Modelling of Changing Geometries for the Excavation Process in Tunnelling with the BEM

Christian Duenser[1,a] and Gernot Beer[1,b]

[1] Institute for Structural Analysis, Graz University of Technology, Lessingstrasse 25/II 8010 Graz, Austria

[a]duenser@tugraz.at, [b]gernot.beer@tugraz.at

Keywords: Tunnelling, Sequential excavation, Boundary Element Method, Corners and Edges, Multiple Regions, Discontinuous Elements

Abstract. The modelling of the sequential excavation process in tunnelling is a complicated task with the Boundary Element Method (BEM), because it depends heavily on changing geometries and changing boundary conditions. One possibility to model the sequential excavation is to use the Multiple Region BEM (MRBEM). The problem of corners and edges has a big influence to the results of this modelling strategy. This will be pointed out and a solution with discontinuous elements will be shown. Another possibility to solve the excavation problem in tunnelling is to use a single region boundary element calculation in combination with internal result computation. Stresses in the interior domain are calculated to obtain the loading for subsequent analysis steps. With this method the problem of corners and edges is largely avoided, but the crucial point is the correct evaluation of the loading for the next excavation step. These methods are shown for tunnelling examples in 2D.

Introduction

The New Austrian Tunnelling Method (NATM) is characterised by a complicated sequence of excavation and installation of support systems. In this context the modelling of the sequential tunnel excavation in an infinite/semi-inifinite domain will be discussed. The numerical method which will be applied is the Boundary Element Method (BEM). Fig. 1 shows a typical excavation with the NATM. As a possible example the tunnel cross section is divided into two parts, top heading and bench. The volumes of material are excavated at different time and location. One way to model the sequential tunnel excavation with the BEM is to use multiple regions [1,2,3]. The technique of multiple regions is shown to be applied successfully for the modelling of the sequential excavation process [4,5].

Another possibility to simulate the sequential excavation is the use of a single region boundary element calculation for each step of excavation [6]. To obtain the loading of each subsequent excavation step stresses in the interior domain are calculated. These two solution strategies are explained in the following sections and an example in 2D is shown.

Multiple Region Boundary Element Method (MRBEM)

As in Fig. 1 shown the model consists of several finite regions which represent the volumes of excavation. These regions are embedded in an infinite region, which represents the infinite extent of the continuum. The excavation process is simulated by deactivating of regions from the model system. The geometry and boundary conditions are changing progressively from excavation step to step. For all regions stiffness matrices are calculated and assembled to a global system of equations, which is solved for the interface displacements.

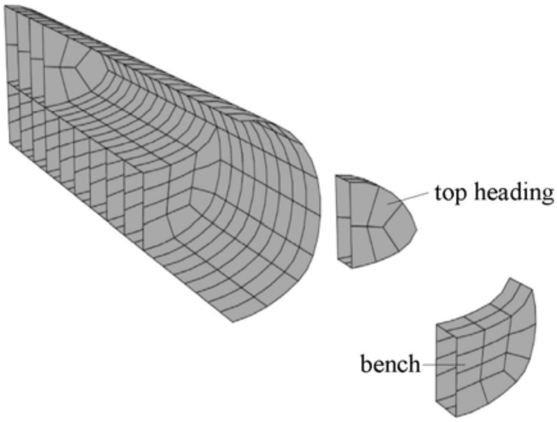

Fig. 1 – Example for a staged excavation process in 3D (only half of the mesh shown)

The stiffness matrix of a region is determined by using the boundary integral equation shown in Eq.1

$$\mathbf{c}(P)\mathbf{u}(P) = \int_S \mathbf{U}(P,Q)\mathbf{t}(Q)dS - \int_S \mathbf{T}(P,Q)\mathbf{u}(Q)dS \tag{1}$$

