Numerical effects in orbit determination for spaceborne gravimetry

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Variational equation approach

For the simpler case of single-satellite observations.

Observations

- Satellite position r (from GNSS)
- Force model $\ddot{r}$ (from accelerometer, background models, . . .)

Unknown variable:

Gravity field parameters

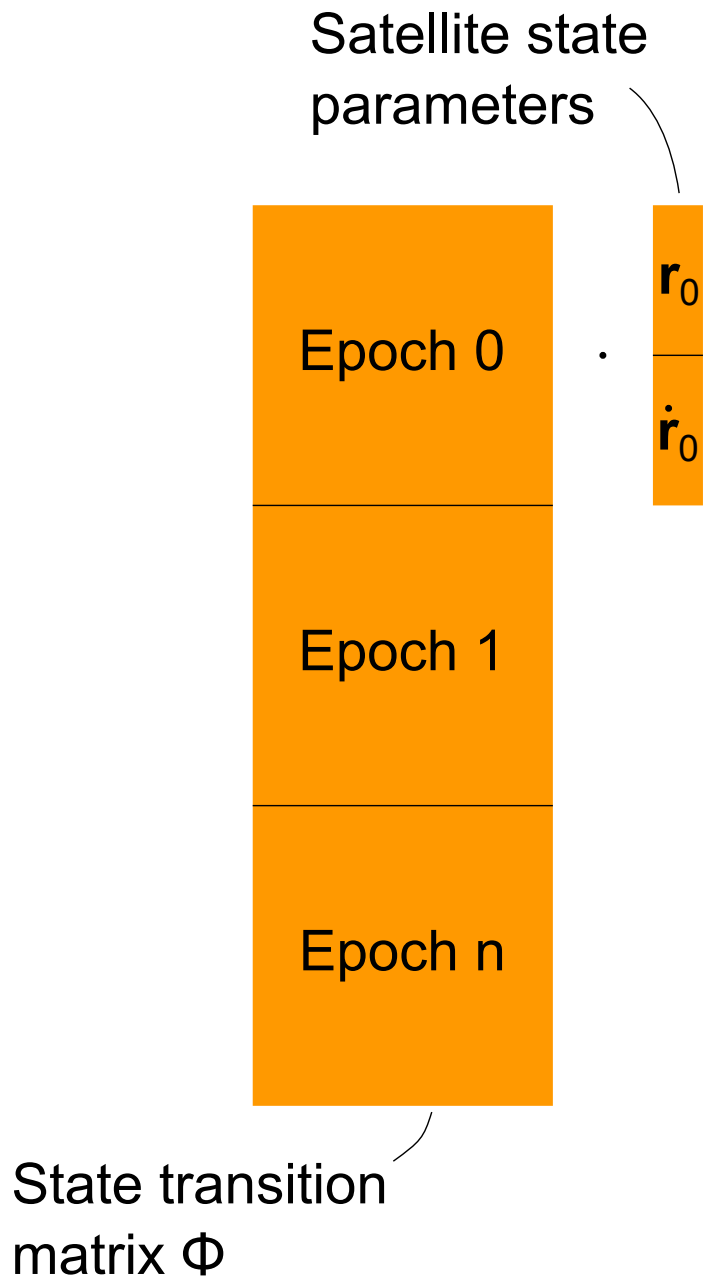
Equation of motion connects position and acting forces:

$$r = \iint \ddot{r}$$

State transition matrix

Connection between satellite state at each epoch and initial state:

- Satellite state vector, e.g. r_0, $\dot{r}_0$.
- Start with undisturbed motion $r(t) = r_0 + t \cdot \dot{r}_0$



State transition matrix

- Integrate forces to velocity and position.
- Use integrated and observed position to solve for satellite state parameters and to adjust state transition matrix.

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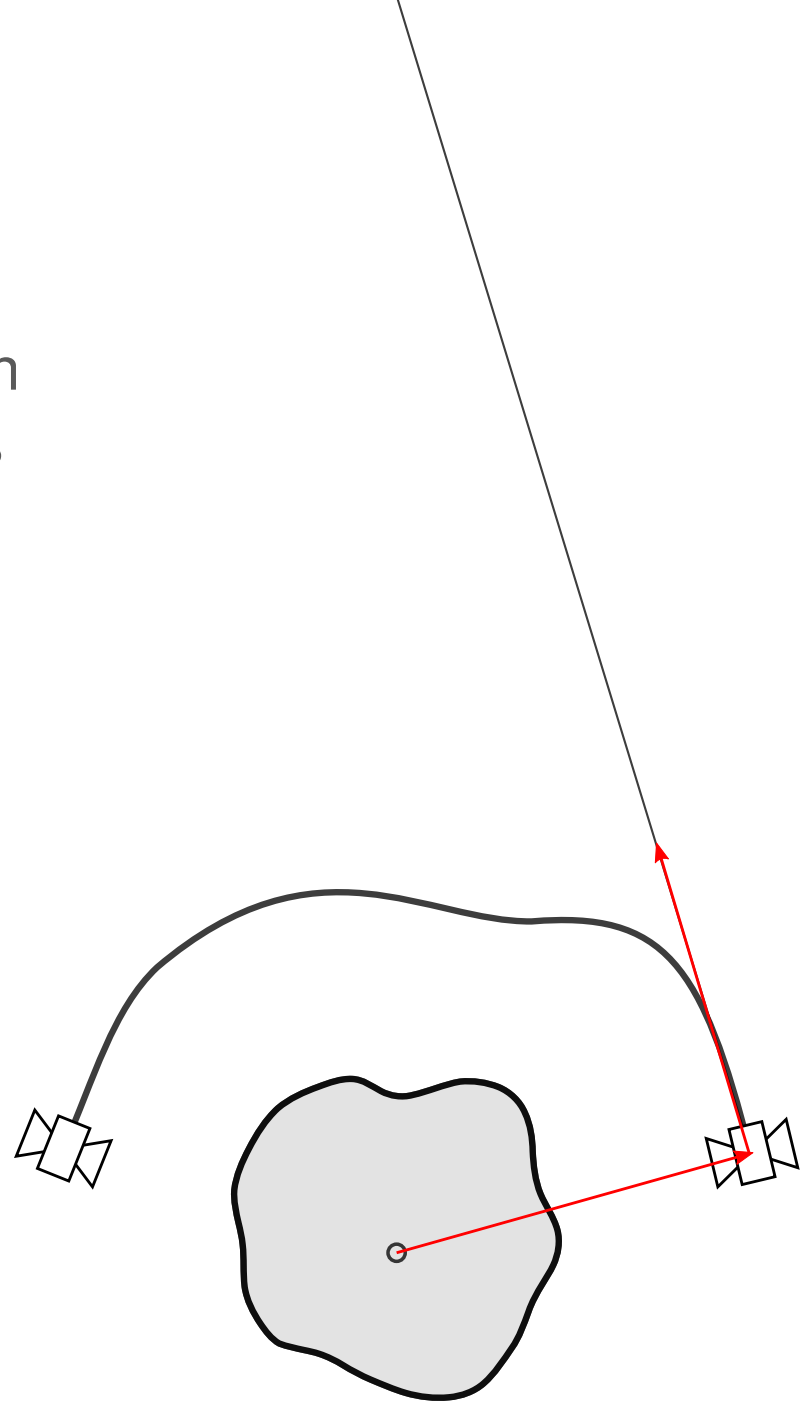
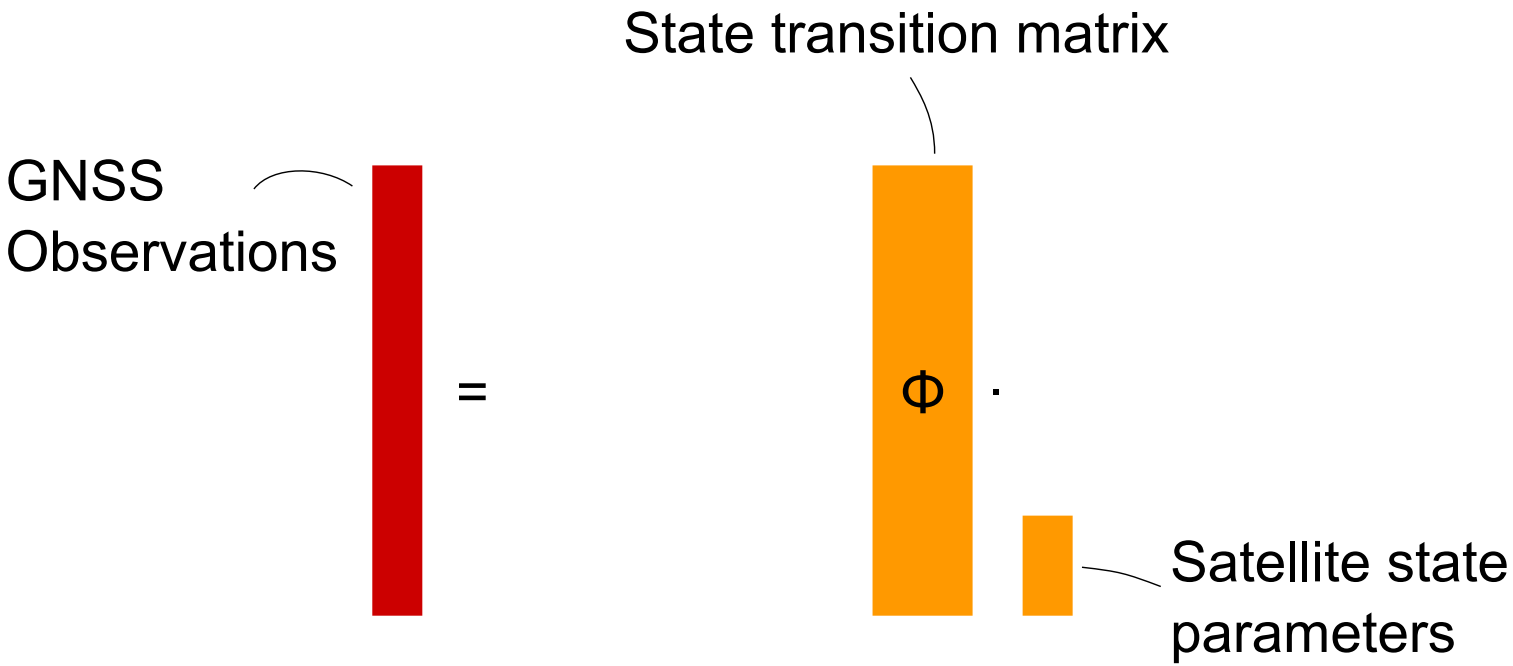
Need to solve for 6 integration constants
$\Rightarrow$ Variational problem.

