Wave Propagation in Water Saturated Soil: Modeling and Numerical Realisation

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Abstract: Biot's theory of porous media governs the wave propagation in a porous elastic solid infiltrated with fluid. Representations of this theory using different sets of unknowns are available. As well a simplified theory neglecting the inertia effects of the relative fluid to solid movement has been proposed. These models are shown and the restrictions of the simplified theory are discussed. For the numerical solution a time domain Boundary Element formulation based on the Convolution Quadrature Method is proposed. This formulation is used to study wave propagation in poroelastic half-spaces in two and three dimensions. The comparison of the complete theory with the simplified theory shows that for typical geomechanical materials the simplification can be done. If, however, the effect of the second slow compressional wave is important only the complete theory is acceptable.

1 Introduction

In many branches of engineering porous materials play an important role, in e.g., the petroleum industry, chemical engineering, soil mechanics, and in recent years also in bio-mechanics. A historical review on the subject of multiphase continuum mechanics identifies two poroelastic theories which have been developed and are used nowadays, namely Biot's theory and the Theory of Porous Media. For more details, the reader is directed to the work of de Boer and Ehlers (1988, 1990) or to the recently published monograph (de Boer, 2000).

Based on the work of von Terzaghi, a theoretical description of porous materials saturated by a viscous fluid was presented by Biot (1941). The dynamic extension was done in two papers, one for the low frequency range (Biot, 1956a) and the other for the high frequency range (Biot, 1956b). A collection of Biot's papers on porous materials has been published by Tolstoy (1991). Among the significant findings in the theory for poroelastodynamics was the identification of three waves for a 3-d continuum, two compressional waves and one shear wave. This extra compressional wave, known as the slow wave, has been experimentally confirmed by Plona (1980). Based on the work of Fillunger, the Theory of Porous Media has been developed. This theory is based on the axioms of continuum theories of mixtures (Truesdell and Toupin, 1960; Bowen, 1976) extended by the concept of volume fractions by Bowen (1980, 1982) and others (Ehlers, 1993b,a). Remarks on the equivalence of both theories, which model the same physical phenomenon, are found in the work of Bowen (1982), Ehlers and Kubik (1994), and Schanz and Diebels (2003). In all these publications, linear versions of both theories are compared. Summarizing the results, it is found that for incompressible constituents both theories are identical if Biot's apparent mass density is set to zero. Further, in case of compressible constituents, the governing mathematical operator is identical but differs in the constant coefficients, i.e., the physical constants used are different. It is still an open question how this gap can be bridged.

Here, Biot's theory is used but the results can be simply transferred to the Theory of Porous Media because the equivalence of the mathematical operator ensures to have the same fundamental solutions however with different material constants.

In the following, first, the different versions of Biot's theory are recalled and the basic equations are mostly given. There is a small comparison of some of these theories with the help of a one-dimensional wave propagation example. Then the poroelastodynamic Boundary Element Method (BEM) is recalled. Certainly, in the framework of numerical

realisations the Finite Element Method (FEM) has also to be mentioned. The standard book on FE formulations for poroelasticity has been presented by Lewis and Schrefler (1998) but it is mainly restricted to consolidation. The most complete presentation for poroelastodynamics but with a strong focus on geomechanics has been published by Zienkiewicz et al. (1999). In principle, this monograph summarizes all the effort this group has been published on this subject. Here, the focus will be on the BEM because it is the natural treatment of semi-infinite domains. The efficiency of BEM in dealing with such domains has long been recognized by researchers and engineers. There is a large number of applications (see, e.g., Beskos (1987, 1997)). A survey on the history of BEM itself is given by Cheng and Cheng (2005). The modeling of poroelastic domains with BEM is reviewed by Chopra (2001).

For modeling a transient behavior, two approaches exist in pure boundary based formulations: solving in the time domain by time-stepping (Mansur, 1983) and solving in the integral transform domain via Laplace or Fourier transform with subsequent inverse transformation (Cruse and Rizzo, 1968). The time stepping approach is sometimes impossible due to the lack of a time dependent fundamental solution which is often only available in the transformed domain. Therefore, first poroelastodynamic BE formulations based on Biot's theory have been published in Laplace domain by Manolis and Beskos (1989) for a displacement based formulation (see the correction of this paper in Manolis and Beskos (1990)). Based on the four (three) unknowns in 3-d (2-d) of the displacement–pressure formulation BEM formulations in frequency domain have been published by Cheng et al. (1991) and Domínguez (1992).

However, it is more natural to work in the real time domain and observe the phenomenon as it evolves. Such a time domain formulation was developed by Wiebe and Antes (1991) with the restriction of vanishing damping between the solid skeleton and the fluid. Another time dependent formulation was proposed by Chen and Dargush (1995) based on the analytical inverse transformation of the Laplace domain fundamental solutions. But, as the authors admit, this formulation is highly CPU-time demanding. Alternatively, Schanz (2001b,a,c) utilizes the Convolution Quadrature Method proposed by Lubich (1988a,b) to establish a time stepping BE formulation for poroelastodynamics based on the Laplace transformed fundamental solutions.

Throughout this paper, the summation convention is applied over repeated indices and Latin indices receive the values 1,2, and 1,2,3 in two-dimensions (2-d) and three-dimensions (3-d), respectively. Commas $()_{,i}$ denote spatial derivatives and dots $\dot{()}$ denote the time derivative. As usual, the Kronecker delta is denoted by δ_{ij}. Further, any index (sub- or superscript) f refers to the interstitial fluid and s to the skeleton.

2 Biot's theory

In Biot's theory a fully saturated material is assumed, i.e., an elastic skeleton with a statistical distribution of interconnected pores is considered. This porosity is denoted by

$$\phi = \frac{V^f}{V} \,, \tag{1}$$

where V^f is the volume of the interconnected pores contained in a sample of bulk volume V. Contrary to these pores the sealed pores are considered as part of the solid. Full saturation is assumed leading to $V = V^f + V^s$ with V^s the volume of the solid.

As the aim of the paper is to model wave propagation phenomena, it is sufficient to formulate a linear kinematic equation. I.e., the relation of the solid/fluid strain to the solid/fluid displacement is chosen linear, respectively

$$\varepsilon^s_{ij} = \frac{1}{2}\left(u^s_{i,j} + u^s_{j,i}\right) \qquad \varepsilon^f_{kk} = u^f_{k,k} \tag{2}$$

assuming small deformation gradients. In (2), the components of the solid and fluid strain tensors are denoted by ε^s_{ij} and ε^f_{kk}, respectively. The corresponding displacements are u^s_i and u^f_i.

In the following, a brief summary of the basic equations will be given written with different degrees of freedom for the compressible and incompressible model as well as for some simplified models.

2.1 Compressible model

If the constitutive equations are formulated for the elastic solid and the interstitial fluid, a partial stress formulation is obtained (Biot, 1955)

$$\sigma^s_{ij} = G\left(u^s_{i,j} + u^s_{j,i}\right) + \left(K - \frac{2}{3}G + \frac{Q^2}{R}\right) u^s_{k,k}\delta_{ij} + Q u^f_{k,k}\delta_{ij} \tag{3a}$$

$$\sigma^f = -\phi p = Q u^s_{k,k} + R u^f_{k,k}\,. \tag{3b}$$

The respective stress tensors are denoted by σ^s_{ij} and $\sigma^f\delta_{ij}$. The elastic skeleton is assumed to be isotropic and homogeneous where the two elastic material constants compression modulus K and shear modulus G refer to the bulk material. The coupling between the solid and the fluid is characterized by the two parameters Q and R. In the above, the sign conventions for stress and strain follow that of elasticity, namely, tensile stress and strain is denoted positive. Therefore, in equation (3b) the pore pressure p is the negative hydrostatic stress in the fluid σ^f.

An alternative representation of the constitutive equation (3) is used in Biot's earlier work (Biot, 1941). There, the total stress $\sigma_{ij} = \sigma^s_{ij} + \sigma^f\delta_{ij}$ is introduced and with Biot's effective stress coefficient $\alpha = \phi\,(1 + Q/R)$ the constitutive equation with the solid displacement u^s_i and the pore pressure p

$$\sigma_{ij} = G\left(u^s_{i,j} + u^s_{j,i}\right) + \left(K - \frac{2}{3}G\right) u^s_{k,k}\delta_{ij} - \alpha\delta_{ij}p \tag{4a}$$

is obtained. Additional to the total stress σ_{ij}, the variation of fluid volume per unit reference volume ζ is introduced

$$\zeta = \alpha u^s_{k,k} + \frac{\phi^2}{R}p\,. \tag{4b}$$

This variation of fluid ζ is defined by the mass balance over a reference volume, i.e., by the continuity equation

$$\dot{\zeta} + q_{i,i} = a \tag{5}$$

with the specific flux $q_i = \phi\left(\dot{u}^f_i - \dot{u}^s_i\right)$ and a source term $a\,(t)$. Equation (5) identify ζ as a kind of strain describing the motion of the fluid relative to the solid. The source term $a\,(t)$ is not motivated by any physical reason but it is needed for the derivation of the fundamental solutions.

The next step is to state the balances of momentum. In any two-phase material there are three possibilities to formulate the balances of momentum: First, the balance of momentum in the solid, second the balance of momentum in the fluid and, third, the balance of momentum for the bulk material. But, it is sufficient to choose two of them.

The first two balances are used by Biot (1956a) resulting in a formulation with the unknowns solid displacement u^s_i and fluid displacement u^f_i

$$\sigma^s_{ij,j} + (1-\phi)\,f^s_i = (1-\phi)\,\rho_s\ddot{u}^s_i - \rho_a\left(\ddot{u}^f_i - \ddot{u}^s_i\right) - \frac{\phi^2}{\kappa}\left(\dot{u}^f_i - \dot{u}^s_i\right) \tag{6a}$$

$$\sigma^f_{,i} + \phi f^f_i = \qquad \phi\rho_f\ddot{u}^f_i + \rho_a\left(\ddot{u}^f_i - \ddot{u}^s_i\right) + \frac{\phi^2}{\kappa}\left(\dot{u}^f_i - \dot{u}^s_i\right)\,. \tag{6b}$$

The first balance equation (6a) is that for the solid skeleton and the second (6b) is that for the interstitial fluid. In equation (6), the body forces in the solid skeleton f^s_i and in the fluid f^f_i are introduced. Further, the respective densities are denoted by ρ_s and ρ_f. To describe the dynamic interaction between fluid and skeleton an additional

density the apparent mass density ρ_a has been introduced by Biot (1956a). It can be written as $\rho_a = C\phi\rho_f$ where C is a factor depending on the geometry of the pores and the frequency of excitation. At low frequency, Bonnet and Auriault (1985) measured $C = 0.66$ for a sphere assembly of glass beads. In higher frequency ranges, a certain functional dependence of C on frequency has been proposed based on conceptual porosity structures, e.g., in Biot (1956b) and Bonnet and Auriault (1985). Another relation to determine this apparent mass density has been suggested by Berryman (1980)

$$\rho_a = \rho_f \left(1 - a_\infty\right), \qquad a_\infty = \frac{1}{2}\left(\frac{1}{\phi} + 1\right) \tag{7}$$

which he found in good agreement with experiments. There, the tortuosity a_∞ describes the morphology of the porous media and may be interpreted as the ratio between the average length of micro-channels in the porous material to the average characteristic distance on the micro-level. I.e., for straight channels it is equal to one and otherwise larger: $1 \leq a_\infty \leq \infty$. In the linear model this quantity is constant (Wilmanski, 2005).