Where $\mathbf{u}$ and $\mathbf{t}$ are the displacements and tractions on the boundary S, respectively, $\mathbf{c}$ is related to the boundary geometry, $\mathbf{U}$ and $\mathbf{T}$ are the fundamental solutions for displacements and tractions, respectively. In discretized form this equation is equivalent to

$$\sum_{e=1}^{E}\sum_{n=1}^{N} \Delta\mathbf{T}_{ni}^{e} \cdot \mathbf{u}_{n}^{e} = \sum_{e=1}^{E}\sum_{n=1}^{N} \Delta\mathbf{U}_{ni}^{e} \cdot \mathbf{t}_{n}^{e} \tag{2}$$

$\Delta\mathbf{U}_{ni}^{e}$ and $\Delta\mathbf{T}_{ni}^{e}$ are the integrated kernel shape function products of the element e for a collocation point (P_i) at point i. $\mathbf{u}_{n}^{e}$ and $\mathbf{t}_{n}^{e}$ are vectors containing all displacements and tractions for the element. Collocating at all boundary nodes results in an equation system. In general the boundary conditions are mixed and the known and unknown values are separated to the right and left side of the equation and this gives the following:

$$\mathbf{B}^N \cdot \begin{Bmatrix} \mathbf{t}_{c0}^N \\ \mathbf{u}_{f0}^N \end{Bmatrix} = \mathbf{f}_0^N; \qquad \mathbf{B}^N \cdot \begin{Bmatrix} \mathbf{t}_{cn}^N \\ \mathbf{u}_{fn}^N \end{Bmatrix} = \mathbf{f}_n^N \qquad for \;\; n = 1,2......Ndofc \tag{3}$$

Matrix $\mathbf{B}^N$ is the assembled left hand side for region N, containing the integrated kernel shape function products related to the unknown boundary conditions. $\mathbf{t}_{c0}^N$ is a part of the unknown vector representing the traction at the interface c and $\mathbf{u}_{f0}^N$ are the displacements due to the loading (excavation tractions) at the Neumann boundary. The second equation of Eq. 3 obtains the same left hand side as the first but the loading are unit displacements at the interface degrees of freedom. Applying unit displacements at all degrees of freedom at the interface results in the stiffness matrix K^N of region N. The final solution $\mathbf{t}_c^N$, the tractions at the interface nodes, and $\mathbf{u}_f^N$, the displacements at the Neumann boundary, can be expressed in terms of $\mathbf{u}_c^N$, the displacements at interface nodes:

$$\begin{Bmatrix} \mathbf{t}_c^N \\ \mathbf{u}_f^N \end{Bmatrix} = \begin{Bmatrix} \mathbf{t}_{c0}^N \\ \mathbf{u}_{f0}^N \end{Bmatrix} + \begin{Bmatrix} \mathbf{K}^N \\ \mathbf{A}^N \end{Bmatrix} \cdot \mathbf{u}_c^N \quad where \quad \begin{Bmatrix} \mathbf{K}^N \\ \mathbf{A}^N \end{Bmatrix} = \begin{Bmatrix} \left[\mathbf{t}_{c1},\ \mathbf{t}_{c2},\ .....\ \mathbf{t}_{cNdofc} \right] \\ \left[\mathbf{u}_{f1}, \mathbf{u}_{f2}, ... \mathbf{u}_{fNdofc} \right] \end{Bmatrix} \tag{4}$$

Using the conditions of equilibrium and compatibility the stiffness matrices $\mathbf{K}^N$ and the vectors $\mathbf{t}_{c0}^N$ of all regions can be assembled into a global system of equation, which can be solved for the unknown interface displacements $\mathbf{u}_c$ as shown in Eq. 5.

$$\mathbf{t}_{c0} + \mathbf{K} \cdot \mathbf{u}_c = 0 \tag{5}$$

$\mathbf{K}$ is the assembled stiffness matrix related to the interface nodes only, $\mathbf{t}_{c0}$ is the assembled vector of tractions at the interface due to the given boundary conditions at the Neumann boundary. The remaining unknowns, displacements at the Neumann boundary and tractions at the interface can be evaluated returning from system to region level, by using Eq. 4.