Transition of Φ from linear motion to perturbed motion.



Connection to potential



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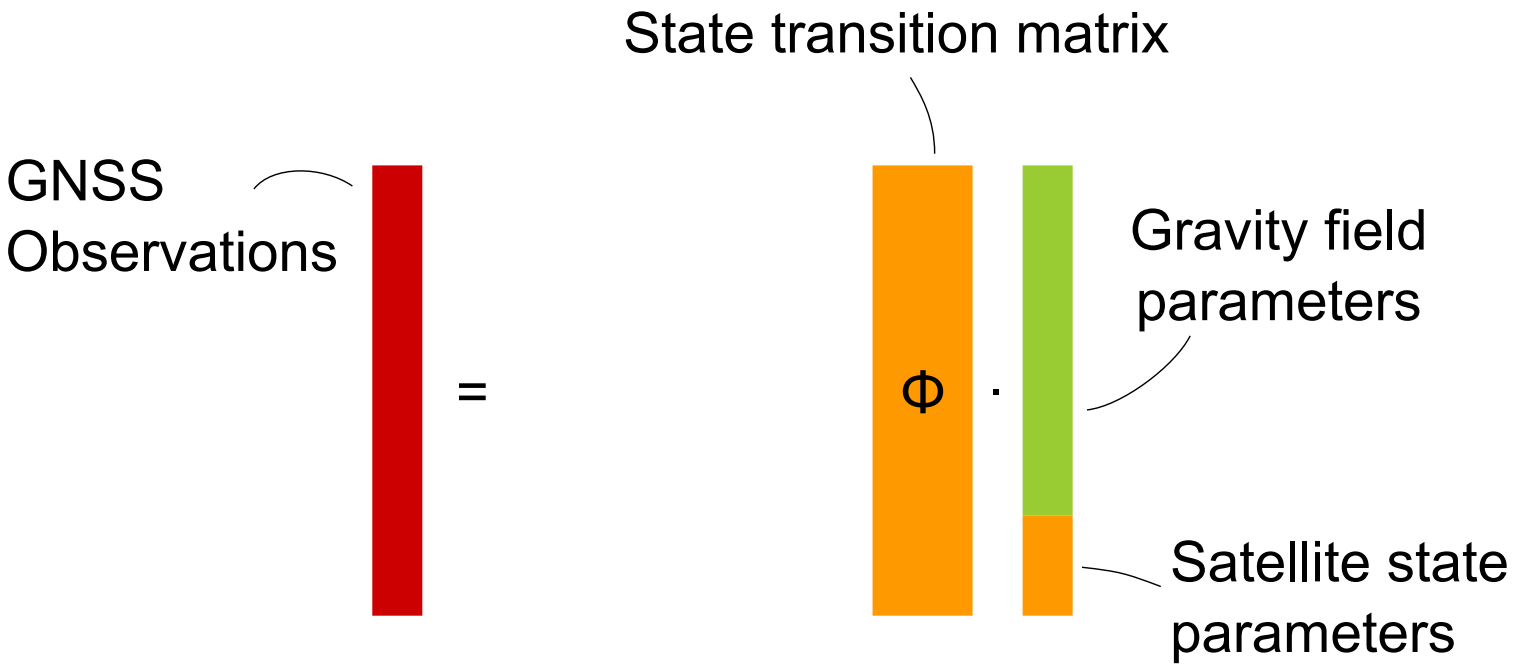

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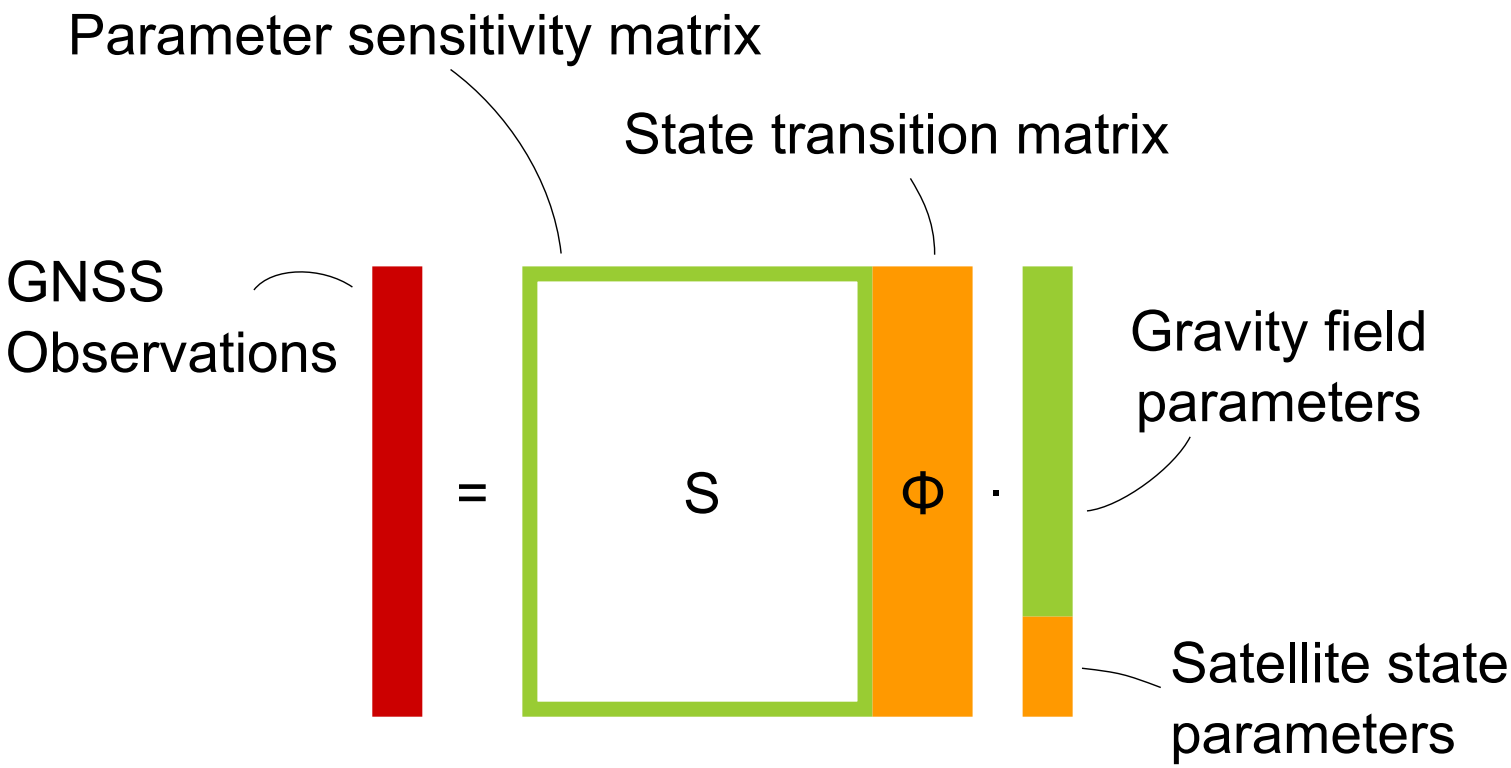

