SIS100 Dipole Magnets
Field Measurements and Representation

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Beam Dynamics meets Magnets II
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Outline

Where to find

- Theory overview: “Cylindrical Circular and Elliptical, Toroidal Circular and Elliptical Multipoles Fields, Potentials and their Measurement for Accelerator Magnets”, Pierre Schnizer, Egbert Fischer, Bernhard Schnizer, arXiv:1410.8090, submitted to PR STAB.
- “Theory and application of plane elliptic multipoles for static magnetic fields” Pierre Schnizer, Bernhard Schnizer, Egbert Fischer, Pavel Akishin, NIMA, vol 607, 2009, p 505-516

Acknowledgement

- The results presented today were achieved within a short time scale.
- This is to thank all people for their dedication:
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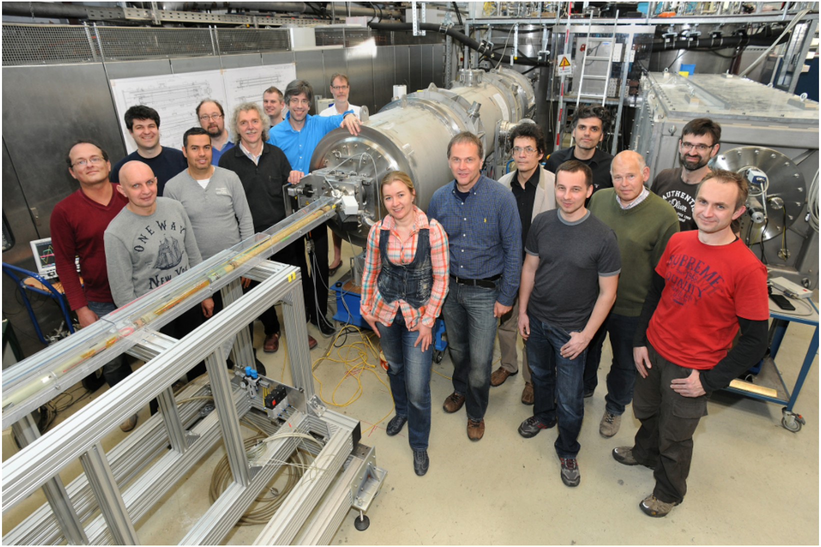
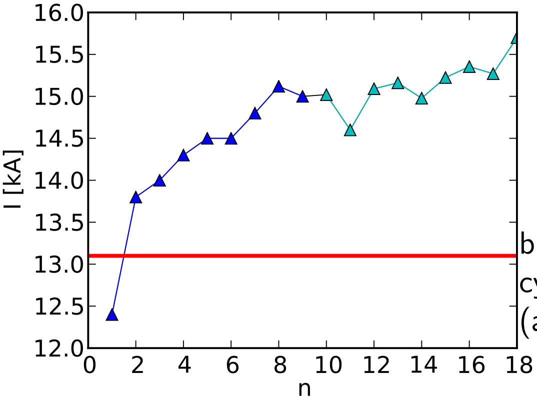
- together with the ones not even mentioned!


ONE WAY

Achieved Results

- Starting from fall last year:
  - test station upgrade
    - upgrade of the power converter (22 V / 20 kA)
    - HTS current leads (tested up to 15.7 kA)
    - overlapping coil probe measurement $\rightarrow$ analysis software developed
  - SIS100 FoS dipole magnet development:
    - high current coil
    - isotropic low loss iron yoke
    - shimming inserts

Magnet Power Test



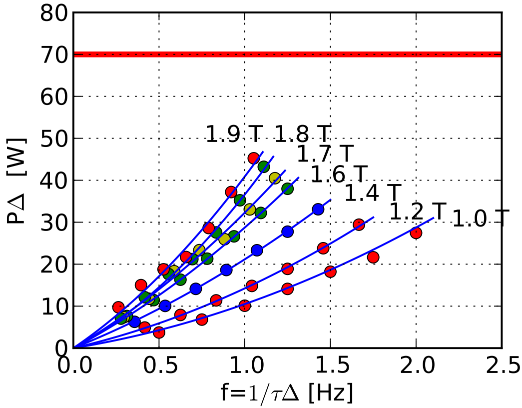
- second quench above nominal
- no detraining

blue. . . first run, cyan. . . second run (after first thermo cycle)