The factor ϕ^2/κ in front of the damping term in (6) is usually denoted by b. Here, the simplification of a frequency independent, respectively time independent, value is taken which is only valid in low frequency range. Further, the above chosen factor ϕ^2/κ is given only in case of circular pores when κ denotes the permeability. More general models with a frequency dependence of κ may be found, e.g., in the works of Biot (1956b); Johnson et al. (1987), or Auriault et al. (1985). Clearly, a frequency dependent factor in front of the damping term in (6) is unphysical. Hence, this term has to be transformed to time domain and then a convolution in time between the respective time-dependent factor and the damping term has to be performed.

The third above mentioned balance of momentum for the mixture is formulated in Biot's earlier work (Biot, 1941) for quasi-statics and in Biot (1956a) for dynamics. This dynamic equilibrium is given by

$$\sigma_{ij,j} + F_i = \rho_s \left(1 - \phi\right) \ddot{u}_i^s + \phi\rho_f \ddot{u}_i^f \,, \tag{8}$$

with the bulk body force per unit volume $F_i = \left(1 - \phi\right) f_i^s + \phi f_i^f$. It is obvious that adding the two partial balances (6a) and (6b) results in the balance of the mixture (8).

In most papers using the total stress formulation, now, the constitutive assumption for the fluid transport in the interstitial space is given by Darcy's law. Here, it is also used, however, with the balance of momentum in the fluid (6b) Darcy's law is already given. Rearranging (6b) and taking the definition of the flux $q_i = \phi\left(\dot{u}_i^f - \dot{u}_i^s\right)$ as well as $\sigma^f = -\phi p$ into account the dynamic version of Darcy's law

$$q_i = -\kappa\left(p_{,i} + \frac{\rho_a}{\phi}\left(\ddot{u}_i^f - \ddot{u}_i^s\right) + \rho_f \ddot{u}_i^f - f_i^f\right) \tag{9}$$

is achieved.

Aiming at the equation of motion, the constitutive equations have to be combined with the corresponding balances of momentum. The kinematic conditions (2) are already inserted in the constitutive equations. Next, the degrees of freedom must be determined. There are several possibilities:

i) To use the solid displacement u_i^s and the fluid displacement u_i^f with six (four) unknowns in 3-d (2-d) (see, e.g., (Biot, 1956a)).

ii) Alternatively the solid displacement u_i^s and the relative fluid to solid displacement weighted by the porosity $w_i = \phi\left(u_i^f - u_i^s\right)$ with also six (four) unknowns in 3-d (2-d) can be used (see, e.g., (Zienkiewicz and Shiomi, 1984)). Beside this relative displacement also sometimes the seepage velocity, i.e., the time derivative of w_i, with or without the porosity as weighting factor is applied.

iii) A combination of the pore pressure p and the solid displacement u_i^s with four (three) unknowns in 3-d (2-d) can be established. As shown by Bonnet (1987), this choice is sufficient.

In the following, the first choice will be denoted by u_i^s-u_i^f-formulation, the second by u_i^s-w_i-formulation, and the third by u_i^s-p-formulation.

u_i^s-u_i^f-formulation First, the equations of motion for a poroelastic body are presented for the unknowns solid displacement u_i^s and fluid displacement u_i^f. Inserting in (6) the constitutive equations (3) written for the partial stress tensors yields a set of equations of motion in time domain

$$\begin{aligned} G u^s_{i,jj} + \left(K + \frac{1}{3}G\right) u^s_{j,ij} + Q\left(\frac{Q}{R} u^s_{j,ji} + u^f_{j,ji}\right) + (1-\phi)\, f_i^s & \\ = (1-\phi)\,\rho_s \ddot{u}_i^s - \rho_a\left(\ddot{u}_i^f - \ddot{u}_i^s\right) - \frac{\phi^2}{\kappa}\left(\dot{u}_i^f - \dot{u}_i^s\right) & \quad (10a) \\ R\left(\frac{Q}{R} u^s_{j,ji} + u^f_{j,ji}\right) + \phi f_i^f = \phi\rho_f \ddot{u}_i^f + \rho_a\left(\ddot{u}_i^f - \ddot{u}_i^s\right) + \frac{\phi^2}{\kappa}\left(\dot{u}_i^f - \dot{u}_i^s\right) . & \quad (10b) \end{aligned}$$

u_i^s-w_i-formulation Second, for this representation in all of the above equations u_i^f is changed to w_i. With this unknown, the dynamic equilibrium for the mixture (9) reads

$$\sigma_{ij,j} + F_i = \rho \ddot{u}_i^s + \rho_f \ddot{w}_i \,, \tag{11}$$

using the bulk density $\rho = \rho_s\,(1-\phi) + \phi\rho_f$. The dynamic version of Darcy's law (9) is then

$$\dot{w}_i = q_i = -\kappa\left(p_{,i} + \rho_f \ddot{u}_i^s + \frac{1}{\phi}\left(\frac{\rho_a}{\phi} + \rho_f\right)\ddot{w}_i - f_i^f\right) . \tag{12}$$

Inserting in the partial balances of momentum (6) the constitutive equations (4) reformulated with w_i and using the time integrated homogeneous form of the continuity equation (5), i.e., $\zeta = -w_{i,i}$, the system of governing equations

$$\begin{aligned} G u^s_{i,jj} + \left(K + \frac{1}{3}G + \alpha^2\frac{R}{\phi^2}\right) u^s_{j,ij} + \alpha\frac{R}{\phi^2} w_{j,ji} + F_i = \rho\ddot{u}_i^s + \rho_f\ddot{w}_i & \quad (13a) \\ \alpha\frac{R}{\phi} u^s_{j,ji} + \frac{R}{\phi} w_{j,ji} + \phi f_i^f = \phi\rho_f \ddot{u}_i^s + \left(\frac{\rho_a}{\phi} + \rho_f\right)\ddot{w}_i + \frac{\phi}{\kappa}\dot{w}_i & \quad (13b) \end{aligned}$$

is obtained. Note that in this formulation the source term $a\,(t)$ is set to zero.

u_i^s-p-formulation Third, the respective equations of motion are presented for the pore pressure p and the solid displacement u_i^s as unknowns. To achieve this formulation the fluid displacement u_i^f or the relative fluid/solid displacement w_i has to be eliminated. In order to do this, Darcy's law (12) is rearranged to obtain w_i. Because w_i appears in (12) as time derivatives this rearrangement is performed in Laplace domain. A transformation to Laplace domain results in

$$\hat{w}_i = -\underbrace{\frac{\kappa\rho_f\phi^2 s^2}{\phi^2 s + s^2\kappa\,(\rho_a + \phi\rho_f)}}_{\beta}\ \frac{1}{s^2\rho_f}\left(\hat{p}_{,i} + s^2\rho_f\hat{u}_i^s - \hat{f}_i^f\right) . \tag{14}$$

In equation (14), the abbreviation β is defined for further usage and $\mathscr{L}\{f\,(t)\} = \hat{f}\,(s)$ denotes the Laplace transform, with the complex variable s. Moreover, vanishing initial conditions for u_i^s and u_i^f are assumed here and in the following. Now, the final set of differential equations for the displacement $\hat{u}_i^s$ and the pore pressure $\hat{p}$ is obtained by inserting the constitutive equations (4) into the Laplace transformed dynamic equilibrium (11) and the continuity equation (5) with $\hat{w}_i$ from equation (14). This leads to the final set of differential equations for the displacement $\hat{u}_i^s$

and the pore pressure $\hat{p}$

$$G\hat{u}^s_{i,jj} + \left(K + \frac{1}{3}G\right)\hat{u}^s_{j,ij} - (\alpha-\beta)\,\hat{p}_{,i} - s^2\,(\rho - \beta\rho_f)\,\hat{u}^s_i = \beta\hat{f}^f_i - \hat{F}_i \tag{15a}$$

$$\frac{\beta}{s\rho_f}\hat{p}_{,ii} - \frac{\phi^2 s}{R}\hat{p} - (\alpha-\beta)\,s\hat{u}^s_{i,i} = -\hat{a} + \frac{\beta}{s\rho_f}\hat{f}^f_{i,i}\,. \tag{15b}$$

Certainly, the same elimination process of w^f_i can be performed in Fourier domain resulting in a similar set of equations.

This set of equations describes the behavior of a poroelastic continuum completely as well as the u^s_i-u^f_i-formulation (10) and the u^s_i-w_i-formulation (13). Contrary to both latter formulations an analytical representation of (15) in time domain is only possible for $\kappa \to \infty$. This represent a negligible friction between solid and interstitial fluid.

2.2 Incompressible models

In a two-phase material not only each constituent, the solid and the fluid, may be compressible on a microscopic level but also the skeleton itself possesses a structural compressibility. If the compression modulus of one constituent is much larger on the microscale than the compression modulus of the bulk material this constituent is assumed to be materially incompressible. A common example for a materially incompressible solid constituent is soil. In this case, the individual grains are much stiffer than the skeleton itself. The respective conditions for such incompressibilities are (Detournay and Cheng, 1993)

$$\frac{K}{K^s} \ll 1 \quad \text{incompressible solid,} \qquad \frac{K}{K^f} \ll 1 \quad \text{incompressible fluid}\,, \tag{16}$$

where K^s denotes the compression modulus of the solid grains and K^f the compression modulus of the fluid. With these conditions it is obvious that three cases exists:

i) only the solid is incompressible,

ii) only the fluid is incompressible, or

iii) the combination of both.

To find the respective constitutive equations for each of these cases the material parameters α, R, and Q have to be rewritten in a different way. Considerations of constitutive relations at micro mechanical level as given by Detournay and Cheng (1993) lead to a more rational model for this purpose

$$\alpha = 1 - \frac{K}{K^s} \tag{17a}$$

$$R = \frac{\phi^2 K^f K^{s2}}{K^f\,(K^s - K) + \phi K^s\,(K^s - K^f)} \tag{17b}$$

$$Q = \frac{\phi\,(\alpha - \phi)\,K^f K^{s2}}{K^f\,(K^s - K) + \phi K^s\,(K^s - K^f)}\,. \tag{17c}$$

Inserting in equations (17) the conditions of incompressibility (16) the three different cases are found:

- Incompressible solid $K/K^s \ll 1$

$$\alpha \approx 1 \qquad R \approx K^f\phi \qquad Q \approx K^f\,(1-\phi) \tag{18}$$

These limiting values can be inserted in the constitutive assumptions (3) or (4), respectively.

- Incompressible fluid $K/K^f \ll 1$

$$\alpha \text{ unchanged} \qquad R \approx \frac{\phi^2 K^s}{1-\phi-\dfrac{K}{K^s}} \qquad Q \approx \frac{\phi\,(\alpha-\phi)\,K^s}{1-\phi-\dfrac{K}{K^s}} \tag{19}$$

  Also in this case, these limiting values can be inserted in the constitutive assumptions (3) or (4), respectively.

- Both constituents are assumed to be incompressible $K/K^s \ll 1$ and $K/K^f \ll 1$

$$\alpha \approx 1 \qquad R \to \infty \qquad Q \to \infty \qquad \text{but} \qquad \frac{Q}{R} = \frac{1-\phi}{\phi} \tag{20}$$

  The relation $R,Q \to \infty$ expresses that the values of R and Q become large, however, due to physical reasons they are in any case limited. But, the condition that R becomes large is used to neglect in (4b) the influence of the pore pressure. This condition and $\alpha = 1$ results in the incompressible constitutive assumptions

$$\sigma_{ij} = G\left(u^s_{i,j} + u^s_{j,i}\right) + \left(K - \frac{2}{3}G\right) u^s_{k,k}\delta_{ij} - \delta_{ij}p \tag{21a}$$

$$\zeta = u^s_{k,k} \tag{21b}$$

  for the total stress formulation. From (21), it is obvious that this special modeling of a porous continuum relates the variation of fluid volume directly to the volumetric solid strain and the pore pressure is added to the solid stress linearly without the weighting factor α.