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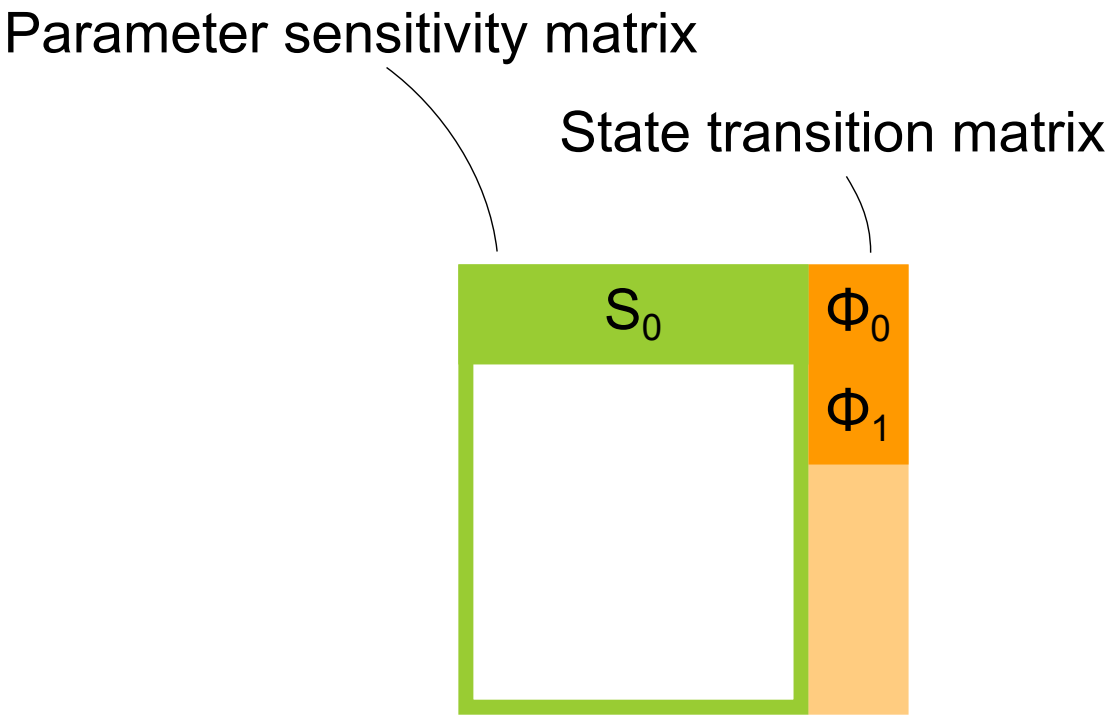

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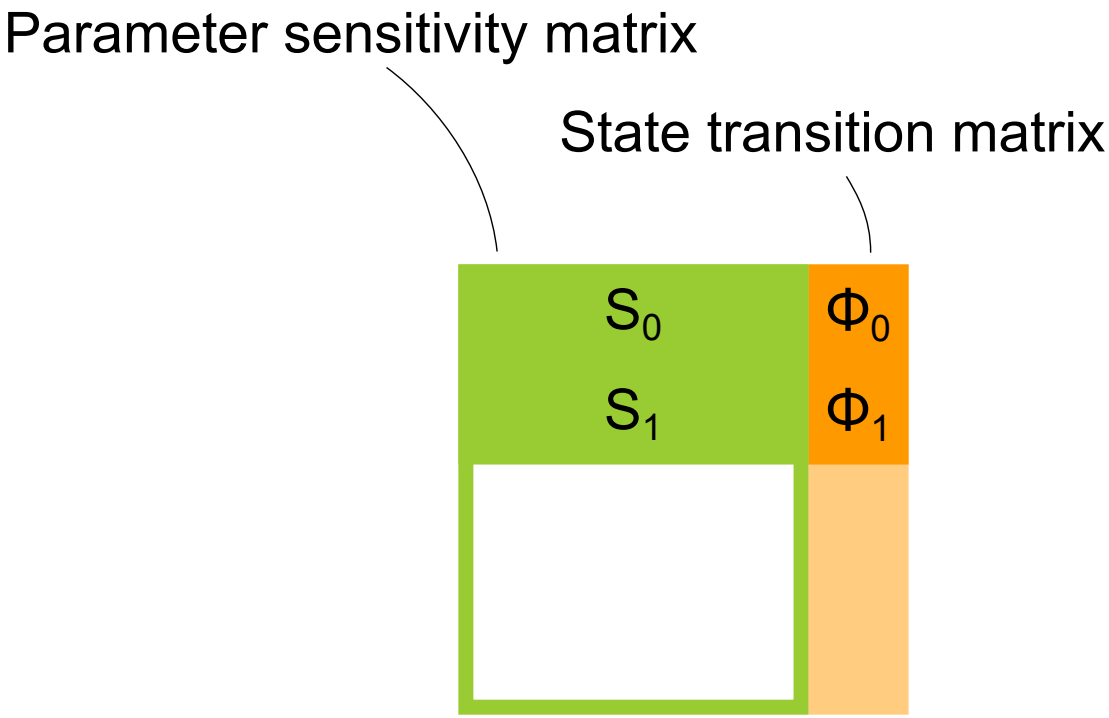

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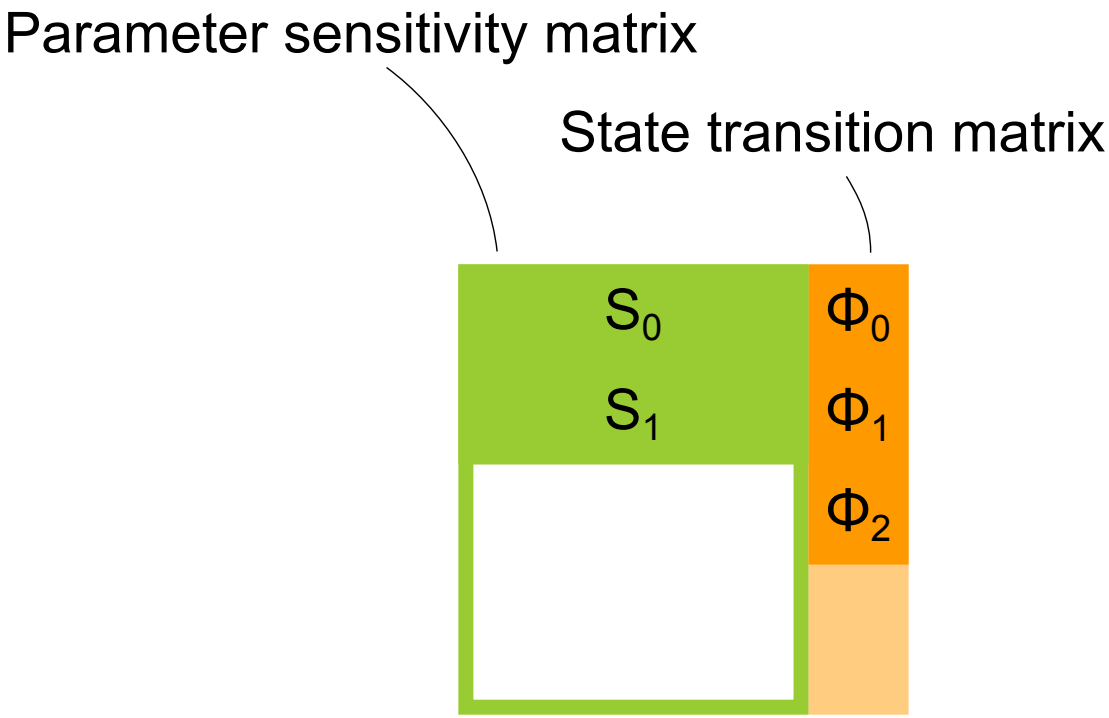