Minimum Cycle test

- 1.1 seconds reached
  $\rightarrow$ machine
  operation cycle
- minimum cycle $\rightarrow$
  to be determined

AC Loss test



- loss considerably lower: 50 W measured ↔ 70 W expected
- reduces load on cryoplant (otherwise @ limit)

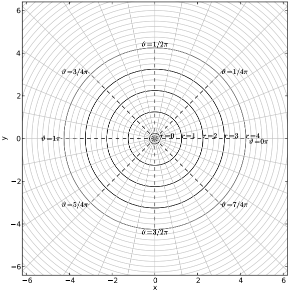
SIS100 FOS Magnet: Site Acceptance test

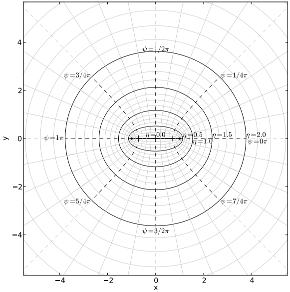
	specified	tested	
current			
I_{max} (DC)	13.1 kA	15.7 kA	
$(\mathrm{d}I/\mathrm{d}t)_{max}$	26.2 kA/s	27 kA/s	$I_{max} = 14$kA
AC Loss			
triangular cycle	70 W	50 W	
shortest op. cycle	1.1 s	1.1 s	

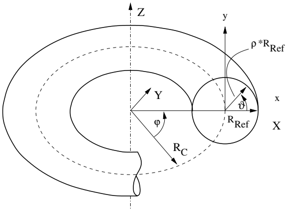
4 Coordinate systems: The systems

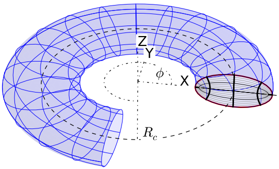
Cylindric circular	standard approach
Cylindric ellliptic	field description using elliptic coordinates
Toroidal circular	solved by R-separation: transformation of differential equation
Toroidal ellliptic	solved by R-separation: transformation of differential equation
Global Toroidal	see LBNL (and others?)

4 Coordinate systems: geometry

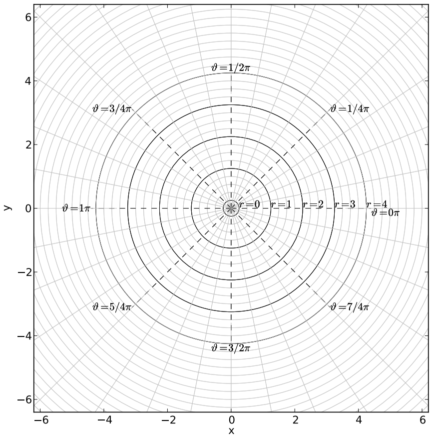


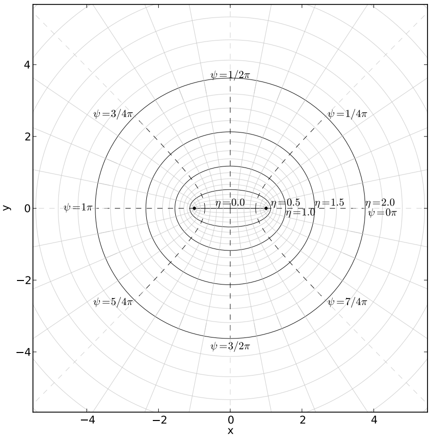


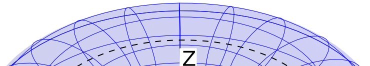




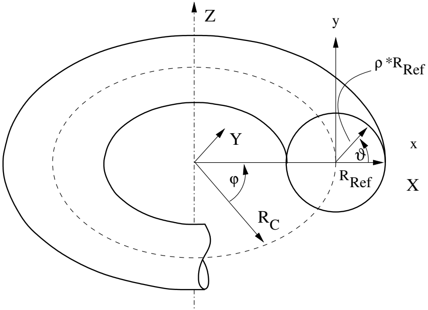
4 Coordinate systems: geometry

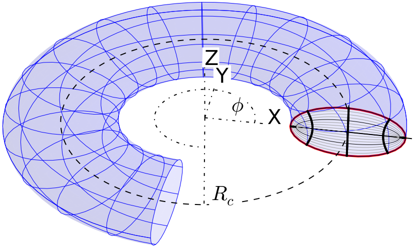






4 Coordinate systems: geometry





Basis Equation

$$\frac{d^2 H}{d\eta^2} - \gamma\, H = 0 \quad \text{and} \quad \frac{\partial^2 \bar{\Psi}}{\partial \psi^2} + \gamma\, \bar{\Psi} = 0.$$