  For the partial stress formulation (3), a different point of view must be considered because inserting the infinite values of Q and R in the constitutive law (3) results in an infinite stress. Biot (1955) has given as condition for incompressible constituents

$$(1-\phi)\,\varepsilon^s_{kk} + \phi\varepsilon^f_{kk} = 0\,, \tag{22}$$

  i.e., it is assumed that the dilatation of the bulk material vanishes. Realizing the relation

$$\frac{Q}{R} = \frac{1-\phi}{\phi} \quad \Rightarrow \qquad \frac{Q}{R}\varepsilon^s_{kk} + \varepsilon^f_{kk} = \frac{Q}{R}u^s_{k,k} + u^f_{k,k} = 0 \tag{23}$$

  also in the partial stress formulation incompressible constituents can be included resulting in the constitutive assumptions

$$\sigma^s_{ij} = G\left(u^s_{i,j} + u^s_{j,i}\right) + \left(K - \frac{2}{3}G\right) u^s_{k,k}\delta_{ij} \tag{24a}$$

$$\sigma^f = -\phi p = R\left(\frac{Q}{R}u^s_{kk} + u^f_{kk}\right) \stackrel{!}{=} 0\,. \tag{24b}$$

  To achieve the zero value in equation (24b), the condition that the value R becomes large but is limited must be used.

  Comparing the incompressible constitutive equations (21) and (24), it becomes obvious that they do not coincide whereas the underlying compressible models are equal. Especially, in (24) an uncoupling of the solid and the fluid in the constitutive assumption occurs in contrast to (21). This is not really a contradiction. Keeping in mind that an incompressible model is always an approximation for the more realistic compressible case, it is clear that different approximations can exist.

It must be remarked that with an assumption of incompressibility different wave propagation phenomena may be lost. Especially in the case with both constituents modeled incompressible the fast compressional wave has an infinite velocity which is unphysical (Schanz and Pryl, 2004). Regarding the assumption of only one incompressible constituent (18) and (19) no special governing equations must be given because only the material data are changed and not the structure of the constitutive law. So, in the following, the expression 'incompressible' will denote the case when both constituents are modeled incompressible. The other two models are sometimes referred to as hybrid models (de Boer, 1998).

u_i^s-u_i^f-incompressible formulation Inserting in (10) the incompressibility condition (22) yields the governing equations

$$G u^s_{i,jj} + \left(K + \frac{1}{3}G\right) u^s_{j,ij} + (1-\phi)\, f_i^s = (1-\phi)\,\rho_s \ddot{u}_i^s - \rho_a \left(\ddot{u}_i^f - \ddot{u}_i^s\right) - \frac{\phi^2}{\kappa}\left(\dot{u}_i^f - \dot{u}_i^s\right) \tag{25a}$$

$$\phi f_i^f = \qquad \phi\rho_f \ddot{u}_i^f + \rho_a \left(\ddot{u}_i^f - \ddot{u}_i^s\right) + \frac{\phi^2}{\kappa}\left(\dot{u}_i^f - \dot{u}_i^s\right) \tag{25b}$$

using the solid displacement u_i^s and fluid displacement u_i^f as unknowns. Due to the uncoupling of the fluid and solid in the constitutive assumptions (24), in equations (25) only the coupling by the acceleration and damping terms remain. Further, the second equation (25b) is no longer an independent equation. As an additional equation the incompressibility condition (22) has to be used.

u_i^s-w_i-incompressible formulation In the derivation of the compressible u_i^s-w_i-formulation the total stress is used. The incompressible form of it (21a) has still the pore pressure as variable. However, for incompressible constituents the relation between the solid strain and the pore pressure is missing. Hence, an u_i^s-w_i-formulation can not be established. This is also true if the second incompressible model used for the u_i^s-u_i^f-formulation is applied. There, the solid displacement is related to the fluid displacement by a constant value (23) such that a relative displacement can no longer be a variable.

u_i^s-p-incompressible formulation Contrary, if the solid displacement and the pore pressure are used as unknowns a sufficient set of differential equations is obtained. Inserting in (15) simply the conditions (20), i.e., setting $\alpha = 1$ and taking the limit $R \to \infty$, the equations of motion under the assumption of incompressible constituents are achieved resulting in

$$G\hat{u}^s_{i,jj} + \left(K + \frac{1}{3}G\right)\hat{u}^s_{j,ij} - (1-\beta)\,\hat{p}_{,i} - s^2\,(\rho - \beta\rho_f)\,\hat{u}_i^s = \beta\hat{f}_i^f - \hat{F}_i \tag{26a}$$

$$\frac{\beta}{s\rho_f}\hat{p}_{,ii} - (1-\beta)\,s\hat{u}^s_{i,i} = -\hat{a} + \frac{\beta}{s\rho_f}\hat{f}^f_{i,i}\,. \tag{26b}$$

The equation for the pore pressure (26b) shows that this variable is no longer a degree of freedom. Integrating of (26b) yields the gradient of the pore pressure which can then be eliminated in (26a). Physically interpreted the pore pressure is in this case only determined by the deformation of the solid skeleton and no longer by any deformation of the fluid.

2.3 Simplified models

The set of governing equations (15) formulated with the unknowns solid displacements u_i and pore pressure p is one possible representation of Biot's linear theory for a poroelastic continuum in the Laplace domain. However, for the FEM, on the one hand, it is advantageous to have a description in the time domain which means the u_i^s-u_i^f-formulation (10) or the u_i^s-w_i-formulation (13) is preferable. But, on the other hand, using the reduced set of unknowns of the u_i^s-p-formulation is preferable under the view point of computer storage. Hence, a u_i^s-p-formulation in time domain is desirable.

The reason why a u_i^s-p–formulation is not possible in time domain is found in the generalized Darcy's law (12) where an elimination of the fluid displacement u_i^f or of the relative displacement w_i is not possible. However, if the second time derivative of the relative fluid to solid displacement w_i is neglected in the equilibrium (11) and in the dynamic version of Darcy's law (12) an elimination is possible. This results in the simplified dynamic equilibrium

$$\sigma_{ij,j} + F_i = \rho\ddot{u}_i^s\,, \tag{27}$$

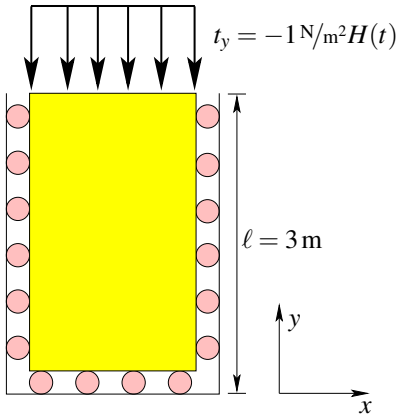


Figure 1: Geometry and boundary conditions of the poroelastic column

and the simplified dynamic version of Darcy's law

$$\dot{w}_i = q_i = -\kappa \left(p_{,i} + \rho_f \ddot{u}_i^s - f_i^f \right) . \tag{28}$$

Now, the simplified balance of momentum for the fluid (28) can be used to replace w_i in the equations (4) and (5). Rearranging them yields the governing set of differential equations for the unknowns solid displacement u_i^s and pore pressure p in the time domain

$$G u_{i,jj}^s + \left(K + \frac{1}{3} G \right) u_{j,ij}^s - \alpha p_{,i} - \rho \ddot{u}_i^s = -F_i \tag{29a}$$

$$\kappa p_{,ii} - \frac{\phi^2}{R} \dot{p} - \alpha \dot{u}_{i,i}^s + \kappa \rho_f \ddot{u}_{i,i}^s = \kappa f_{i,i}^f . \tag{29b}$$

This simplification has been published by Zienkiewicz et al. (1980). Beside this simplified version another even more simplified version exists (Zienkiewicz and Shiomi, 1984). There, also the inertia of the solid displacement in Darcy's law (28) is neglected, i.e., a static version of Darcy's law is used to formulate the balance of momentum for the fluid.

In Zienkiewicz et al. (1980), the limitations of the different simplifications are discussed with the help of an analytical 1-d example. Summarizing their results, in soil mechanics or geomechanical applications with mostly low frequency acceleration the complete Biot theory does not significantly differ from the simplified form. To demonstrate the effect of the simplifications on wave propagation phenomena the semi-analytical solution of a poroelastic column as depicted in Figure 1 is calculated. Due to the boundary conditions a one-dimensional problem is given which is solved in Laplace domain using standard techniques for ordinary differential equations (for details see Schanz and Cheng (2000) in case of the complete Biot model and Schanz and Struckmeier (2005) for the simplified model). The inverse transformation is performed with the Convolution Quadrature Method (Lubich, 1988a,b). The used material data are those of a soil which are taken from literature (Kim and Kingsbury, 1979) and are given in Table 1. The displacement of the midpoint at the top of the column is plotted versus time in Figure 2 for

Table 1: Material data of a water saturated soil (taken from Kim and Kingsbury (1979))

	$K\,(\mathrm{N/m^2})$	$G\,(\mathrm{N/m^2})$	$\rho\,(\mathrm{kg/m^3})$	$\rho_f\,(\mathrm{kg/m^3})$	ϕ	$R\,(\mathrm{N/m^2})$	α	$\kappa\,(\mathrm{m^4/Ns})$
soil	$2.1 \cdot 10^8$	$9.8 \cdot 10^7$	1884	1000	0.48	$1.2 \cdot 10^9$	0.98	$3.55 \cdot 10^{-9}$

Biot's theory (denoted 'Biot'), for the simplified theory (29) (denoted 'simple poro'), and for those equations with a static Darcy law (denoted 'static Darcy'). The differences are visible but as well it is obvious that they are small. Especially, the 'simple poro' model does not deviate too much from the full solution.