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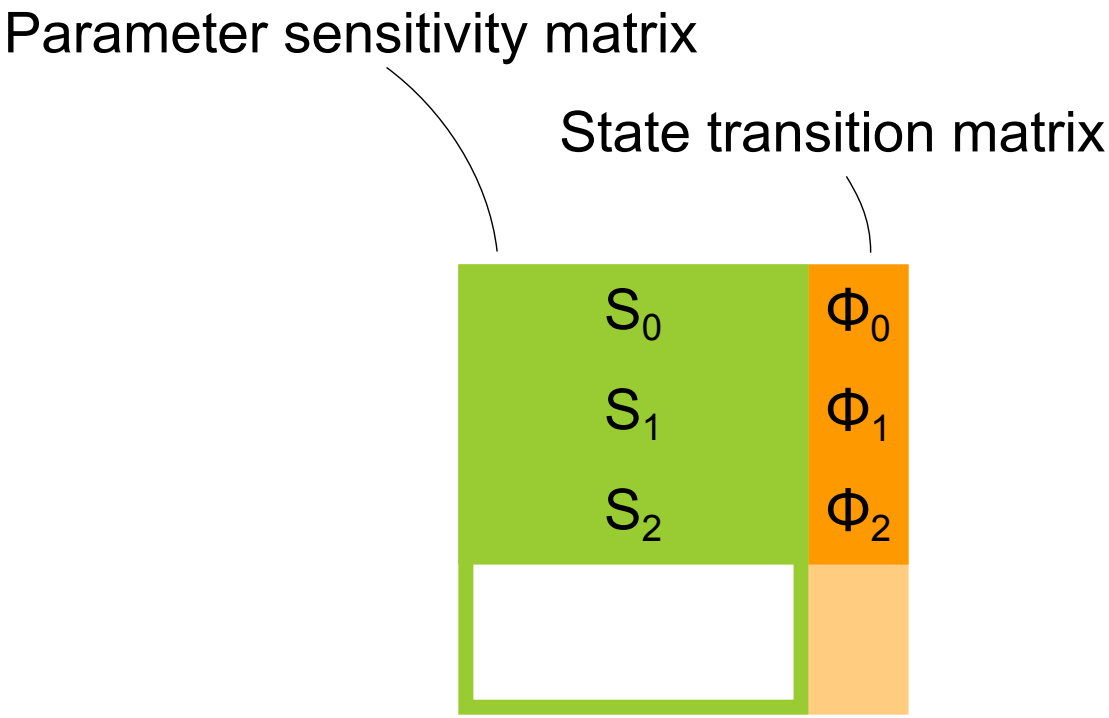

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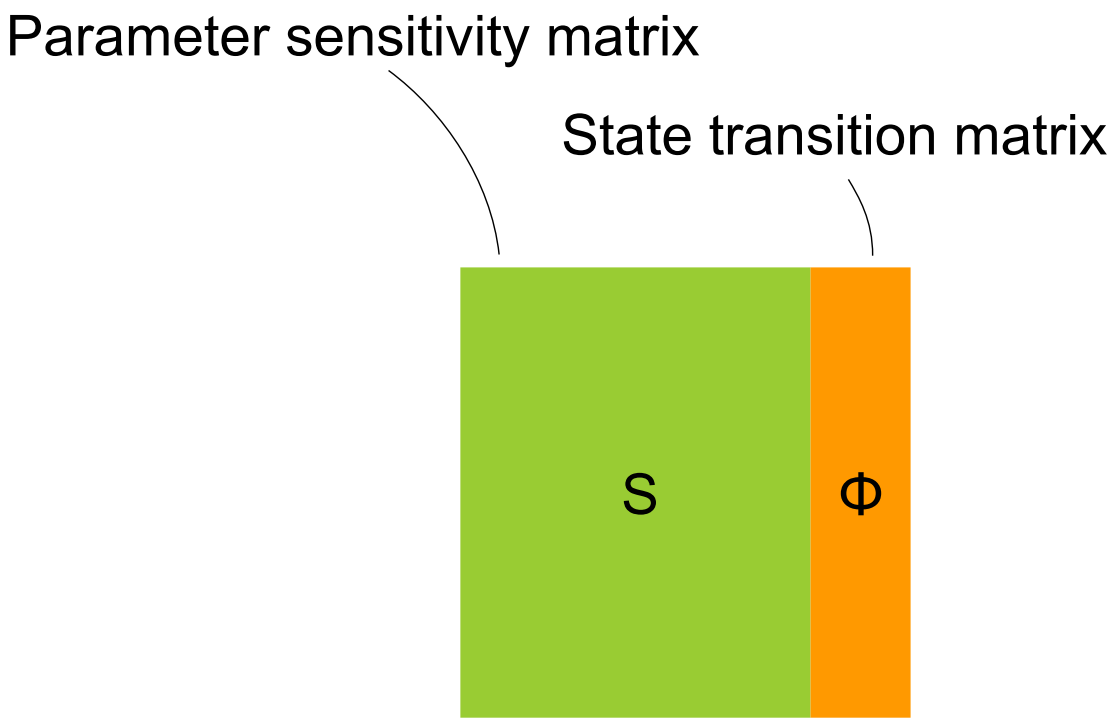

