Common normals of two ellipses/ellipsoids and the 'one hand clapping problem'

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In the case of collision prediction for moving objects complicated shapes are often replaced by geometrically simplier ones like quadrics or, more specifically, ellipses and ellipsoids. In particular, we discuss the following tasks:

1. Given two ellipses k_1 and k_2 in a common plane or two ellipsoids Φ_1 and Φ_2 in 3-space find all common normals of k_1 and k_2 or Φ_1 and Φ_2, respectively.

2. 'One hand clapping'; 2-dimensional case: Let k_1 and k_2 be ellipses in a common plane ε and let $O \in \varepsilon$ be a point. If the second ellipse k_2 is rotated about O find all instances of k_2 which are tangent to k_1.

3. 'One hand clapping'; 3-dimensional case: Let Φ_1 and Φ_2 be ellipsoids and let a be a straight line. Find all instances of the second ellipsoid Φ_2 tangent to Φ_1 if Φ_2 is rotated about the axis a.

Those kinds of problems always lead to a set of algebraic equations whose solutions can be found by Groebner bases methods. But it turns out that by investing appropriate geometric considerations those Groebner bases methods can either be completely avoided or at least the number of variables and the degrees of the occurring polynomials can be reduced.

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