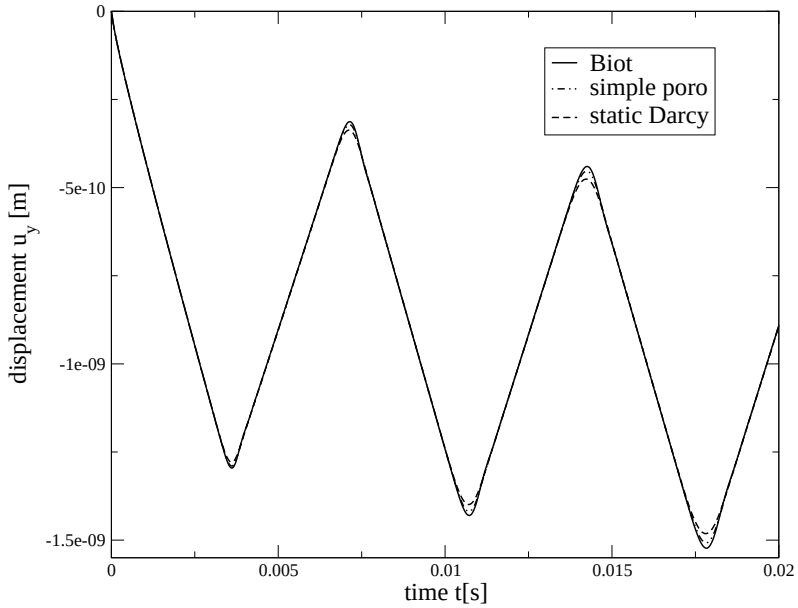
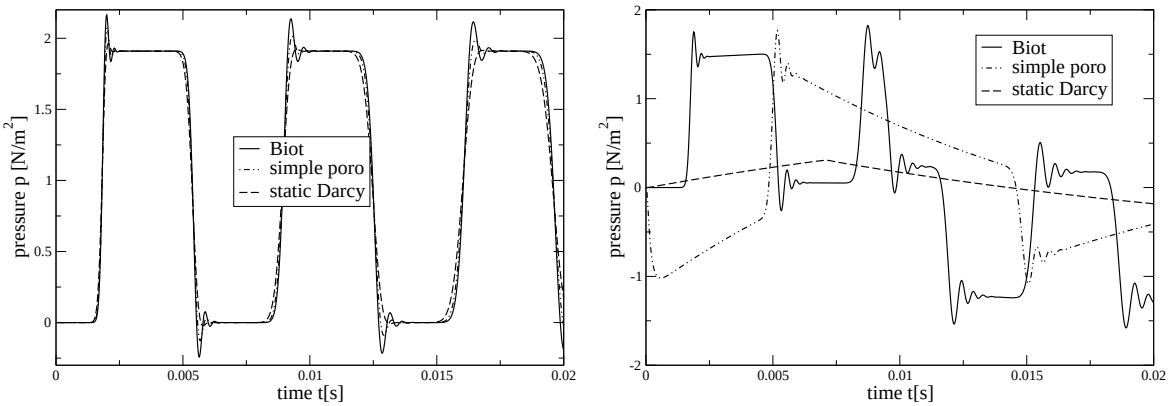


Figure 2: Comparison Biot's theory and the simplified versions: Displacement response of a poroelastic column due to a Heaviside stress load



(a) $\kappa = 3.55 \cdot 10^{-9}\,\mathrm{m}^4/\mathrm{Ns}$ (b) $\kappa = 1 \cdot 10^{-5}\,\mathrm{m}^4/\mathrm{Ns}$

Figure 3: Comparison Biot's theory and the simplified versions: Pressure response of a poroelastic column at the bottom for different permeabilities κ

The main differences in these theories concern the slow compressional wave. This highly dispersive wave can be made visible if the permeability is increased, i.e., the viscosity of the fluid is reduced. The pressure result at the bottom of the column is plotted versus time in Figure 3. The response corresponding to the realistic permeability is shown in picture 3(a) and for the increased permeability in picture 3(b). In both cases, a difference in the models is visible. Whereas in the realistic case only the pressure jumps at the moment of the wave arrival are not so sharp for the simplified model, for the reduced permeability only the full Biot model shows a correct behavior with visible wave fronts. Both other simplified theories result in a completely unphysical behavior.

For completeness it should be remarked that Bowen and Lockett (1983) have studied under what conditions a consolidation theory, i.e., all inertia terms are neglected, can be used. It has been found that even in the long time behavior for, e.g., harmonic loadings the inertia can not be neglected.

3 Boundary Element Method

The poroelastodynamic BE formulation based on the Convolution Quadrature Method is given next for the u_i^s-p-formulation. The same methodology for the simplified Biot model has been published in Schanz and Struckmeier (2005) and for the incompressible u_i^s-p-formulation in Pryl and Schanz (2006).

3.1 Boundary integral equation

The boundary integral equation for dynamic poroelasticity in Laplace domain can be obtained using either the corresponding reciprocal work theorem (Cheng et al., 1991) or the weighted residuals formulation (Domínguez, 1992). Here, the approach starting with the weighted residual statement is presented.

Weighted residuals The poroelastodynamic integral equation can be derived directly by equating the inner product of (15), written in matrix form with operator matrix

$$\mathbf{B} = \begin{bmatrix} G\nabla^2 + \left(K + \frac{1}{3}G\right)\partial_i\partial_j - s^2(\rho - \beta\rho_f) & -(\alpha-\beta)\partial_i \\ -s(\alpha-\beta)\partial_j & \dfrac{\beta}{s\rho_f}\nabla^2 - \dfrac{\phi^2 s}{R} \end{bmatrix} \tag{30}$$

and the matrix of the fundamental solutions $\mathbf{G}$ to a null vector, i.e.,

$$\int_\Omega \mathbf{G}^T\mathbf{B}\begin{bmatrix}\hat{u}_i^s \\ \hat{p}\end{bmatrix} \mathrm{d}\Omega = \mathbf{0} \qquad \text{with} \qquad \mathbf{G} = \begin{bmatrix}\hat{U}_{ij}^s & \hat{U}_i^f \\ \hat{P}_j^s & \hat{P}^f\end{bmatrix}, \tag{31}$$

where the integration is performed over a domain Ω with boundary Γ and vanishing body forces F_i and sources a are assumed. By this inner product, essentially, the error in satisfying the governing differential equations (15) is forced to be orthogonal to $\mathbf{G}$. According to the theory of Green's formula and using partial integration the operator $\mathbf{B}$ is transformed from acting on the vector of unknowns $[\hat{u}_i^s\ \hat{p}]^T$ to the matrix of fundamental solutions $\mathbf{G}$. These steps are easier understood looking at equation (31) written in index notation, i.e., at three integral equations for the solid ($j = 1,2,3$ in 3-d)

$$\begin{aligned}\int_\Omega \Bigg[& G\hat{u}_{i,kk}^s\hat{U}_{ij}^s + \left(K + \frac{1}{3}G\right)\hat{u}_{k,ik}^s\hat{U}_{ij}^s - (\alpha-\beta)\,\hat{p}_{,i}\hat{U}_{ij}^s - s^2(\rho-\beta\rho_f)\,\hat{u}_i^s\hat{U}_{ij}^s \\ & + \frac{\beta}{s\rho_f}\hat{p}_{,kk}\hat{P}_j^s - \frac{\phi^2 s}{R}\hat{p}\hat{P}_j^s - (\alpha-\beta)\,s\hat{u}_{k,k}^s\hat{P}_j^s\Bigg]\mathrm{d}\Omega = 0\end{aligned} \tag{32}$$

and at one integral equation for the fluid

$$\begin{aligned}\int_\Omega \Bigg[& G\hat{u}_{i,kk}^s\hat{U}_i^f + \left(K + \frac{1}{3}G\right)\hat{u}_{k,ik}^s\hat{U}_i^f - (\alpha-\beta)\,\hat{p}_{,i}\hat{U}_i^f - s^2(\rho-\beta\rho_f)\,\hat{u}_i^s\hat{U}_i^f \\ & + \frac{\beta}{s\rho_f}\hat{p}_{,kk}\hat{P}^f - \frac{\phi^2 s}{R}\hat{p}\hat{P}^f - (\alpha-\beta)\,s\hat{u}_{k,k}^s\hat{P}^f\Bigg]\mathrm{d}\Omega = 0\,.\end{aligned} \tag{33}$$

In the above integral equations, either one or two differentiations have to be transformed by either one or two partial integrations. To show the principal procedure, parts of integral equations (32) and (33) are exemplary presented in detail. The remaining partial integrations for the other parts in integral equations (32) and (33) can be performed analogously.

First, an integral with one differentiation in the kernel leads to (n_k is the outward normal vector)

$$\int_\Omega (\alpha-\beta)\,s\hat{u}_{k,k}^s\hat{P}^f\,\mathrm{d}\Omega = \int_\Gamma (\alpha-\beta)\,s\hat{u}_k^s n_k\hat{P}^f\,\mathrm{d}\Gamma - \int_\Omega (\alpha-\beta)\,s\hat{u}_k^s\hat{P}_{,k}^f\,\mathrm{d}\Omega \tag{34}$$

while an integral with two differentiation is transformed to

$$\begin{aligned}\int_\Omega G\hat{u}^s_{i,kk}\hat{U}^s_{ij}\,\mathrm{d}\Omega &= \int_\Gamma G\hat{u}^s_{i,k}n_k\hat{U}^s_{ij}\,\mathrm{d}\Gamma - \int_\Omega G\hat{u}^s_{i,k}\hat{U}^s_{ij,k}\,\mathrm{d}\Omega \\ &= \int_\Gamma G\hat{u}^s_{i,k}n_k\hat{U}^s_{ij}\,\mathrm{d}\Gamma - \int_\Gamma G\hat{u}^s_i\hat{U}^s_{ij,k}n_k\,\mathrm{d}\Gamma + \int_\Omega G\hat{u}^s_i\hat{U}^s_{ij,kk}\,\mathrm{d}\Omega\,.\end{aligned} \tag{35}$$

In both integrations by parts, the divergence theorem is used. Obviously, one integration by parts changes the sign of the resulting domain integral while it remains unchanged in the case of two integration by parts, i.e., the operator $\mathbf{B}$ is transformed into its adjoint operator $\mathbf{B}^*$. This yields the following system of integral equations given in matrix notation as

$$\int_\Gamma \begin{bmatrix}\hat{U}^s_{ij} & -\hat{P}^s_j\\ \hat{U}^f_i & -\hat{P}^f\end{bmatrix}\begin{bmatrix}\hat{t}_i\\ \hat{q}\end{bmatrix}\mathrm{d}\Gamma - \int_\Gamma \begin{bmatrix}\hat{T}^s_{ij} & \hat{Q}^s_j\\ \hat{T}^f_i & \hat{Q}^f\end{bmatrix}\begin{bmatrix}\hat{u}^s_i\\ \hat{p}\end{bmatrix}\mathrm{d}\Gamma = -\int_\Omega (\mathbf{B}^*\mathbf{G})^T\begin{bmatrix}\hat{u}^s_i\\ \hat{p}\end{bmatrix}\mathrm{d}\Omega = \begin{bmatrix}\hat{u}^s_j\\ \hat{p}\end{bmatrix}\,. \tag{36}$$

To solve the domain integral in equation (36) for $\mathbf{y}\in\Omega$, the definition of fundamental solutions

$$\mathbf{B}^*\mathbf{G} + \mathbf{I}\delta(\mathbf{x}-\mathbf{y}) = \mathbf{0} \tag{37}$$

and the property of the Dirac distribution $\delta(\mathbf{x}-\mathbf{y})$ is used. Additionally, the traction vector $\hat{t}_i = \hat{\sigma}_{ij}n_j$ and the normal flux $\hat{q} = -\frac{\beta}{s\rho_f}\left(\hat{p}_{,i} + \rho_f s^2\hat{u}^s_i\right)n_i$ is introduced, and the abbreviations

$$\hat{T}^s_{ij} = \left[\left(\left(K-\frac{2}{3}G\right)\hat{U}^s_{kj,k} + \alpha s\hat{P}^s_j\right)\delta_{i\ell} + G\left(\hat{U}^s_{ij,\ell} + \hat{U}^s_{\ell j,i}\right)\right]n_\ell \tag{38a}$$

$$\hat{Q}^s_j = \frac{\beta}{s\rho_f}\left[\hat{P}^s_{j,i} - \rho_f s\hat{U}^s_{ji}\right]n_i \tag{38b}$$

$$\hat{T}^f_i = \left[\left(\left(K-\frac{2}{3}G\right)\hat{U}^f_{k,k} + \alpha s\hat{P}^f\right)\delta_{i\ell} + G\left(\hat{U}^f_{i,\ell} + \hat{U}^f_{\ell,i}\right)\right]n_\ell \tag{38c}$$

$$\hat{Q}^f = \frac{\beta}{s\rho_f}\left[\hat{P}^f_{,j} - \rho_f s\hat{U}^f_j\right]n_j \tag{38d}$$

are used, where (38a) and (38b) can be interpreted as being the adjoint term to the traction vector $\hat{t}_i$ and the flux $\hat{q}$, respectively. With the fundamental solutions given in appendix A, the integral representation deduced starting from the weighted residuals is completely given.