$$\Psi(\eta,\psi)/\Psi_0 = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \frac{\cosh(n\eta)}{\cosh(n\eta_0)} \cos(n\psi) + b_n \frac{\sinh(n\eta)}{\sinh(n\eta_0)} \sin(n\psi) \right],$$

$$\mathbf{B} = B_y(\eta + \mathrm{i}\psi) + \mathrm{i}\, B_x(\eta + \mathrm{i}\psi) = \frac{\mathbf{E_1}}{2} + \sum_{n=2}^{\infty} \mathbf{E_n} \frac{\cosh[(n-1)(\eta + \mathrm{i}\psi)]}{\cosh((n-1)\ \eta_0)}.$$

$$\begin{aligned} \mathbf{E}_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} \mathbf{B_0}\left(\mathbf{z} = e \cosh(\eta_0 + \mathrm{i}\psi)\right) \cos((n-1)\ \psi)\ d\psi, \\ &= \frac{1}{2\pi} \int_{-\pi}^{\pi} \left[\mathbf{B_0}\left(\mathbf{z} = e \cosh(\eta_0 + \mathrm{i}\psi)\right) + \mathbf{B_0}\left(\mathbf{z} = e \cosh(\eta_0 - \mathrm{i}\psi)\right)\right]\ e^{\mathrm{i}(n-1)\psi}\ d\psi. \end{aligned} \tag{1}$$

Fields with elliptic coordinates

- our original proposal [1]: Cartesian field components $B_y + \mathtt{i} B_x$ on elliptic coordinates η, ψ
- (Classical) vector analysis: elliptic field components, elliptic coordinates, metric elements, vector operators...
- how do they relate: apply local tangents to transform ... [2]

Relation Elliptic / Cylindric

- given in NIMA [1]
- alternative recursive method by Franchetti / Pena) [3]
- or even simpler Chebyshev polynoms :

$$\begin{aligned} T_n(x) &= \cos\left(n \arccos x\right) \quad \text{für} \quad x \in [-1, 1] \\ T_n(x) &= \cosh\left(n\,\mathrm{arcosh}(x)\right) \quad \text{für} \quad |x| > 1 \end{aligned}$$

(courtesy of V. Marousov)

- or : integral [4, 5],

$$\cosh(n\mathbf{w}) = 2^{(n-1)} \cosh^n \mathbf{w} + \sum_{m=1}^{[n/2]} (-1)^m \, \frac{n}{m} \binom{n-m-1}{m-1} 2^{(n-2m-1)} \cosh^{n-2m} \mathbf{w}$$

- always a finite sum: m elliptic $\rightarrow$ n + 1 circular

Elliptic multipoles on elliptic coordiantes

Field $\eta + \mathrm{i}\psi = \mathbf{w}$: Tangents:

$$B_\psi + \mathrm{i} B_\eta = e \underbrace{\frac{\sinh w}{\sqrt{\cosh^2\eta - \cos^2\psi}}}_{h_t} (By + \mathrm{i} B_x) \tag{2}$$

$$h_t \left(B_\psi + \mathrm{i} B_\eta\right) = \left(\hat{B}_\psi + \mathrm{i}\hat{B}_\eta\right) = \hat{\mathbf{B}}(\mathbf{w})$$

$$\hat{\mathbf{B}}(\mathbf{w}) = e \sinh[\mathbf{w}] \left[\frac{\hat{\mathbf{E}}_1}{2} + \sum_{m=2}^{M-1} \cosh\left[(m-1)\mathbf{w}\right]\right] \tag{3}$$