The excavation is modelled by deactivating regions, this means removing stiffness matrices from the equation system [5]. This implies that the geometry of the system and the boundary conditions of some regions are changing, for instance from interface conditions to Neumann conditions. For these regions the stiffness matrices have to be updated each calculation step before the assembly process starts.

A problem, which arises is the one of corners and edges [2,5,6,7]. In the following an example (shown in Fig. 2) is shown where this problem is pointed out. This example concerns an excavation in 2D and models a tunnel in longitudinal section. Of course this means an excavation of infinite extend out of plane, which is not a real tunnel excavation, but the problem of corners can be shown clearly. The example shown in Fig. 2 consists of 10 regions which are sequentially removed from the system. At the beginning most of the boundary elements belong to interface boundaries. The regions are of rectangular shape and exhibit geometrical corners. If adjacent elements of these corners belong to the interface, Dirichlet boundary conditions are applied for the evaluation of the stiffness matrices. The resultant tractions at these corners are multi-valued in reality, this means different on both sides of the corner node. If continuous elements are used the boundary integral equation is able to deliver only one distinct vector of tractions at the corner node, therefore the tractions are unique. As the resultant tractions at the interface belong to the loading of the subsequent load step erroneous results will be achieved.

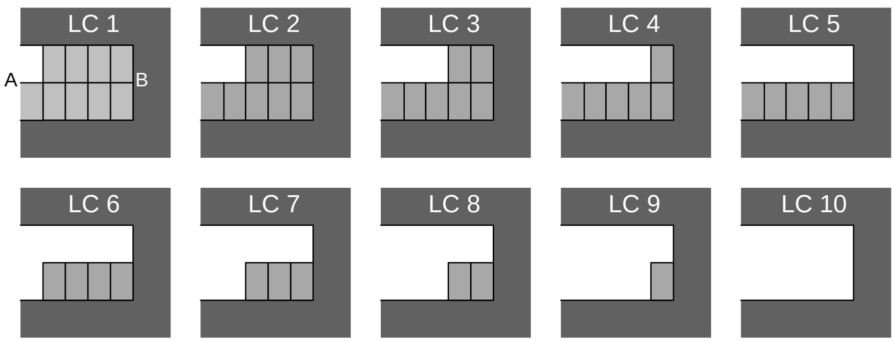

Fig. 2 – Example for a sequential excavation process in 2D

In the diagram of Fig. 3 vertical displacements for load case 1 to 5 at the line $\overline{AB}$ (shown in Fig.2) are shown. The dashed lines show the results for calculations with continuous quadratic elements. It can be seen that the results, except for load case 1, are erroneous and theses errors accumulate from step to step. With the use of continuous elements the problem of corners and edges is neglected, especially at nodes of the interface where the traction is singular. Doing the same calculations with discontinuous quadratic elements gives excellent results [2]. These results are validated with single region boundary element calculations as well as with finite element calculations.

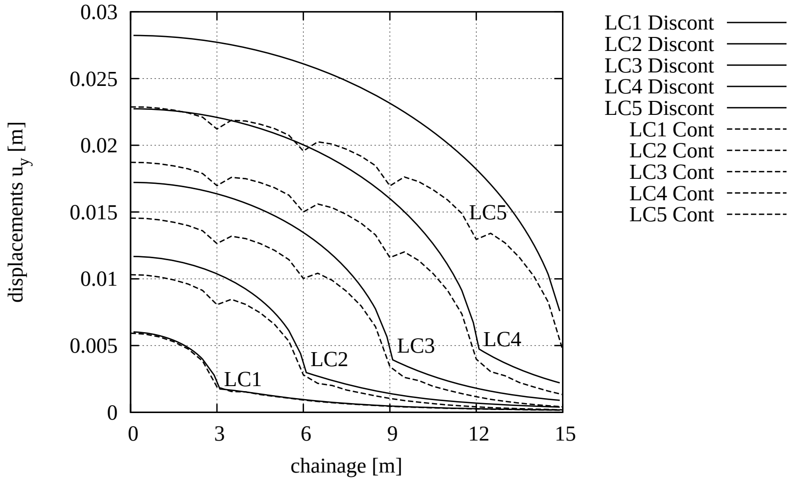

Fig. 3 – Vertical displacements for load case 1 to 5 along the line $\overline{AB}$ (shown in Fig. 2)