Singular integral equation When moving $\mathbf{y}$ to the boundary Γ to determine the unknown boundary data, it is necessary to know the behavior of the fundamental solutions when $r = |\mathbf{y}-\mathbf{x}|$ tends to zero, i.e., when an integration point $\mathbf{x}$ approaches a collocation point $\mathbf{y}$. Six of the eight fundamental solutions, four in $\mathbf{G}$ and four calculated by equations (38), are singular. The order of their singularity can be determined by series representations. This leads to

$$\hat{P}^s_i,\ \hat{U}^f_i = \mathscr{O}\left(r^0\right) \tag{39a}$$

$$\hat{U}^s_{ij} = \underbrace{\frac{1+\nu}{8\pi E(1-\nu)}\left\{r_{,i}r_{,j} + \delta_{ij}(3-4\nu)\right\}\frac{1}{r}}_{\text{elastostatic fundamental solution}} + \mathscr{O}\left(r^0\right) \tag{39b}$$

$$\hat{P}^f = \frac{\rho_f s}{4\pi\beta}\frac{1}{r} + \mathscr{O}\left(r^0\right) \tag{39c}$$

$$\hat{Q}^s_j = \frac{1+\nu}{8\pi E(1-\nu)}\left\{\alpha(1-2\nu)(r_{,n}r_{,j} - n_j) - 2\beta(1-\nu)(r_{,n}r_{,j} + n_j)\right\}\frac{1}{r} + \mathscr{O}\left(r^0\right) \tag{39d}$$

$$\hat{T}^f_i = \frac{\rho_f s^2}{8\pi\beta}\left\{(\alpha-\beta)\frac{1-2\nu}{1-\nu}r_{,i}r_{,n} + n_i\frac{\alpha+\beta(1-2\nu)}{1-\nu}\right\}\frac{1}{r} + \mathscr{O}\left(r^0\right) \tag{39e}$$

$$\hat{T}^s_{ij} = \underbrace{\frac{-1}{8\pi(1-\nu)}\left\{\left[(1-2\nu)\delta_{ij} + 3r_{,i}r_{,j}\right]r_{,n} - (1-2\nu)(r_{,j}n_i - r_{,i}n_j)\right\}\frac{1}{r^2}}_{\text{elastostatic fundamental solution}} + \mathscr{O}\left(r^0\right) \tag{39f}$$

$$\hat{Q}^f = \underbrace{-\frac{r_{,n}}{4\pi r^2}}_{\text{acoustic fundamental solution}} + \mathscr{O}\left(r^0\right) . \tag{39g}$$

In equations (39), it is shown that the fundamental solutions are either regular (39a), weakly singular (39b) - (39e), or strongly singular (39f) and (39g). The strongly singular parts in the kernel functions (39f) and (39g) are known from elastostatics and acoustics, respectively. Therefore, shifting in (36) point $\mathbf{y}$ to the boundary Γ results in the boundary integral equation

$$\int_\Gamma \begin{bmatrix} \hat{U}_{ij}^s & -\hat{P}_j^s \\ \hat{U}_i^f & -\hat{P}^f \end{bmatrix} \begin{bmatrix} \hat{t}_i \\ \hat{q} \end{bmatrix} \mathrm{d}\Gamma = \oint_\Gamma \begin{bmatrix} \hat{T}_{ij}^s & \hat{Q}_j^s \\ \hat{T}_i^f & \hat{Q}^f \end{bmatrix} \begin{bmatrix} \hat{u}_i^s \\ \hat{p} \end{bmatrix} \mathrm{d}\Gamma + \begin{bmatrix} c_{ij} & 0 \\ 0 & c \end{bmatrix} \begin{bmatrix} \hat{u}_i^s \\ \hat{p} \end{bmatrix} \tag{40}$$

with the integral free terms c_{ij} and c known from elastostatics and acoustics, respectively, and with the Cauchy principal value integral $\oint$. A transformation to time domain gives, finally, the time dependent integral equation for poroelasticity

$$\begin{aligned} &\int_0^t \int_\Gamma \begin{bmatrix} U_{ij}^s(t-\tau,\mathbf{y},\mathbf{x}) & -P_j^s(t-\tau,\mathbf{y},\mathbf{x}) \\ U_i^f(t-\tau,\mathbf{y},\mathbf{x}) & -P^f(t-\tau,\mathbf{y},\mathbf{x}) \end{bmatrix} \begin{bmatrix} t_i(\tau,\mathbf{x}) \\ q(\tau,\mathbf{x}) \end{bmatrix} \mathrm{d}\Gamma\,\mathrm{d}\tau = \\ &\qquad \int_0^t \oint_\Gamma \begin{bmatrix} T_{ij}^s(t-\tau,\mathbf{y},\mathbf{x}) & Q_j^s(t-\tau,\mathbf{y},\mathbf{x}) \\ T_i^f(t-\tau,\mathbf{y},\mathbf{x}) & Q^f(t-\tau,\mathbf{y},\mathbf{x}) \end{bmatrix} \begin{bmatrix} u_i(\tau,\mathbf{x}) \\ p(\tau,\mathbf{x}) \end{bmatrix} \mathrm{d}\Gamma\,\mathrm{d}\tau + \begin{bmatrix} c_{ij}(\mathbf{y}) & 0 \\ 0 & c(\mathbf{y}) \end{bmatrix} \begin{bmatrix} u_i(t,\mathbf{y}) \\ p(t,\mathbf{y}) \end{bmatrix} . \end{aligned} \tag{41}$$

In the 2-d case, all equations are similar (i,j=1,2) but the singularities are as in the elastodynamic and acoustic 2-d formulation (see, Schanz and Pryl (2004)).

3.2 Boundary element formulation

A boundary element formulation is achieved following the usual procedure. First, the boundary surface Γ is discretised by E iso-parametric elements Γ_e where F polynomial shape functions $N_e^f(\mathbf{x})$ are defined. Hence, the following ansatz functions are used with the time dependent nodal values $u_i^{ef}(t)$,$t_i^{ef}(t)$, $p^{ef}(t)$, and $q^{ef}(t)$

$$\begin{aligned} u_i(\mathbf{x},t) &= \sum_{e=1}^{E}\sum_{f=1}^{F} N_e^f(\mathbf{x})\, u_i^{ef}(t) \quad t_i(\mathbf{x},t) = \sum_{e=1}^{E}\sum_{f=1}^{F} N_e^f(\mathbf{x})\, t_i^{ef}(t) \\ p(\mathbf{x},t) &= \sum_{e=1}^{E}\sum_{f=1}^{F} N_e^f(\mathbf{x})\, p^{ef}(t) \quad q(\mathbf{x},t) = \sum_{e=1}^{E}\sum_{f=1}^{F} N_e^f(\mathbf{x})\, q^{ef}(t) . \end{aligned} \tag{42}$$

In equations (42), the shape functions of all four variables are denoted by the same function $N_e^f(\mathbf{x})$ indicating the same approximation level of all variables. This is not mandatory but usual (for different combinations of shape functions see Pryl and Schanz (2006)). Inserting these ansatz functions (42) in the time dependent integral equation (41) yields

$$\begin{aligned} \begin{bmatrix} c_{ij}(\mathbf{y}) & 0 \\ 0 & c(\mathbf{y}) \end{bmatrix} \begin{bmatrix} u_i(\mathbf{y},t) \\ p(\mathbf{y},t) \end{bmatrix} = \sum_{e=1}^{E}\sum_{f=1}^{F} \Bigg\{ &\int_0^t \int_\Gamma \begin{bmatrix} U_{ij}^s(t-\tau,\mathbf{y},\mathbf{x}) & -P_j^s(t-\tau,\mathbf{y},\mathbf{x}) \\ U_i^f(t-\tau,\mathbf{y},\mathbf{x}) & -P^f(t-\tau,\mathbf{y},\mathbf{x}) \end{bmatrix} N_e^f(\mathbf{x}) \begin{bmatrix} t_i^{ef}(\tau) \\ q^{ef}(\tau) \end{bmatrix} \mathrm{d}\Gamma\,\mathrm{d}\tau \\ &- \int_0^t \oint_\Gamma \begin{bmatrix} T_{ij}^s(t-\tau,\mathbf{y},\mathbf{x}) & Q_j^s(t-\tau,\mathbf{y},\mathbf{x}) \\ T_i^f(t-\tau,\mathbf{y},\mathbf{x}) & Q^f(t-\tau,\mathbf{y},\mathbf{x}) \end{bmatrix} N_e^f(\mathbf{x}) \begin{bmatrix} u_i^{ef}(\tau) \\ p^{ef}(\tau) \end{bmatrix} \mathrm{d}\Gamma\,\mathrm{d}\tau \Bigg\} . \end{aligned} \tag{43}$$

Next, a time discretization has to be introduced. Since no time dependent fundamental solutions are known, the Convolution Quadrature Method (briefly summarized in appendix B) is the most effective method compared to the possibility inverting the Laplace domain fundamental solutions at every collocation point in every time step using a series expansion (Chen and Dargush, 1995).

Hence, after dividing the time period t in N intervals of equal duration Δt, i.e., $t = N\Delta t$, the convolution integrals between the fundamental solutions and the nodal values in (43) are approximated by the Convolution Quadrature Method, i.e., the quadrature formula (58) is applied to the integral equation (43). This results in the following boundary element time stepping formulation for $n = 0, 1, \ldots, N$

$$\begin{bmatrix} c_{ij}(\mathbf{y}) & 0 \\ 0 & c(\mathbf{y}) \end{bmatrix} \begin{bmatrix} u_i(\mathbf{y},n\Delta t) \\ p(\mathbf{y},n\Delta t) \end{bmatrix} = \sum_{e=1}^{E} \sum_{f=1}^{F} \sum_{k=0}^{n} \left\{ \begin{bmatrix} \omega_{n-k}^{ef}\left(\hat{U}_{ij}^{s},\mathbf{y},\Delta t\right) & -\omega_{n-k}^{ef}\left(\hat{P}_{j}^{s},\mathbf{y},\Delta t\right) \\ \omega_{n-k}^{ef}\left(\hat{U}_{i}^{f},\mathbf{y},\Delta t\right) & -\omega_{n-k}^{ef}\left(\hat{P}^{f},\mathbf{y},\Delta t\right) \end{bmatrix} \begin{bmatrix} t_i^{ef}(k\Delta t) \\ q^{ef}(k\Delta t) \end{bmatrix} - \begin{bmatrix} \omega_{n-k}^{ef}\left(\hat{T}_{ij}^{s},\mathbf{y},\Delta t\right) & \omega_{n-k}^{ef}\left(\hat{Q}_{j}^{s},\mathbf{y},\Delta t\right) \\ \omega_{n-k}^{ef}\left(\hat{T}_{i}^{f},\mathbf{y},\Delta t\right) & \omega_{n-k}^{ef}\left(\hat{Q}^{f},\mathbf{y},\Delta t\right) \end{bmatrix} \begin{bmatrix} u_i^{ef}(k\Delta t) \\ p^{ef}(k\Delta t) \end{bmatrix} \right\} \tag{44}$$

with the weights corresponding to (60), e.g.,

$$\omega_{n-k}^{ef}\left(\hat{U}_{ij}^{s},\mathbf{y},\Delta t\right) = \frac{\mathscr{R}^{-(n-k)}}{L} \sum_{\ell=0}^{L-1} \int_{\Gamma} \hat{U}_{ij}^{s}\left(\frac{\gamma\left(e^{i\ell\frac{2\pi}{L}}\mathscr{R}\right)}{\Delta t},\mathbf{y},\mathbf{x}\right) N_e^f(\mathbf{x})\,\mathrm{d}\Gamma\; e^{-i(n-k)\ell\frac{2\pi}{L}}\,. \tag{45}$$

Note, the calculation of the integration weights is only based on the Laplace transformed fundamental solutions which are available. Therefore, with the time stepping procedure (44) a boundary element formulation for poroelastodynamics is given without time dependent fundamental solutions.