Potential by integration in $\mathbf{w}$ or in η, ψ

$$\mathbf{\Phi}^{\mathbf{e}}(\mathbf{w}) = -\frac{e}{2}\Bigg[\hat{\mathbf{E}}_1 \cosh\mathbf{w} + \frac{1}{2}\hat{\mathbf{E}}_2 \cosh(2\mathbf{w}) + \sum_{m=2}^{M-1} \hat{\mathbf{E}}_{m+1}\left(\frac{1}{m+1}\cosh[(m+1)\mathbf{w}] - \frac{1}{m-1}\cosh[(m-1)\mathbf{w}]\right)\Bigg]. \tag{4}$$

Similar results as for differential forms [2]

Elliptic multipoles on elliptic coordiantes: $\mathbf{E_1}$

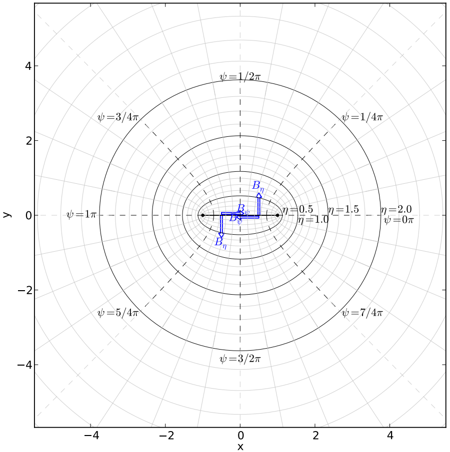
That's why metric still considered important ...
For E_1 $\rightarrow$ direct translation from elliptic to circular harmonics:

$$B_\psi + \mathrm{i} B_\eta = e \frac{\sinh w}{h_t} (By + \mathrm{i} B_x)$$

$$= \frac{e \sinh w}{h_t} \mathbf{E_1}$$

$$= e \underbrace{\frac{\sinh w}{\sqrt{\cosh^2 \eta - \cos^2 \psi}}}_{h_t} (B_1 + \mathrm{i} A_1)$$

Degenerates line for $\eta = 0$ Poles $\pm\mathrm{i}$: $B_x = \pm B_\eta$, $B_y = \pm B_\psi$



Elliptic multipoles on elliptic coordiantes: $\mathbf{E_1}$

etric still considered

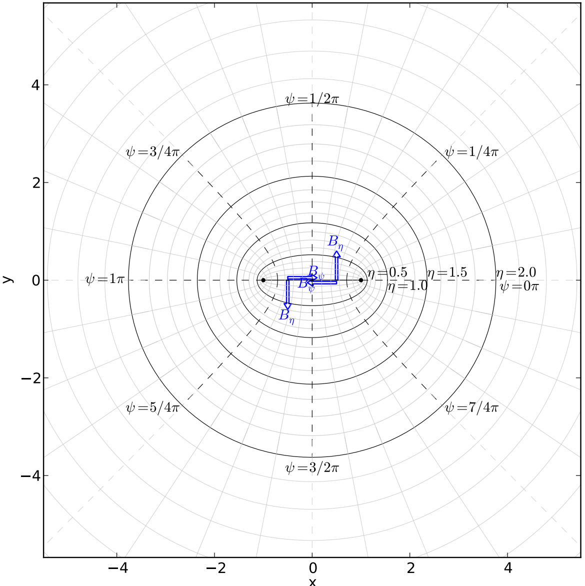
ect translation from
ular harmonics:

$$\frac{\sinh w}{h_t}(By + \mathrm{i}B_x)$$

$$\frac{\ldots w}{\ldots - \cos^2\psi}(B_1 + \mathrm{i}A_1)$$

t

ne for $\eta = 0$ Poles
η, $B_y = \pm B_\psi$



Elliptic multipoles

- elliptic coordinates
- but cylindric field components $B_y(\eta + \mathrm{i}\psi) + \mathrm{i}\, B_x(\eta + \mathrm{i}\psi)$
- $B_y + \mathrm{i}\, B_x \Leftrightarrow h_t\left(\hat{B}_\psi + \mathrm{i}\, \hat{B}_\eta\right)$, both analytic complex functions
  Cauchy Riemann $\Leftrightarrow \Delta\Phi = 0$ (as defined in Cartesian Coordinates)
- can be calculated by Fourier Transform
- Translation to circular using analytical matrix