Single region boundary element method

The method discussed before requires a predefinition of the complete geometry of the tunnel excavation problem. From beginning the calculation has to deal with all the regions, interfaces, etc. which in subsequent load cases will be part of the excavation process. This means, that the size of the equation system is determined by the number of all degrees of freedom at the entire geometry and it nearly remains the same for every load step of calculation. Now only a single boundary element region is used to represent the actual excavation surface [6]. Stresses are calculated at points inside the domain, provided that the boundary conditions are known. For the respective load step excavation loads are determined with this approach.

The algorithm is explained for the same example as in the previous section. Fig. 4 shows the excavation sequence for 10 load cases. The excavated parts of each step, where the loading is applied, are indicated by hatched areas. With this method it is necessary to discretize the excavated tunnel surface only. A single boundary element region is sufficient for the discretization of the respective load step which can be seen in Fig. 4.

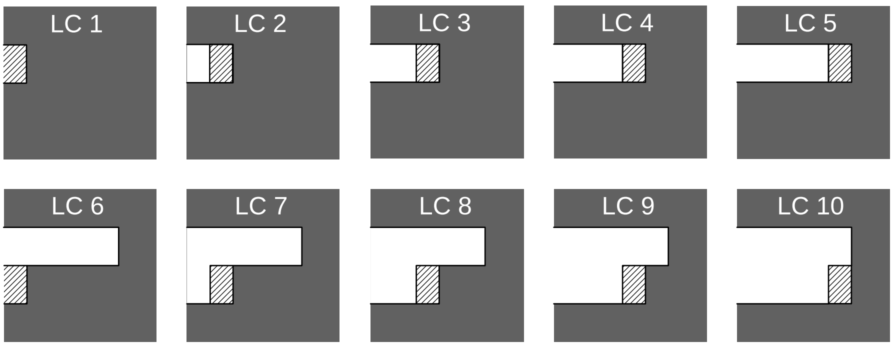

Fig. 4 – Example for a sequential excavation process in 2D

The crucial part is the determination of excavation tractions for the current load step. This is explained next for load case LC4. In Fig. 5 the excavation steps from LC1 until LC4 are shown. The region to be excavated for LC4 (indicated by hatched areas) is shown in the sketches for each of the previous load cases. The resultant displacement field and stress field for LC4 is the accumulation of incremental results of all previous load cases. This implies that the previous load cases have to be considered for the determination of the loading for LC4.

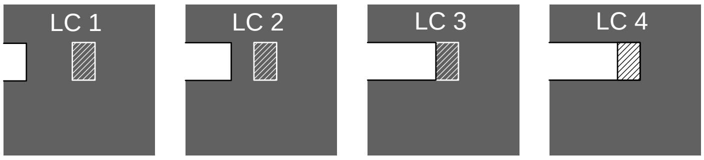

Fig. 5 – Excavation of LC4

The excavation tractions are calculated with internal stress evaluation. The stress at an internal point is calculated by the following integral equation:

$$\boldsymbol{\sigma}(P_i) = \int_S \mathbf{R}(P,Q)\mathbf{t}(Q)dS - \int_S \mathbf{S}(P,Q)\mathbf{u}(Q)dS \qquad (6)$$

where $\boldsymbol{\sigma}(P_i)$ is the stress at an internal Point P_i, $\mathbf{R}$ and $\mathbf{S}$ are the fundamental solutions for stress, $\mathbf{t}$ and $\mathbf{u}$ are the boundary traction and displacement values, respectively. The stress is evaluated at the same points for load case 1 to 3. For load case 1 and 2 all points are internal points as shown in Fig. 5. For these load cases there is no difficulty in the evaluation of the stress. For LC 3 some of the points of the excavated volume are boundary points. Because of the sharp corners at point A and B (shown in Fig. 6) the stress is infinite and a calculation directly at these points is not possible. To overcome this problem the stress is evaluated inside the adjacent element very near to the boundary, at an intrinsic coordinate of value $\xi = -0,90$. The stress is extrapolated to the boundary according to the parabolic shape function of the element. This is shown in Fig. 6.