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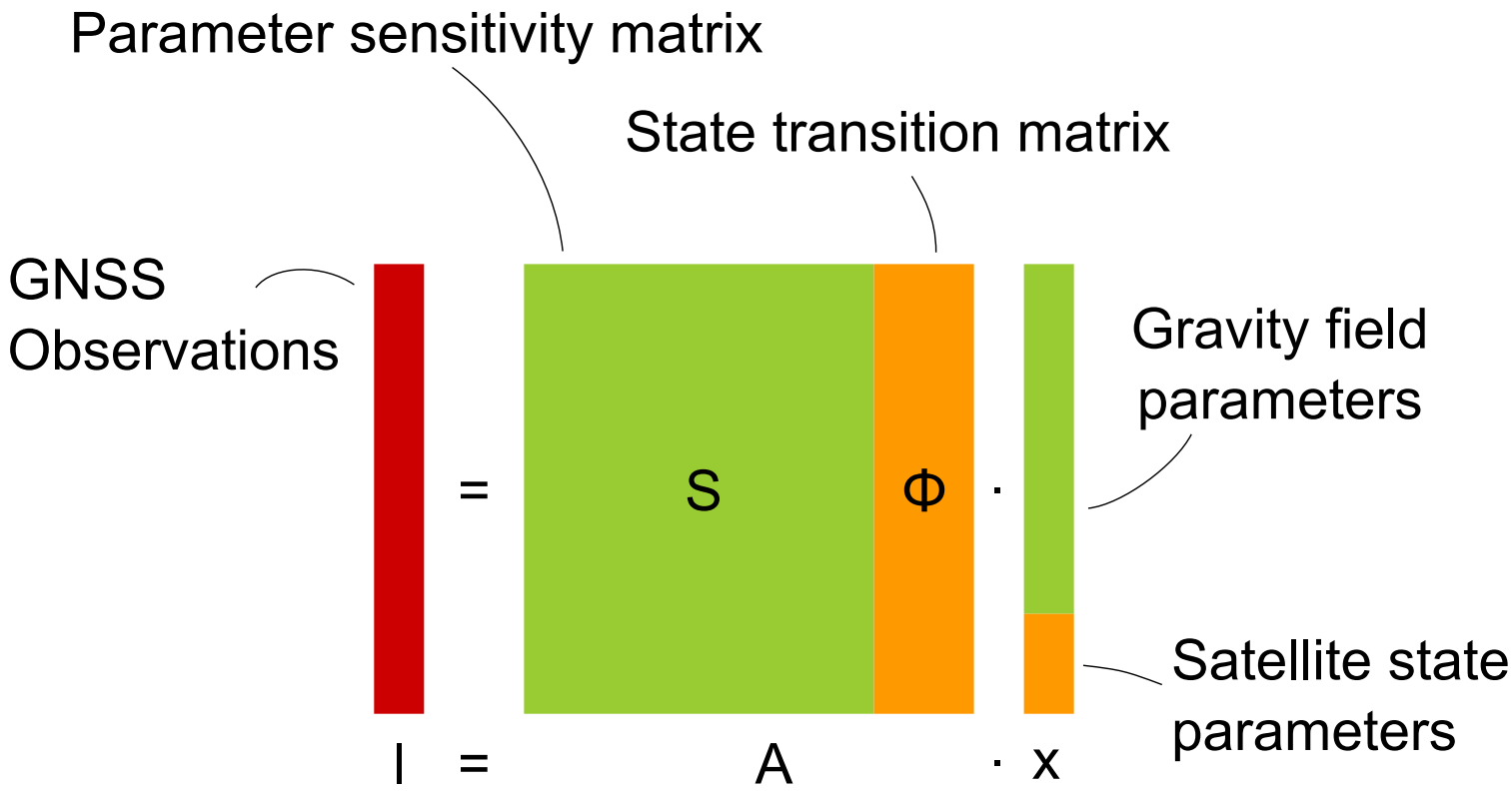
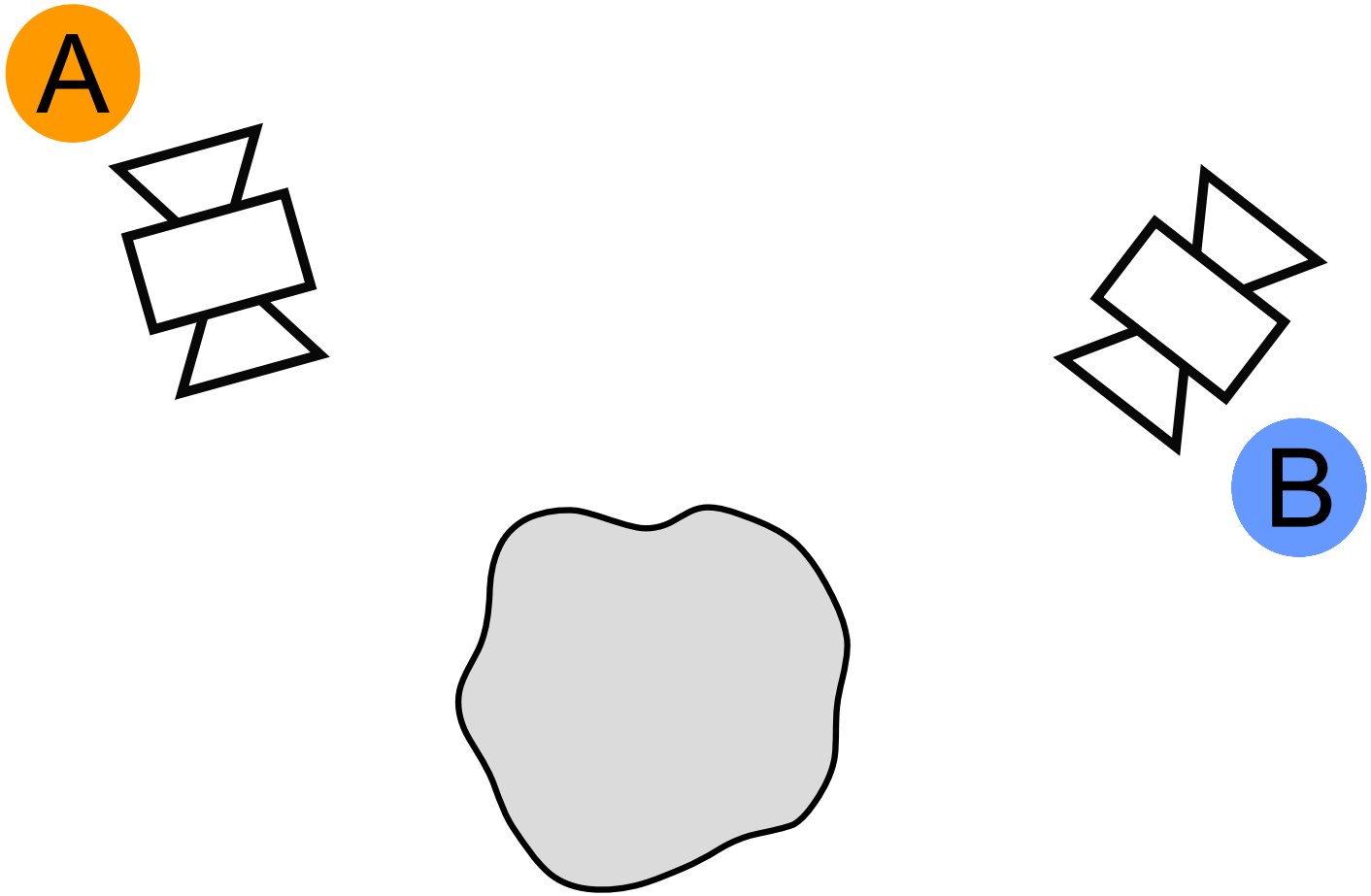


- Complete linearized observation equations for position observations.
- The observation equations represent a dynamic orbit.

Ranging measurement from dynamic orbit

For range rate observation: Projection of differential velocity onto baseline.



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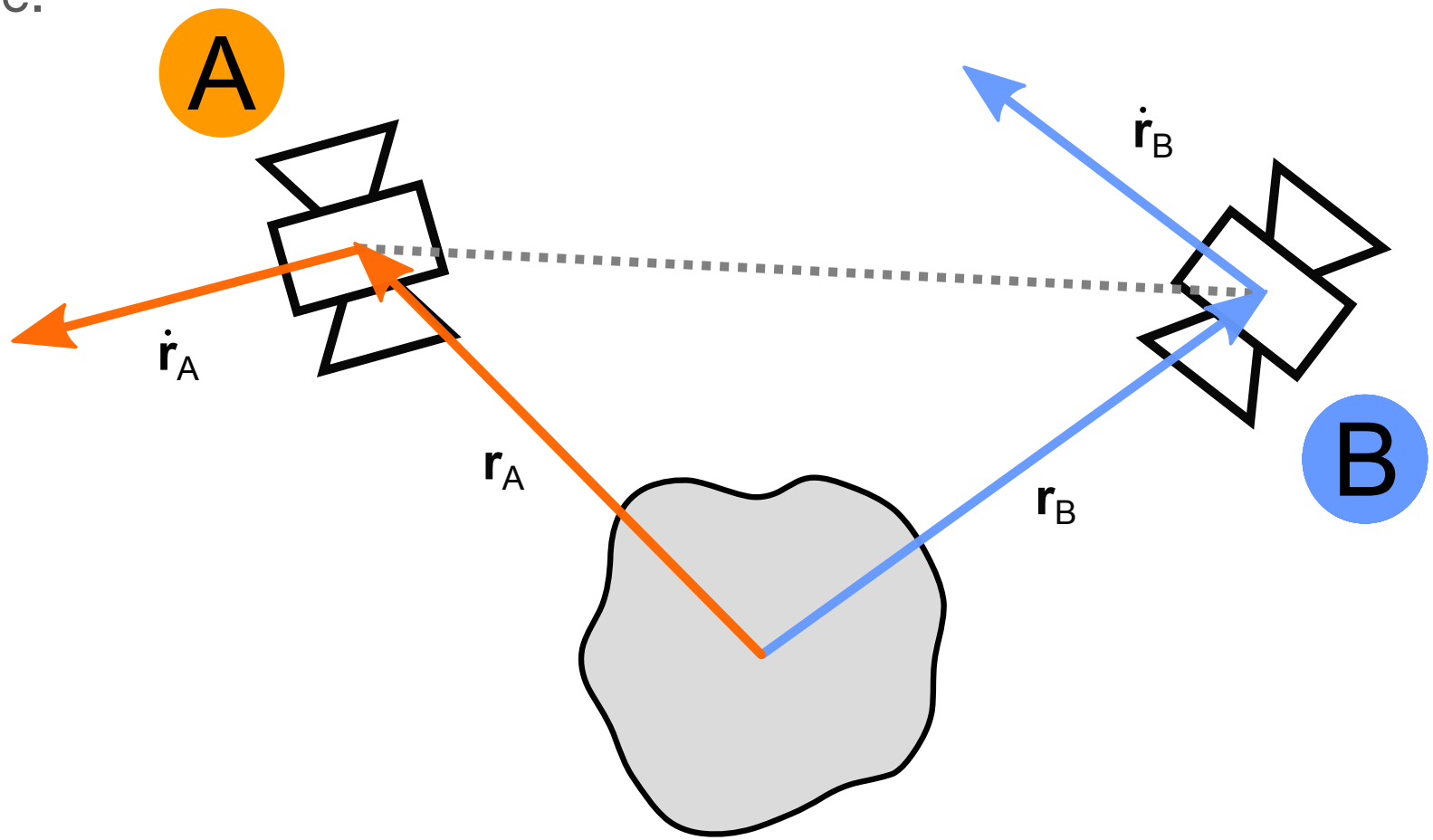