To calculate the integration weights ω_{n-k}^{ef} in (44), spatial integration over the boundary Γ has to be performed. Because the essential constituents of the Laplace transformed fundamental solutions are exponential functions, i.e., the integrand is smooth, the regular integrals are evaluated by standard Gaussian quadrature rule, while the weakly singular parts of the integrals in (44) are regularized by polar coordinate transformation. The strongly singular integrals in (44) are equal to those of elastostatics or acoustics, respectively, and, hence, the regularization methods known from these theories can be applied, e.g., the method suggested by Guiggiani and Gigante (1990). Moreover, to obtain for equation (44) a system of algebraic equations, collocation is used at every node of the shape functions $N_e^f(\mathbf{x})$.

According to $t - \tau = (n-k)\Delta t$, the integration weights ω_{n-k}^{ef} are only dependent on the difference $n-k$. This property is analogous to elastodynamic time domain BE formulations (see, e.g., Domínguez (1993)) and can be used to establish a recursion formula ($m = n-k$)

$$\omega_0(\mathbf{C})\,\mathbf{d}^n = \omega_0(\mathbf{D})\,\bar{\mathbf{d}}^n + \sum_{m=1}^{n} \left(\omega_m(\mathbf{U})\,\mathbf{t}^{n-m} - \omega_m(\mathbf{T})\,\mathbf{u}^{n-m}\right) \quad n = 1,2,\ldots,N \tag{46}$$

with the time dependent integration weights ω_m containing the Laplace transformed fundamental solutions $\mathbf{U}$ and $\mathbf{T}$, respectively (see, equation (45)). Similarly, $\omega_0(\mathbf{C})$ and $\omega_0(\mathbf{D})$ are the corresponding integration weights of the first time step related to the unknown and known boundary data in time step n $\mathbf{d}^n$ and $\bar{\mathbf{d}}^n$, respectively. Finally, a direct equation solver is applied. Clearly, the 2-d formulation does not differ in this abstract notation.

Dimensionless variables An aspect in the numerical implementation is the choice of dimensionless variables. The easiest choice is to normalize the variables on the total time t_{max}, on the largest distance in the mesh r_{max}, and on a material constant like the Young's modulus E. This has been implemented and results in

$$\begin{aligned} \tilde{x}_i &= \frac{x_i}{r_{max}} \quad \tilde{t} = \frac{t}{t_{max}} \quad \tilde{E} = 1 \quad \tilde{R} = \frac{R}{E} \\ \tilde{\rho} &= \frac{r_{max}^2}{t_{max}^2 E}\rho \quad \tilde{\rho_f} = \frac{r_{max}^2}{t_{max}^2 E}\rho_f \quad \tilde{\kappa} = \frac{t_{max}E}{r_{max}^2}\kappa\,. \end{aligned} \tag{47}$$

Another, more complicated way, is to normalize all material data as suggested by Chen and Dargush (1995). Both these choices and a few other have been implemented and compared by Schanz and Kielhorn (2005). The

comparison is mainly based on the condition numbers of the system matrices for several different geometries, material data, and discretisation. In most cases the choice (47) yields the best results. However, there are a few constellations where the choice of Chen and Dargush is superior. It should also be remarked that a calculation without dimensionless variables is mostly not possible at all, i.e., the inversion of the matrix of the first time step fails.

4 Wave propagation in a poroelastic half space

To demonstrate that the results of the u_i^s-p-formulation with neglect of the derivative of the seepage velocity are similar to the results of Biot's complete theory, the displacement response and the pore pressure distribution of a poroelastic half space in 2-d and 3-d is compared, respectively. The material data in both test examples are those of a soil (see table 1).

4.1 2-d model of a poroelastic half space

First, the half space is modeled in 2-d with a strip of 51 m length, where 51 linear elements are used (see figure 4). The simulated half-space is loaded by a vertical total stress vector $t_y = -1000\,\mathrm{N/m^2}\,(H(t) - H(\Delta t))$ at an area of 1 m

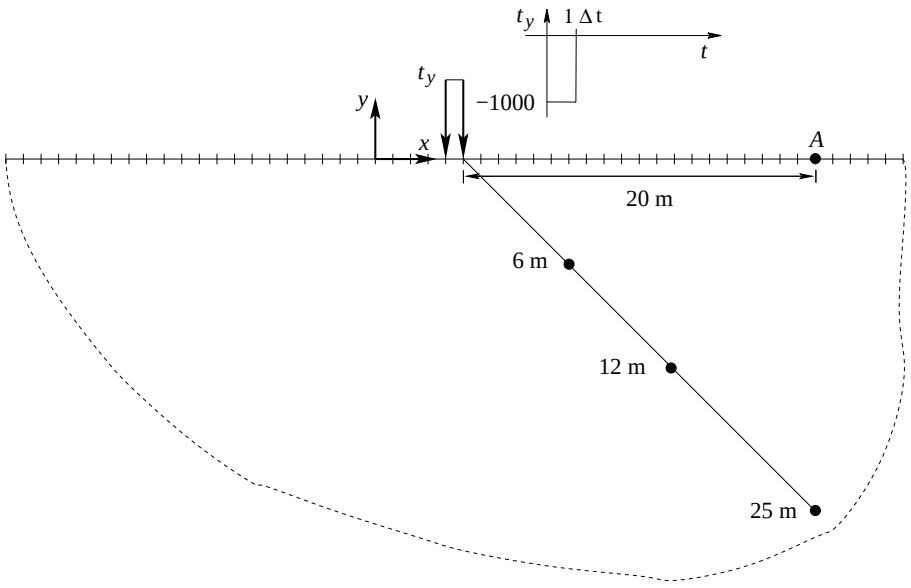
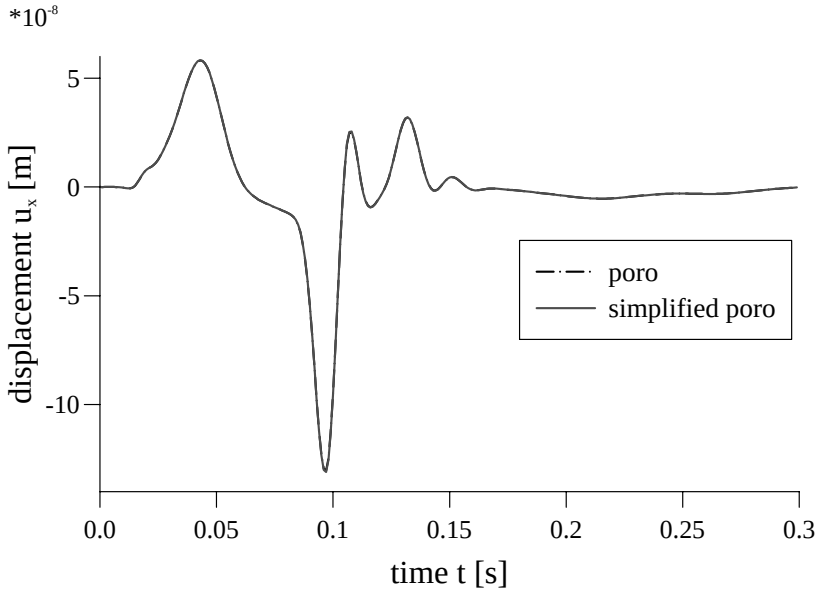


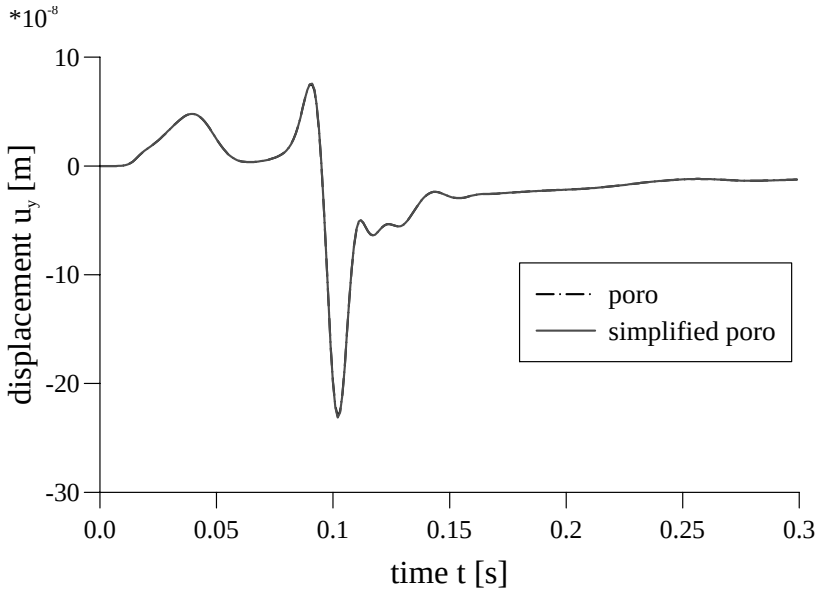
Figure 4: Poroelastic half space in 2-d: mesh and loading

and the remaining surface is traction free. The load simulates an impulse by keeping the load over one time step. The free surface is assumed to be permeable, i.e., the pore pressure is zero all over the surface.

First, the time history of the displacement at point A is presented. In figure 5, the calculated horizontal and vertical displacement at point A is plotted versus time for both formulations. The u_i^s-p-formulation with neglect of the derivative of the seepage velocity are denoted 'simplified poro' and the original Biot u_i^s-p-formulation is denoted 'poro'. Clearly, the arrival of the fast compressional wave at $t \approx 0.01\,\mathrm{s}$ and of the Rayleigh wave at $t \approx 0.09\,\mathrm{s}$ can be observed. As expected the slow compressional wave is not visible due to the dispersion effects and the shear wave is covered by the Rayleigh wave. In both coordinate directions no differences are visible between both formulations. The differences in the displacement amplitudes are approximately of the order $\mathcal{O}\left(10^{-3}\right)$.



(a) horizontal



(b) vertical

Figure 5: Vertical and horizontal displacement at point A

Additionally the pore pressure distribution under the surface is observed by variation of the depth from -6 m to -20 m. The various locations are depicted in figure 4. The time histories of the pore pressure are presented in figure 6 for both formulations. As before in the displacement results, no significant differences between the simplified formulation and Biot's equations are found. In all three depths the arrival of the fast compressional wave is observed as a more or less wide peak. After some oscillations of the numerical solution the pore pressure decreases to zero as expected result for an impulse load.