Toroidal Circular Multipoles

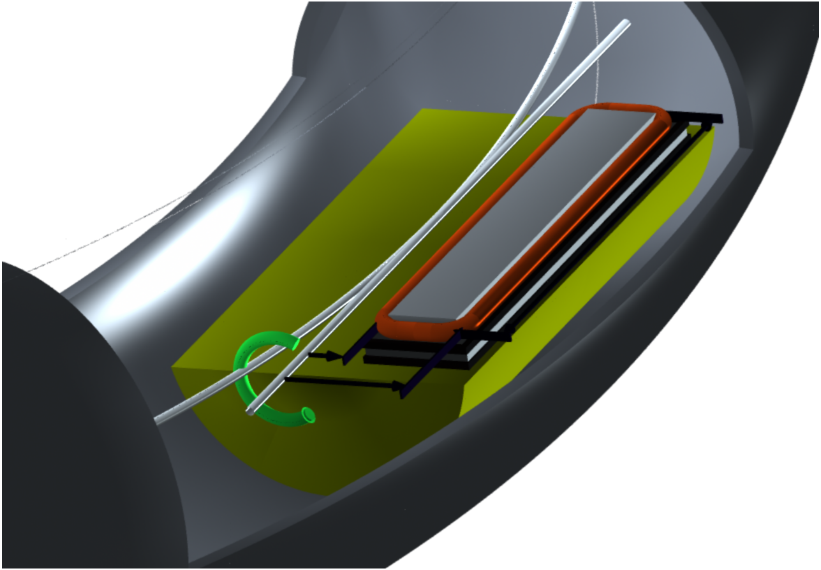
- Local toroidal coordinates [6, 4], R-separable
- Transform $\Phi \to \sqrt{h}\Phi$; $h = 1 + \varepsilon\rho\cos\vartheta = 1 + x\,R_C$, $eps = R_{Ref}/rc$ neglect ε^2 $\to$Laplace Operator of polar coordinates
- solve field expressions: $\mathbf{B}^{\mathbf{t}}(\mathbf{z}) = \sum_{m=1}^{M}\left[R_m\ \mathbf{T}^{\mathrm{n}}_{\mathbf{m}}(\mathbf{z})\ + \mathrm{i}\,S_m\ \mathbf{T}^{\mathrm{s}}_{\mathbf{m}}(\mathbf{z})\right]$.

$$\begin{pmatrix}\mathbf{T}^{\mathrm{n}}_{\mathbf{m}}(\mathbf{z})\\ \mathbf{T}^{\mathrm{s}}_{\mathbf{m}}(\mathbf{z})\end{pmatrix} = \left(\frac{\mathbf{z}}{R_{Ref}}\right)^{m-1}\left(1 - \varepsilon\frac{1}{2}\frac{\mathrm{Re}\,(\mathbf{z})}{R_{Ref}}\right) - \varepsilon\frac{1}{2m}\begin{pmatrix}\mathrm{Im}\left[\left(\frac{\mathbf{z}}{R_{Ref}}\right)^{m}\right]\mathrm{i}\\ \mathrm{Re}\left[\left(\frac{\mathbf{z}}{R_{Ref}}\right)^{m}\right]\end{pmatrix}.$$

- different basis functions for *normal* and *skew* field expansions

R_C torus larger radius, R_{Ref} reference radius, $\mathbf{z} = x + \mathrm{i}y$

Coil probe within torus



Coil probe within torus: Conversion

$$\begin{pmatrix} \vec{R}_\mu \\ \vec{S}_\mu \end{pmatrix} = \left(\begin{array}{c|c} G^{nn}_{\nu,\mu} & G^{ns}_{\nu,\mu} \\ \hline G^{sn}_{\nu,\mu} & G^{ss}_{\nu,\mu} \end{array} \right) \begin{pmatrix} \vec{B}_\nu \\ \vec{A}_\nu \end{pmatrix} . \tag{5}$$

$$\begin{pmatrix} G^{nn} \\ G^{ns} \\ G^{sn} \\ G^{ss} \end{pmatrix} = I + \mathcal{L}^{dr} + \varepsilon \left(-U + \mathcal{L}^{L} - \mathcal{L}^{sk} + \mathrm{i}\mathcal{L}^{R2} \right) + \varepsilon \begin{pmatrix} \mathrm{Re}\left[\mathcal{L}^{R2_0}\right] \\ -\mathrm{Im}\left[\mathcal{L}^{R2}\right] \end{pmatrix} \tag{6}$$

feed down... $\mathcal{L}^{dr}_{\nu,\mu} = \binom{\nu-1}{\mu-1} \left(\frac{\mathbf{d_z}}{R_{Ref}} \right)^{\nu-\mu} * \mathcal{L}_{\nu,\mu} - I$

feed down 2... $\mathcal{L}^{dr}_{\nu,\mu} = R_{Ref} \frac{\mathrm{d}}{\mathrm{d}\mathbf{d_z}} \mathcal{L}^{dr}$, $\mathbf{d_z} = (d_x + \mathrm{i} d_y)/R_{Ref}$