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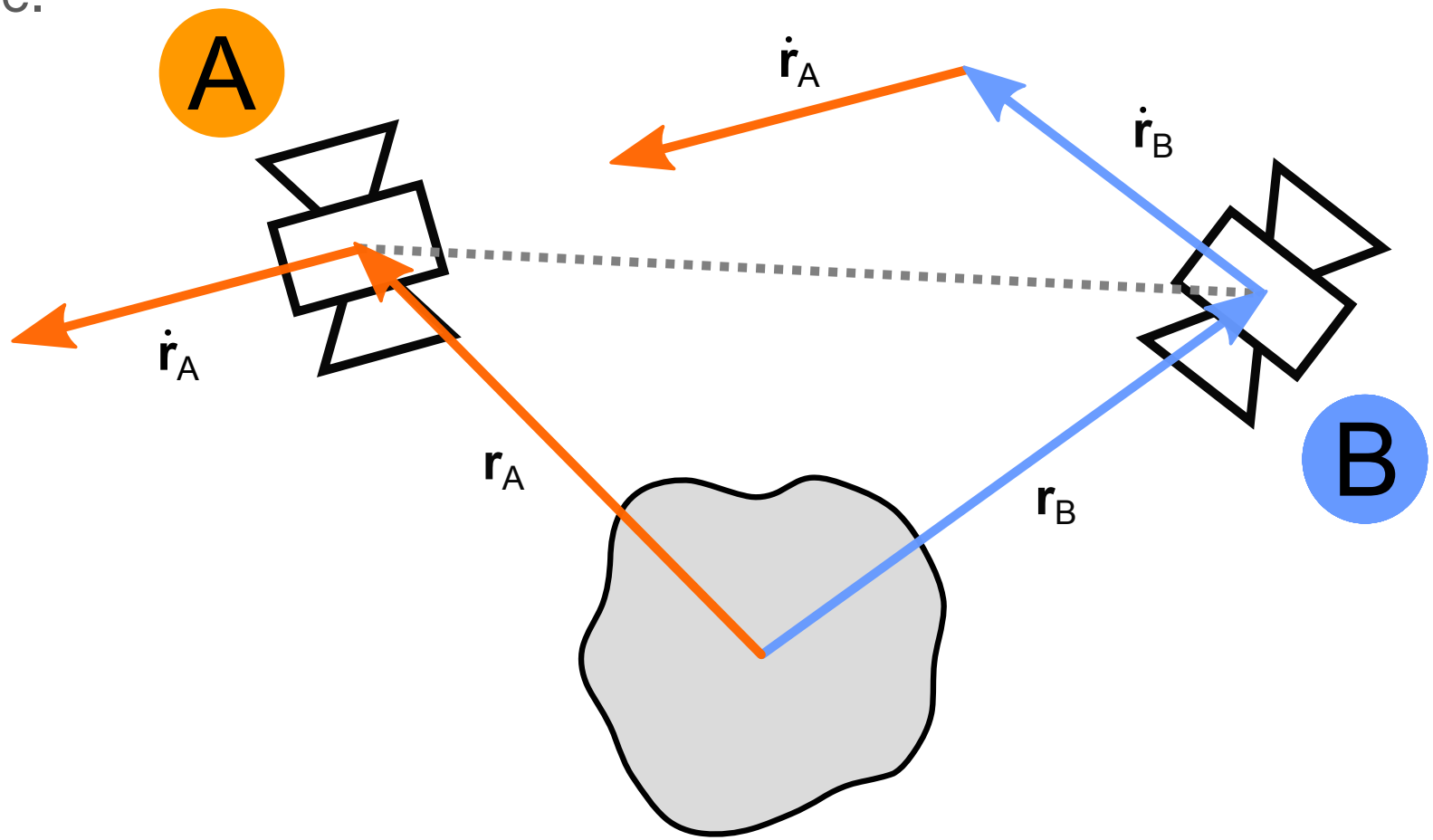

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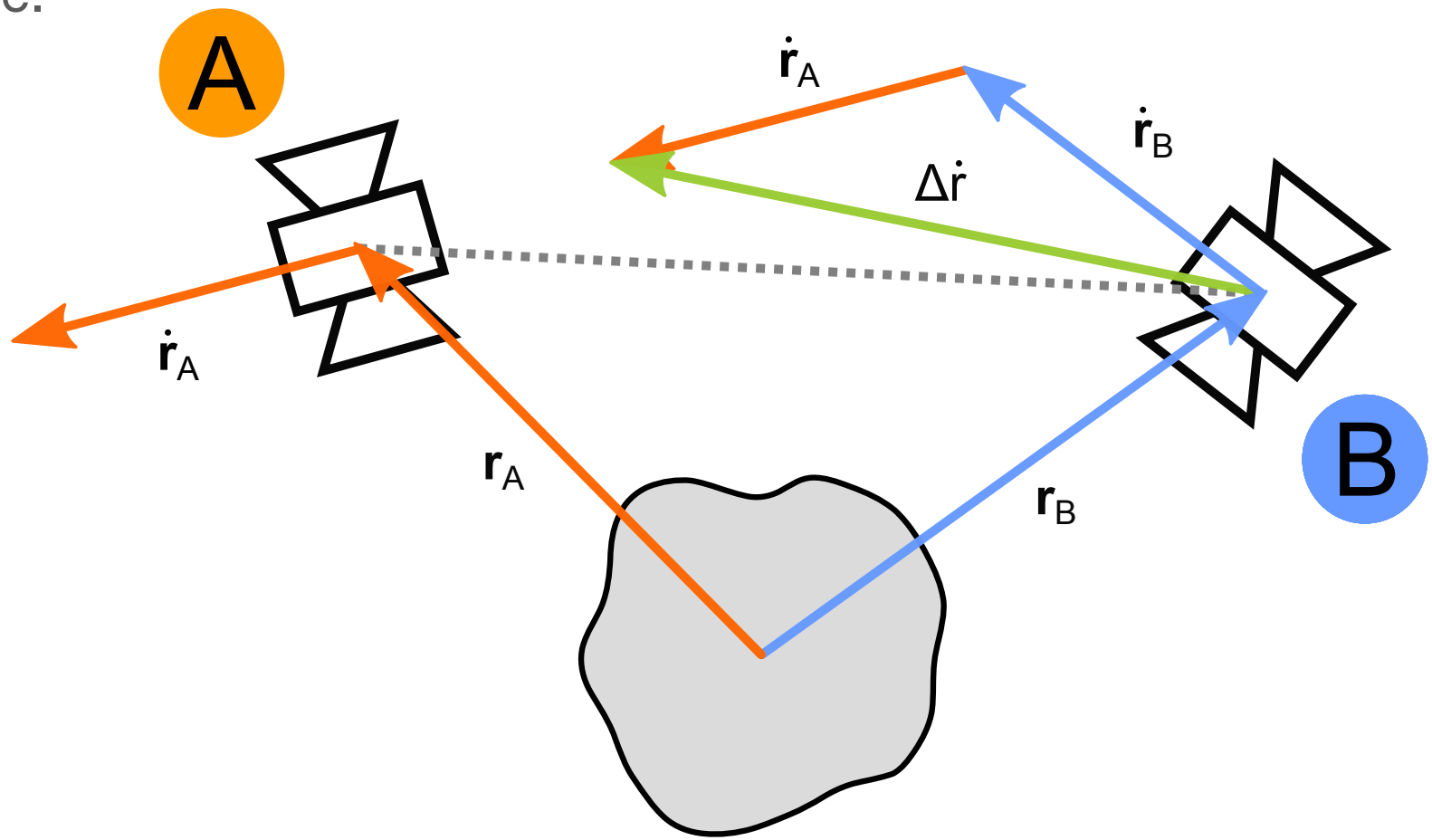