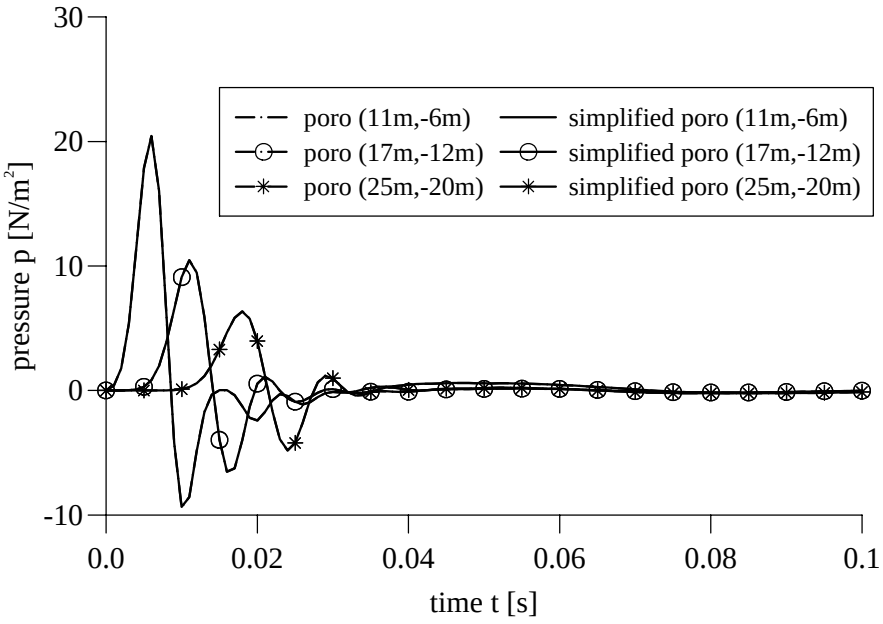


Figure 6: Pore pressure distribution below the surface at different points

4.2 3-d model of a poroelastic half space

For the 3-d model of the half space a strip of 33 m $\times$ 6 m has been discretised with 396 triangular linear elements on 238 nodes (see figure 7). Different to the 2-d simulation, the half space is loaded by a vertical total stress vector

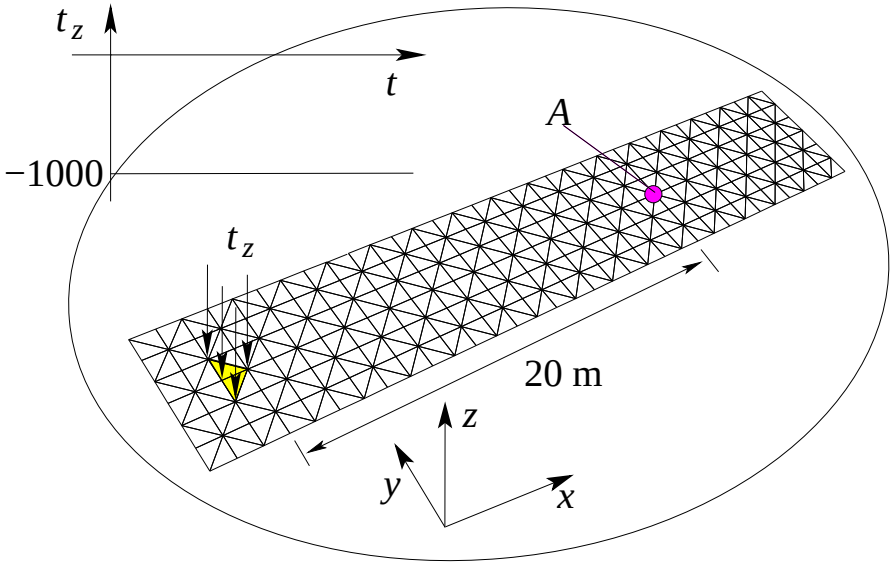
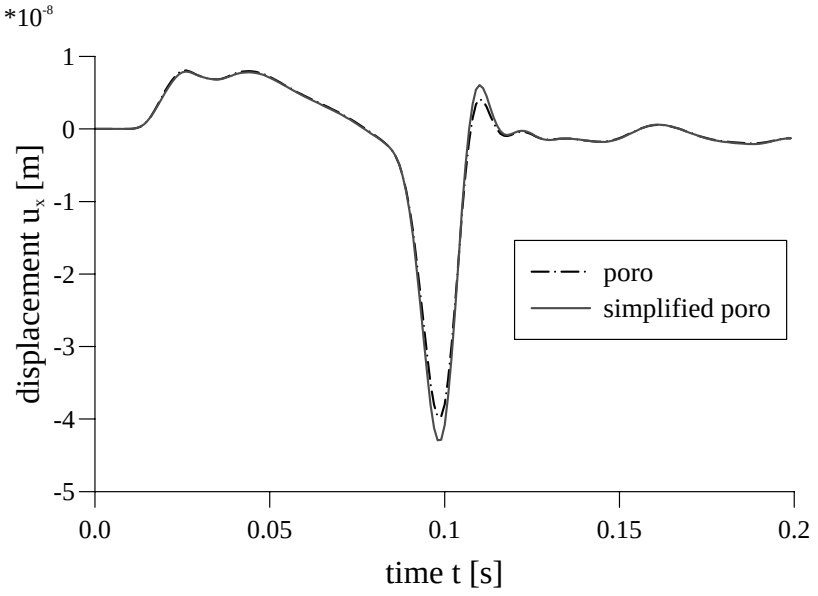


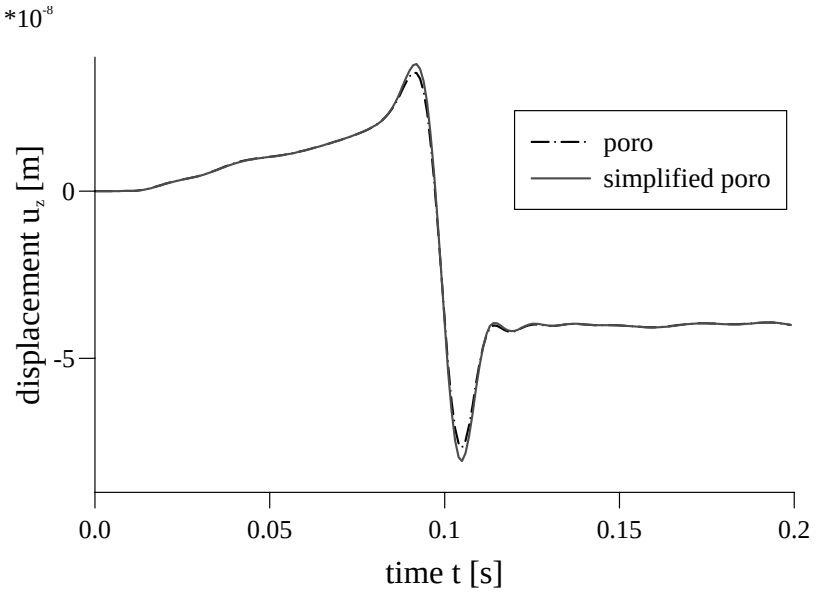
Figure 7: Poroelastic half space in 3-d: mesh and loading

$t_z = -1000\,\mathrm{N/m^2} H(t)$ at an area of $1\,\mathrm{m}^2$ which is kept constant over the whole observation period. The remaining surface is traction free and assumed to be permeable, i.e., the pore pressure is zero all over the surface.

In figure 8, the calculated horizontal and vertical displacement is plotted versus time at point A. Different to the 2-d example in 3-d some differences between the simplified theory and Biot's theory are visible. However, these



(a) horizontal



(b) vertical

Figure 8: Vertical and horizontal displacement at point A

differences are very small and in the range which can also be affected by numerics, i.e., also a change in the time step size can result in differences of the same order. So, in principle it can be concluded that also in the 3-d calculation both formulations give the same result.

The pore pressure distribution in different depths comparable to the study in 2-d is presented in figure 9. There, the pore pressure is depicted versus time in a depth of 6 m, 12 m, and 20 m. Due to the larger distance from the excitation point the fast compressional wave needs different times to reach the chosen points. Also different to the 2-d calculation the pore pressure does not vanish after the passage of the wave because the load is kept over the total observation period. Further, the pore pressure reduces with increasing depth as expected.

Finally, this comparison shows that the simplified theory can be used for the chosen material, a soil and a rock, and

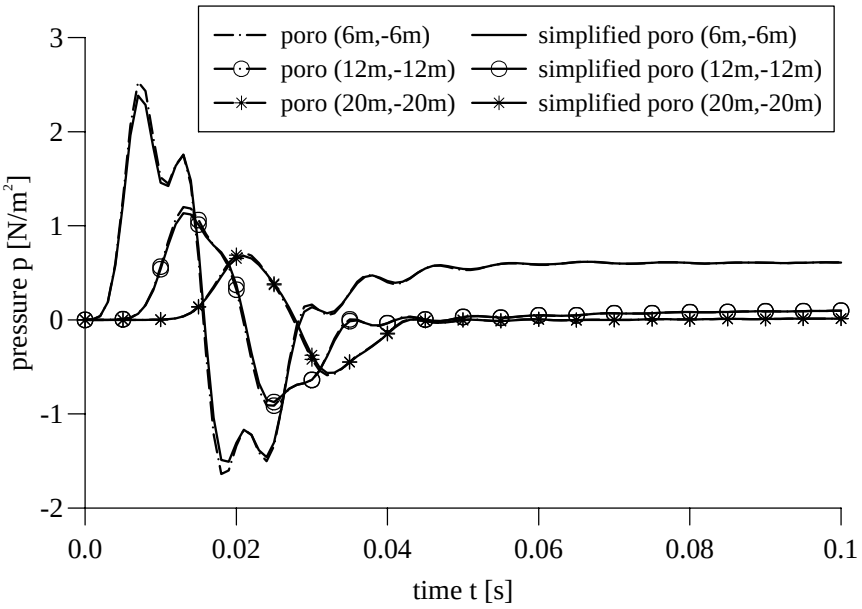


Figure 9: Pore pressure distribution below point A

the presented excitations. There is no significant difference to Biot's complete theory. This confirms the results presented by Zienkiewicz et al. (1980).

5 Conclusions

Different formulations of Biot's theory utilizing either solid and fluid displacements or solid displacements and pore pressure as unknowns are recalled. As well two incompressible models are presented. For the realisation of a time domain formulation with the reduced set of unknowns – solid displacements and pore pressure – a simplified theory is given.

To solve 2-d and 3-d problems a poroelastodynamic BE formulation is presented based on the Convolution Quadrature Method. Hence, this methodology can use the available Laplace domain fundamental solutions. This is true for the complete Biot theory as well as for the simplified version.

In the numerical examples, the complete Biot theory is compared with the simplified version with view on treating wave propagation problems. For the investigated material (soil) both theories give nearly the same results. However, the slow compressional wave is not modelled correctly by the simplified theory. But, the effects of this wave type can only be observed if the permeability is changed. Hence, it can be concluded that the simplification is not allowed for all materials.