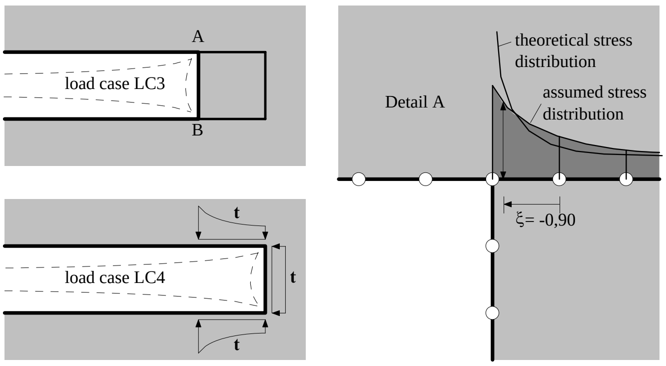

Fig. 6 – Treatment of singular point

The traction vector $\mathbf{t}$ is calculated by multiplying the stress $\boldsymbol{\sigma}$ with the outward normal vector $\mathbf{n}$ to the excavation surface. The resulting traction at the excavation surface for LC4 is the sum of tractions obtained by internal stress calculation for LC1 to LC 3 completed by the tractions due to the virgin stress field. Once the loading tractions are found the solution for the current load step is evaluated by a single region boundary element calculation using Eq. 3. The results of the vertical displacements along the crown of the tunnel are shown in Fig. 7. These results are compared with the reference solution. As can be seen in Fig. 7, it seems that some loading is lost from one load case to the other. The reason for this is the inaccurate evaluation of the tractions near to the singular points A and B, shown in Fig. 6.

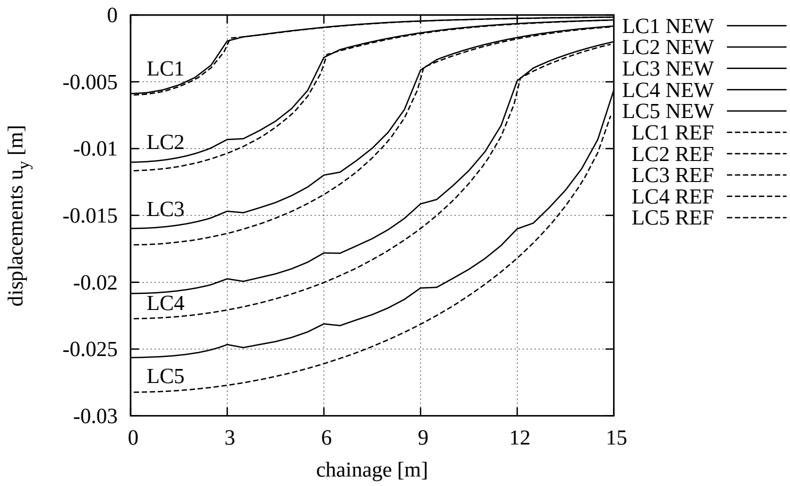

Fig. 7 – Vertical displacements for load case 1 to 5 along at tunnel crown

Conclusion

The main advantage of the single region approach against the conventional method of domain decomposition (MRBEM) is that the effort for the solution is much less. The simulation starts with a very small BEM mesh and this will extend from excavation step to step. Additional effort has to be spent for the correct evaluation of the stress distribution near to the boundary, where singular values of stress appear. Currently the implementation for 3D into the computer code BEFE++ is an ongoing task. A reasonable comparison of the efficiency to the conventional method of MRBEM only makes sense for a 3D example.

Acknowledgement

The work reported here is supported by the European Commission, under its 6th framework program, within the integrated project TUNCONSTRUCT.

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