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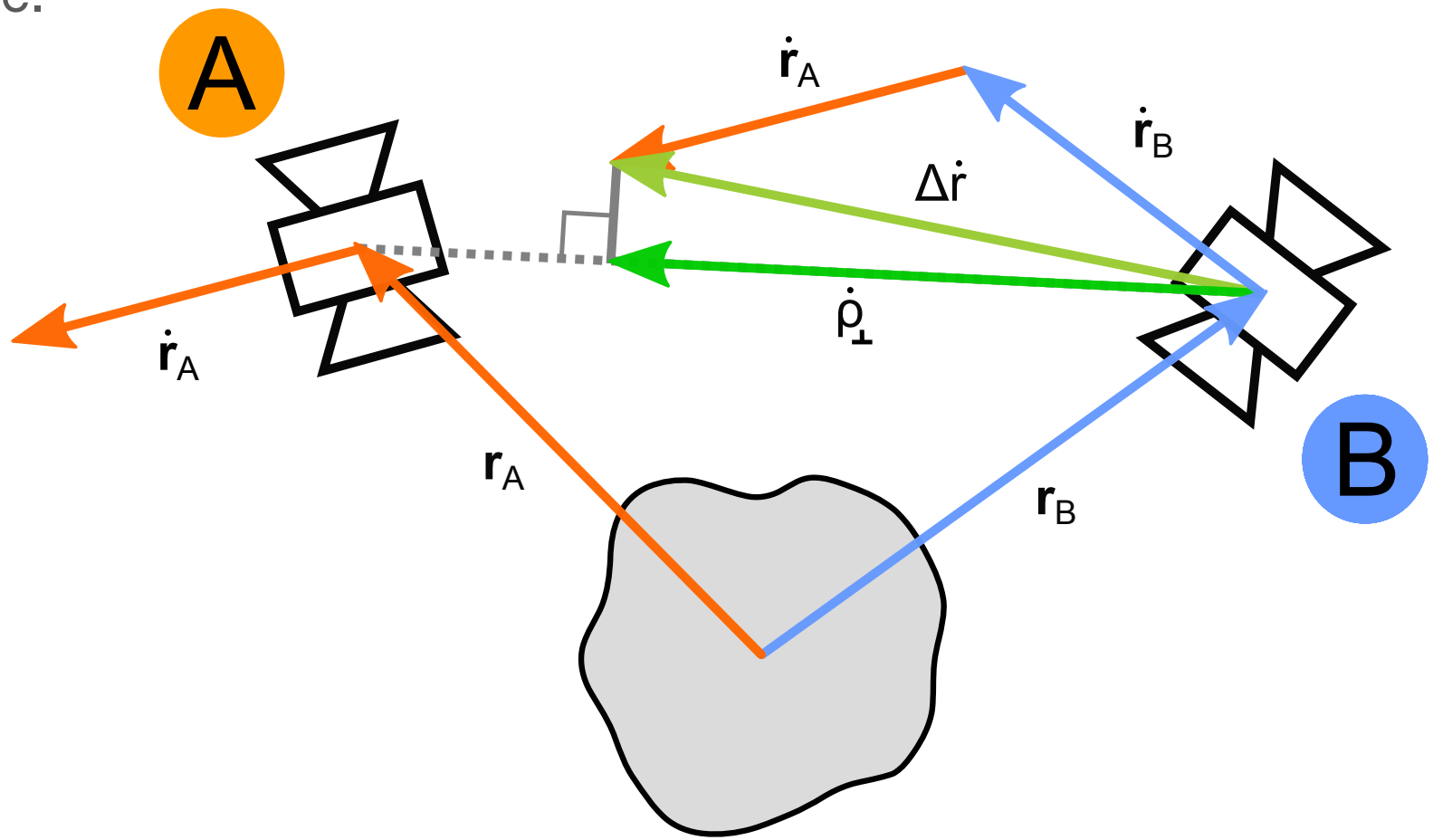
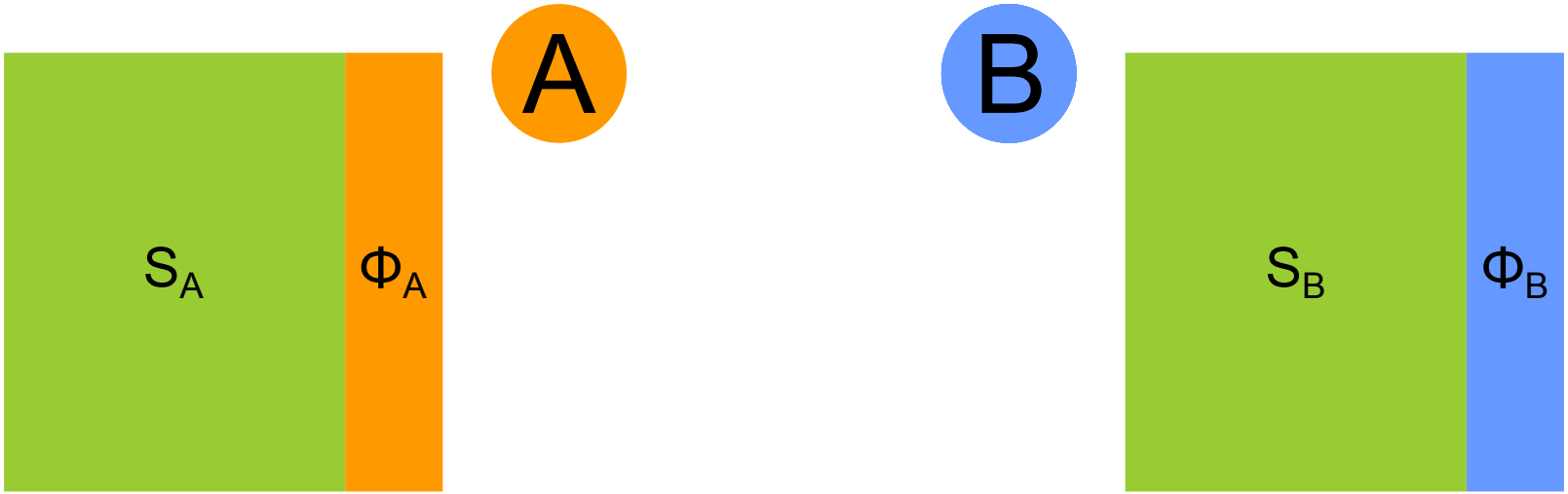


Observation equations for SST



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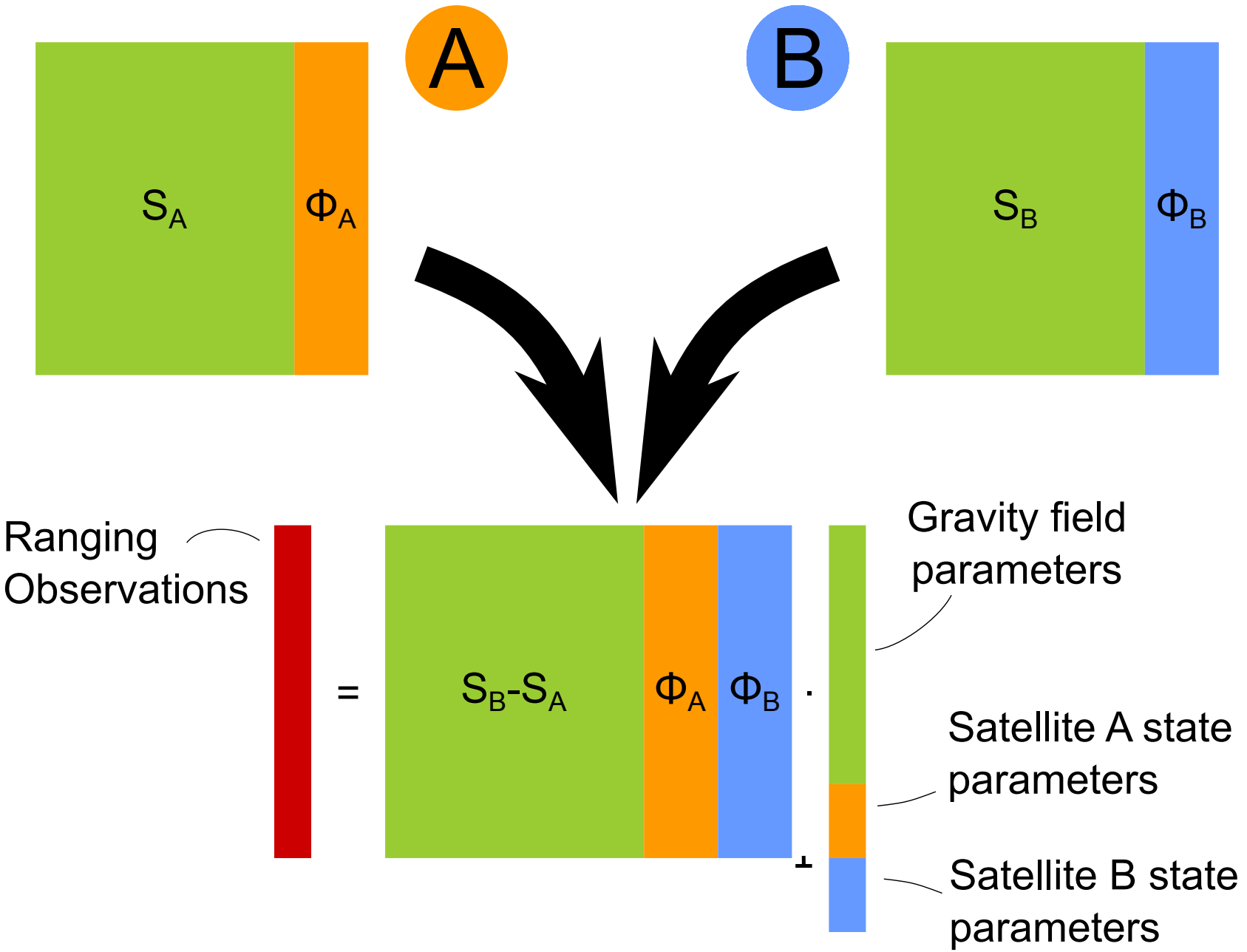
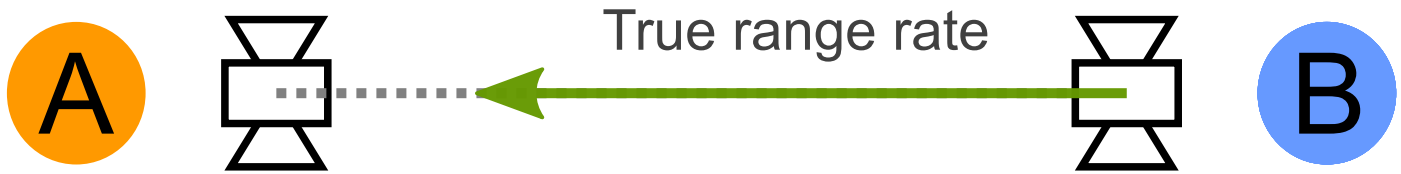