A Poroelastodynamic fundamental solutions

The explicit expressions of the poroelastodynamic fundamental solutions are given in the following. The four elements of the matrix **G** are the displacements caused by a Dirac force in the solid:

$$\hat{U}^s_{ij} = \frac{1}{4\pi r\left(\rho-\beta\rho_f\right)s^2}\left[R_1\frac{\lambda_4^2-\lambda_2^2}{\lambda_1^2-\lambda_2^2}e^{-\lambda_1 r} - R_2\frac{\lambda_4^2-\lambda_1^2}{\lambda_1^2-\lambda_2^2}e^{-\lambda_2 r} + \left(\delta_{ij}\lambda_3^2 - R_3\right)e^{-\lambda_3 r}\right] \tag{48}$$

with $R_k = \frac{3r_{,i}r_{,j}-\delta_{ij}}{r^2} + \lambda_k\frac{3r_{,i}r_{,j}-\delta_{ij}}{r} + \lambda_k^2 r_{,i}r_{,j}$ and $\lambda_4^2 = \frac{s^2\left(\rho-\beta\rho_f\right)}{K+\frac{4}{3}G}$. The pressure caused by the same load is

$$\hat{P}^s_j = \frac{(\alpha-\beta)\,s\rho_f r_{,j}}{4\pi\beta\left(K+\frac{4}{3}G\right)r\left(\lambda_1^2-\lambda_2^2\right)}\left[\left(\lambda_1+\frac{1}{r}\right)e^{-\lambda_1 r} - \left(\lambda_2+\frac{1}{r}\right)e^{-\lambda_2 r}\right]. \tag{49}$$

For a Dirac source in the fluid the respective displacement solution is

$$\hat{U}^f_i = s\hat{P}^s_i \tag{50}$$

and the pressure

$$\hat{P}^f = \frac{s\rho_f}{4\pi r\beta\left(\lambda_1^2-\lambda_2^2\right)}\left[\left(\lambda_1^2-\lambda_4^2\right)e^{-\lambda_1 r} - \left(\lambda_2^2-\lambda_4^2\right)e^{-\lambda_2 r}\right]. \tag{51}$$

The roots $\lambda_i,\ i = 1,2,3$ are

$$\lambda^2_{1,2} = \frac{s^2}{2}\left[\frac{\phi^2\rho_f}{\beta R} + \frac{\rho-\beta\rho_f}{K+\frac{4}{3}G} + \frac{\rho_f(\alpha-\beta)^2}{\beta\left(K+\frac{4}{3}G\right)} \pm \sqrt{\left(\frac{\phi^2\rho_f}{\beta R} + \frac{\rho-\beta\rho_f}{K+\frac{4}{3}G} + \frac{\rho_f(\alpha-\beta)^2}{\beta\left(K+\frac{4}{3}G\right)}\right)^2 - 4\frac{\phi^2\rho_f\left(\rho-\beta\rho_f\right)}{\beta R\left(K+\frac{4}{3}G\right)}}\right] \tag{52}$$

$$\lambda_3^2 = \frac{s^2\left(\rho-\beta\rho_f\right)}{G}. \tag{53}$$

In the derivation of the poroelastodynamic boundary integral equation (36) several abbreviations (38) corresponding to an 'adjoint' traction or flux are introduced. First, the 'adjoint' traction solution is presented. However, due to the extensive expression only parts are given

$$\hat{T}^s_{ij} = \left[\left(\left(K-\frac{2}{3}G\right)\hat{U}^s_{kj,k} + \alpha s\hat{P}^s_j\right)\delta_{i\ell} + G\left(\hat{U}^s_{ij,\ell} + \hat{U}^s_{\ell j,i}\right)\right]n_\ell \tag{54}$$

$$\begin{aligned}\hat{U}^s_{kj,k}\delta_{i\ell}n_\ell = \frac{r_{,j}n_i}{4\pi r s^2\left(\rho-\beta\rho_f\right)\left(\lambda_1^2-\lambda_2^2\right)}\Bigg[&e^{-\lambda_1 r}\left(\frac{1}{r}+\lambda_1\right)\lambda_1^2\left(\lambda_2^2-\lambda_4^2\right)\\ &- e^{-\lambda_2 r}\left(\frac{1}{r}+\lambda_2\right)\lambda_2^2\left(\lambda_1^2-\lambda_4^2\right)\Bigg]\end{aligned}$$

$$\begin{aligned}\left(\hat{U}^s_{ij,\ell}+\hat{U}^s_{\ell j,i}\right)n_\ell = \frac{1}{4\pi r s^2\left(\rho-\beta\rho_f\right)}\Bigg[&\frac{R_5 6}{r^3}\left(\frac{\lambda_4^2-\lambda_2^2}{\lambda_1^2-\lambda_2^2}e^{-\lambda_1 r} - \frac{\lambda_4^2-\lambda_1^2}{\lambda_1^2-\lambda_2^2}e^{-\lambda_2 r} - e^{-\lambda_3 r}\right)\\ &+\frac{R_5 6}{r^2}\left(\frac{\lambda_4^2-\lambda_2^2}{\lambda_1^2-\lambda_2^2}\lambda_1 e^{-\lambda_1 r} - \frac{\lambda_4^2-\lambda_1^2}{\lambda_1^2-\lambda_2^2}\lambda_2 e^{-\lambda_2 r} - \lambda_3 e^{-\lambda_3 r}\right)\\ &+\frac{R_6 2}{r}\left(\frac{\lambda_4^2-\lambda_2^2}{\lambda_1^2-\lambda_2^2}\lambda_1^2 e^{-\lambda_1 r} - \frac{\lambda_4^2-\lambda_1^2}{\lambda_1^2-\lambda_2^2}\lambda_2^2 e^{-\lambda_2 r} - \lambda_3^2 e^{-\lambda_3 r}\right)\\ &-2r_{,n}r_{,i}r_{,j}\left(\frac{\lambda_4^2-\lambda_2^2}{\lambda_1^2-\lambda_2^2}\lambda_1^3 e^{-\lambda_1 r} - \frac{\lambda_4^2-\lambda_1^2}{\lambda_1^2-\lambda_2^2}\lambda_2^3 e^{-\lambda_2 r} - \lambda_3^3 e^{-\lambda_3 r}\right)\\ &-\lambda_3^2\left(\delta_{ij}r_{,n}+r_{,i}n_j\right)\left(\lambda_3+\frac{1}{r}\right)e^{-\lambda_3 r}\Bigg]\end{aligned}$$

with $R_5 = r_{,j}n_i + r_{,i}n_j + r_{,n}(\delta_{ij} - 5r_{,i}r_{,j})$ and $R_6 = r_{,j}n_i + r_{,i}n_j + r_{,n}(\delta_{ij} - 6r_{,i}r_{,j})$. The other explicit expressions are:

$$\hat{Q}_j^s = \frac{n_i}{4\pi r(\rho - \beta\rho_f)s^2}\left[\frac{e^{-\lambda_1 r}}{\lambda_1^2 - \lambda_2^2}R_1\left(\beta\lambda_2^2 - \alpha\lambda_4^2\right) - \frac{e^{-\lambda_2 r}}{\lambda_1^2 - \lambda_2^2}R_2\left(\beta\lambda_1^2 - \alpha\lambda_4^2\right) + \beta e^{-\lambda_3 r}\left(R_3 - \delta_{ij}\lambda_3^2\right)\right] \tag{55}$$

$$\begin{aligned}\hat{T}_i^f = \frac{s^2\rho_f}{4\pi r\beta\left(\lambda_1^2 - \lambda_2^2\right)}\Bigg[&\frac{n_j(\alpha-\beta)2G}{K + \frac{4}{3}G}\left(R_2 e^{-\lambda_2 r} - R_1 e^{-\lambda_1 r}\right) \\ &+ n_i e^{-\lambda_2 r}\left(\frac{(\alpha-\beta)\left(K - \frac{2}{3}G\right)}{K + \frac{4}{3}G}\left(\frac{2}{r^2} + \frac{2\lambda_2}{r} + \lambda_2^2\right) - \alpha\left(\lambda_2^2 - \lambda_4^2\right)\right) \\ &- n_i e^{-\lambda_1 r}\left(\frac{(\alpha-\beta)\left(K - \frac{2}{3}G\right)}{K + \frac{4}{3}G}\left(\frac{2}{r^2} + \frac{2\lambda_1}{r} + \lambda_1^2\right) - \alpha\left(\lambda_1^2 - \lambda_4^2\right)\right)\Bigg]\end{aligned} \tag{56}$$

$$\begin{aligned}\hat{Q}^f = \frac{r_{,n}}{4\pi r\left(\lambda_1^2 - \lambda_2^2\right)}\Bigg[&\left(\lambda_2 + \frac{1}{r}\right)\left(\lambda_2^2 - \lambda_4^2\frac{\rho - \alpha\rho_f}{\rho - \beta\rho_f}\right)e^{-\lambda_2 r} \\ &- \left(\lambda_1 + \frac{1}{r}\right)\left(\lambda_1^2 - \lambda_4^2\frac{\rho - \alpha\rho_f}{\rho - \beta\rho_f}\right)e^{-\lambda_1 r}\Bigg]\end{aligned} \tag{57}$$

B Convolution Quadrature Method

The 'Convolution Quadrature Method' developed by Lubich numerically approximates a convolution integral for $n = 0, 1, \ldots, N$

$$y(t) = \int_0^t f(t-\tau)\,g(\tau)\,\mathrm{d}\tau \quad \rightarrow \quad y(n\Delta t) = \sum_{k=0}^{n}\omega_{n-k}(\Delta t)\,g(k\Delta t), \tag{58}$$

by a quadrature rule whose weights are determined by the Laplace transformed function $\hat{f}$ and a linear multistep method. This method was originally published in Lubich (1988a) and Lubich (1988b). Application to the boundary element method may be found in Schanz and Antes (1997b). Here, a brief overview of the method is given.

In formula (58), the time t is divided in N equal steps Δt. The weights $\omega_n(\Delta t)$ are the coefficients of the power series

$$\hat{f}\left(\frac{\gamma(z)}{\Delta t}\right) = \sum_{n=0}^{\infty}\omega_n(\Delta t)\,z^n \tag{59}$$

with the complex variable z. The coefficients of a power series are usually calculated with Cauchy's integral formula. After a polar coordinate transformation, this integral is approximated by a trapezoidal rule with L equal steps $\frac{2\pi}{L}$. This leads to

$$\omega_n(\Delta t) = \frac{1}{2\pi i}\int_{|z|=\mathscr{R}}\hat{f}\left(\frac{\gamma(z)}{\Delta t}\right)z^{-n-1}\,\mathrm{d}z \approx \frac{\mathscr{R}^{-n}}{L}\sum_{\ell=0}^{L-1}\hat{f}\left(\frac{\gamma\left(\mathscr{R}e^{i\ell\frac{2\pi}{L}}\right)}{\Delta t}\right)e^{-in\ell\frac{2\pi}{L}}, \tag{60}$$

where $\mathscr{R}$ is the radius of a circle in the domain of analyticity of $\hat{f}(z)$.

The function $\gamma(z)$ is the quotient of the characteristic polynomials of the underlying multistep method, e.g., for a BDF 2, $\gamma(z) = \frac{3}{2} - 2z + \frac{1}{2}z^2$. The used linear multistep method must be $A(\alpha)$-stable and stable at infinity (Lubich, 1988b). Experience shows that the BDF 2 is the best choice (Schanz, 1999). Therefore, it is used in all calculations in this paper.

If one assumes that the values of $\hat{f}(z)$ in (60) are computed with an error bounded by ε, then the choice $L = N$ and $\mathscr{R}^N = \sqrt{\varepsilon}$ yields an error in ω_n of size $\mathscr{O}\left(\sqrt{\varepsilon}\right)$ (Lubich, 1988a). Several tests conducted by the author lead to the conclusion that the parameter $\varepsilon = 10^{-10}$ is the best choice for the kind of functions dealt with in this paper (Schanz and Antes, 1997a). The assumption $L = N$ leads to a order of complexity $\mathscr{O}\left(N^2\right)$ for calculating the N coefficients $\omega_n(\Delta t)$. Due to the exponential function at the end of formula (60) this can be reduced to $\mathscr{O}(N \log N)$ using the technique of the Fast Fourier Transformation (FFT).

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