ALONG THE JET AXIS Z, PLOTTED IN RADIAL DIRECTION Y

derivative of density fluctuations in the flame was recorded by scanning half of the flame in a measurement grid consisting of 16 measure points in horizontal and 29 in vertical direction, giving a total number of 464 measure points with a grid size of 2mm. The corresponding phase resolved and Abel-transformed distribution of density fluctuations at an excitation frequency of 212Hz can be seen in FIGURE 6. The phase information of the density fluctuations was obtained by computing a cross correlation between a trigger signal of the siren and the LIV measurement data. This figure shows strong fluctuations of density propagating downstream in the axial direction of the flame. In previous research [10] it was seen that this PPM swirl-stabilized flame changes shape at this perturbation frequency. This happens because the highly reactive gas mixture tends to burn further upstream at phase angles where the velocity fluctuation reduces the overall velocity. These fluctuations in velocities are caused by the periodic modulation of the incoming flow by the siren. This effect results in an axial extension of the reaction zone. From FIGURE 6 it can be seen that the density fluctuations propagate with flow velocity. Therefore the wavelength of the densityfluctuations is in the range of about 15mm and much smaller than the wavelength of the acoustic perturbations caused by the flame. FIGURE 6 shows that nearly two periods of density fluctuations fit in the flame (blue and red regions). By integrating those local fluctuations over the measured area with respect to their sign they partially cancel out each other, resulting in lower global



FIGURE 7: COMPARISON OF SOUND POWER OBTAINED FROM PP-PROBES COMPARED TO THE SOUND POWER FROM LIV MEASUREMENT

fluctuations over the whole flame area. The acoustic wavelength at 212Hz is in a magnitude of meters and therefore much longer then the distance between the single oscillators in the flame which can be seen in FIGURE 6. So the flame acts as a spatially coherent source. This agrees with the acoustic monopole assumption by Strahle.

Finally, a quantitative comparison of the calculated sound power from both measurement techniques was done. FIGURE 7 shows the total sound power as a function of frequency. The maximum power, calculated from the pp-probe measurements was observed at 212Hz with 4.03 10<sup>-06</sup>W. The highest value of sound power calculated from LIV data was found at 213Hz with 6.22 10<sup>-06</sup>W. A shift of 1Hz and the first peak at 210Hz might be due to a change in boundary conditions during the LIV scanning time. The recording of all 464 positions with LIV lasted for several hours, while the acoustic measurements where finished within a short time. Long term fluctuations of the point of operation might explain this change in frequency. The higher value of sound power obtained from LIV measurements can be explained partly by the factor of  $4\pi r^2$  in eq. (2). Here a spherical control surface surrounding the flame is defined, whereas the microphone measurements only covered half of a sphere. It was not possible to correct the result for the only partial sound reflections from the base plate of the combustor. Therefore, not the whole sound power emitted by the flame was recorded with the microphones, resulting in a lower value of sound power than LIV.

Sources of error in the LIV measurements are the constant value of the Gladstone-Dale constant, the signal processing and noise in the LIV measurements. In eq. (12) the Gladstone-Dale constant is used as a constant factor to link fluctuations in the refractive index to density fluctuations. Local changes in gas composition can alter the Gladstone-Dale constant. This effect is not considered here, since in a PPM flame the burned gas composition shows only a slight influence on the Gladstone-Dale constant. [2].

A major influence on the result was found when choosing the FFT-filter function. Since the flame was excited at one frequency, it was necessary to determine the exact amplitude at this frequency while other frequencies were not of interest. Flat top windows have a wider bandwidth than other filters and therefore an accurate estimation of the amplitude [23] whereby the frequency resolution suffers. In signal processing flat top windows are commonly used for calibration purposes.

## CONCLUSION

This paper shows that LIV is capable to predict sound power in the far field of an excited flame, by recording local density fluctuations with some discrepancies compared to microphone measurements. The sound power calculated from LIV data is about 50% higher than that obtained from microphone measurements. This can be explained by noise in the LIV signal, assumptions in the analytical computation of sound power, from both the LIV and the microphone signal and the fact that the microphone measurements did not cover a whole sphere around the flame. It was also observed that the preferred acoustic propagation direction lies between 40° and 80° to the burner axis in downstream direction, meaning a slight deviation from a uniform distribution of sound intensity. This deviation might be explained by the refraction of sound in the temperature field of the flame. However this effect does not alter the integral sound power emitted by the flame. For future research the scanning time of the LIV must be reduced. For this purpose the joint project between Graz University of Technology and Technische Universität Dresden aims towards a full-field laser vibrometry making LIV measurements in flames ready for industrial research. This technology will be based on a high-speed camera system with acousto-optical modulation of the reference beam. Such a system will decrease the measurement time dramatically, offering spatially resolved density fluctuations within a flame in shorter measurement time. Whenever test rigs offer optical access to confined or unconfined flames, this technique will put a strong focus on industry-related experimental research in the field of modern combustion technologies for turbomachinery.

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