$\mathcal{L}^{L}_{\nu,\mu} = \frac{L^2}{3R^2_{Ref}} \mathcal{L}^{dr2}$

$\mathcal{L}^{sk}_{\nu,\mu} = \frac{1}{4\,(\mu+1)} * \frac{K_{\mu+2}}{K_\mu} * \mathcal{L}^{dr2}$

$\mathcal{L}^{R2}_{\nu,\mu} =$

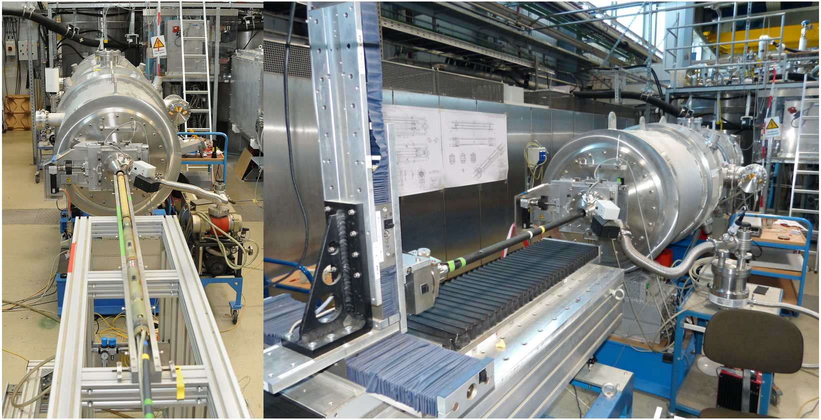
$\frac{1}{4\nu} \left(\frac{\nu\,\mu}{\nu-\mu+1} * \mathcal{L}_{\nu,\mu} + \delta_{\mu,1} \right) * \left[\frac{d_y}{R_{Ref}} - \left(\frac{2-\mu+2\nu}{\mu} * \mathcal{L}_{\nu,\mu} - \nu\delta_{\mu,1} \right) \mathrm{i} \frac{d_x}{R_{Ref}} \right] * \left(\mathcal{L}^{dr} + I \right) .$

$\mathcal{L}^{R2_0}_{\nu,\mu} = \frac{1}{2\nu} \left(\frac{\mathbf{d_z}}{R_{Ref}} \right)^{\nu} \delta_{\mu,1}.$

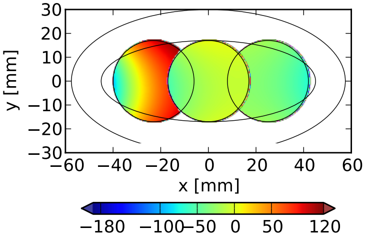
Summary

- elliptic multipoles:
  - Cartesian vector field on elliptic coordinates, complex representation,
  - $\leftrightarrow$ mapping to B_η, B_ψ;
  - $\hat{\mathbf{B}}(w)/ht$ relation to real space coordinates! η_0: $B_y = \pm B_\psi$
- Local Toroidal multipoles: approximate solution: $\lim_{\varepsilon \to 0}$ cylindric circular multipoles
- good for machines like SIS100, SIS300
- allows estimate rotating coil probe artefacts, deducing reasonable coil probe length

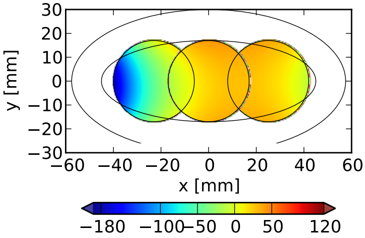
Measuring: mole $\leftrightarrow$ mapper



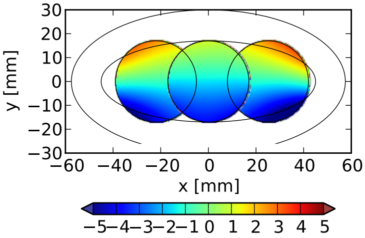
Correcting coil measurements



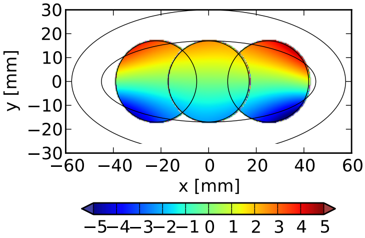
end field: raw



end field: corrected

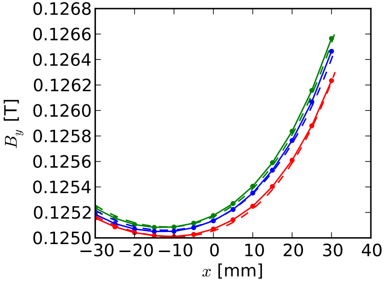


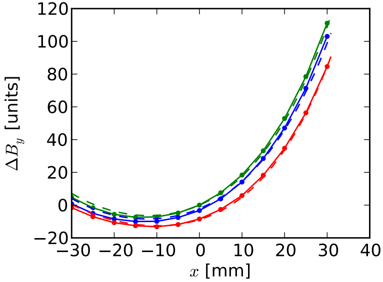
central field: raw



central field: corrected

End field comparison 1500 A





B_y ΔB_y

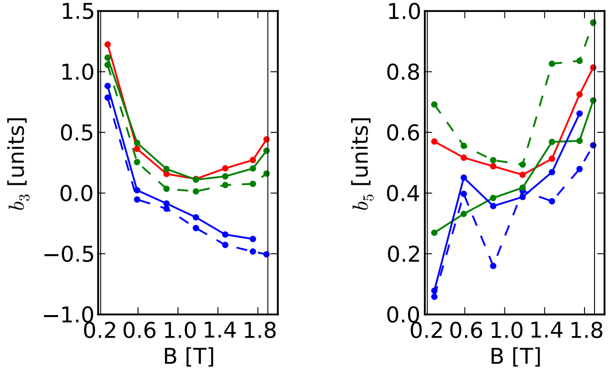
solid lines. . . mapper, dashed lines. . . coil,

blue. . . y = + 10 mm, green. . . y = 0, red. . . y = - 10 mm

- Data quite matching:
  - taking different coordinate systems into account
  - inverting ΔB_y, searching (twisted cable on patch panel)

"injection field level"

First allowed harmonics



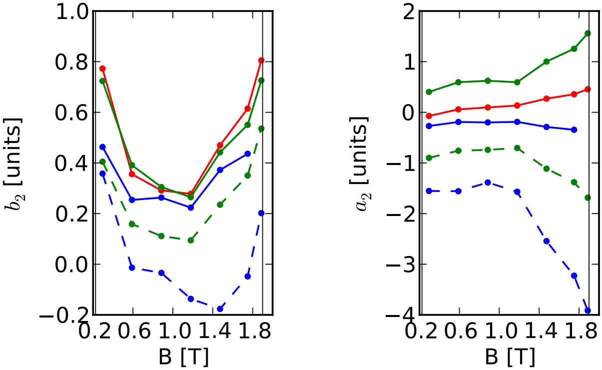
b_3 b_5

$R_{Ref} = 40$ mm

solid lines...mapper, dashed lines...coil,

blue...y = + 10 mm, green...y = 0, red...y = - 10 mm

Not allowed Harmonic I/IV



b_2 a_2

R_{Ref} = 40 mm large skew harmonics!

solid lines. . . mapper, dashed lines. . . coil,

blue. . . y = + 10 mm, green. . . y = 0, red. . . y = - 10 mm

Conclusion

- Theory:
  - Alternative coordiante systems
  - elliptic multipoles: allow description of fields within elliptic beam pipe
  - Local toroidal coordinates: follow particle trajectory within a bent
  - approximation: simpler systems than Global toroidal ones (if approximation acceptable)
  - can deduce measurements for rotating coil probes
- Application
  - on SIS00 magnet:
    - detected skew harmonics
    - gave reliable results ↔ an error that could have been missed

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