Reduction of ranging observations



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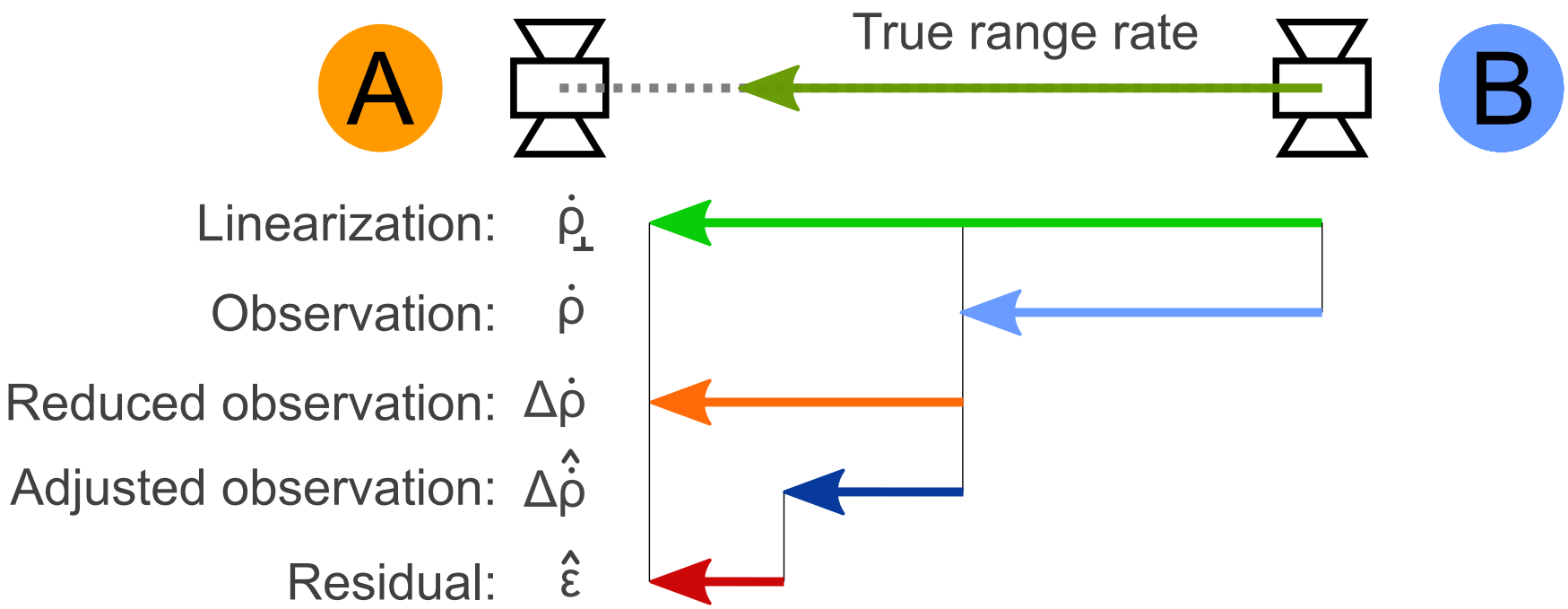

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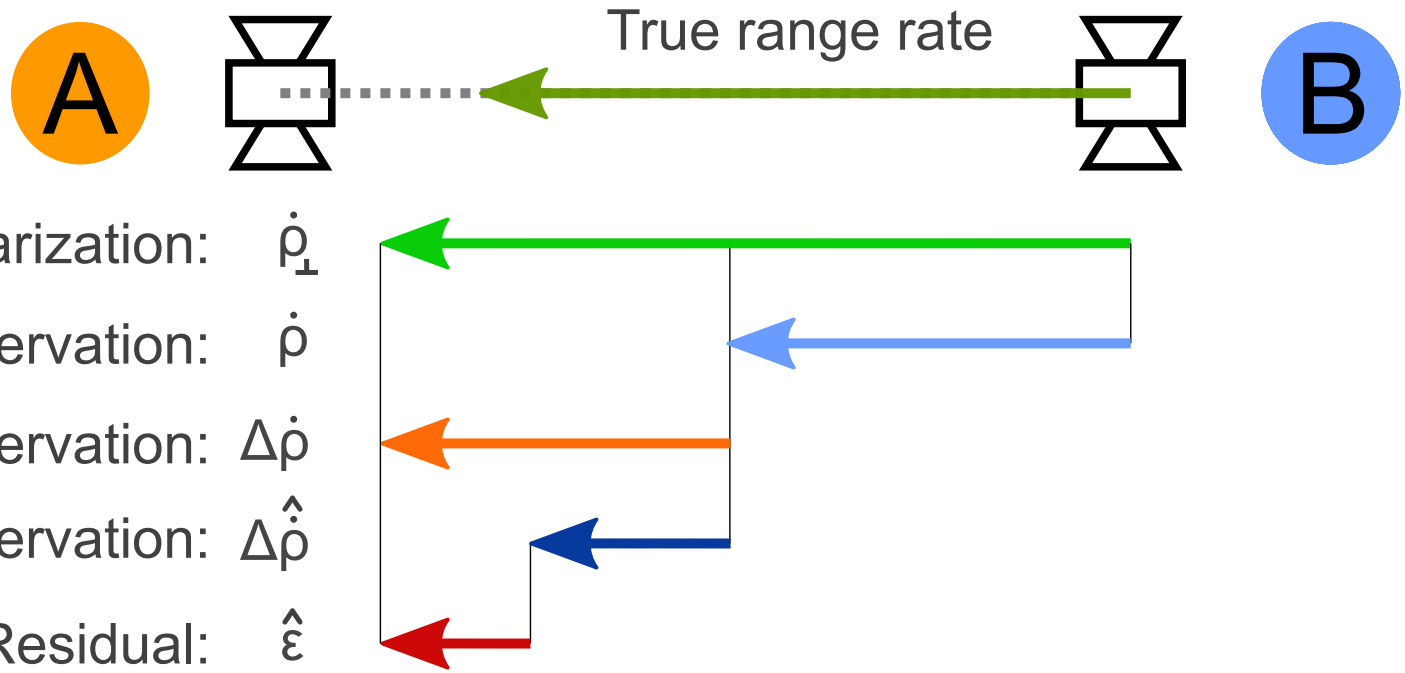

Linearization impacts reduced observation and residuals.

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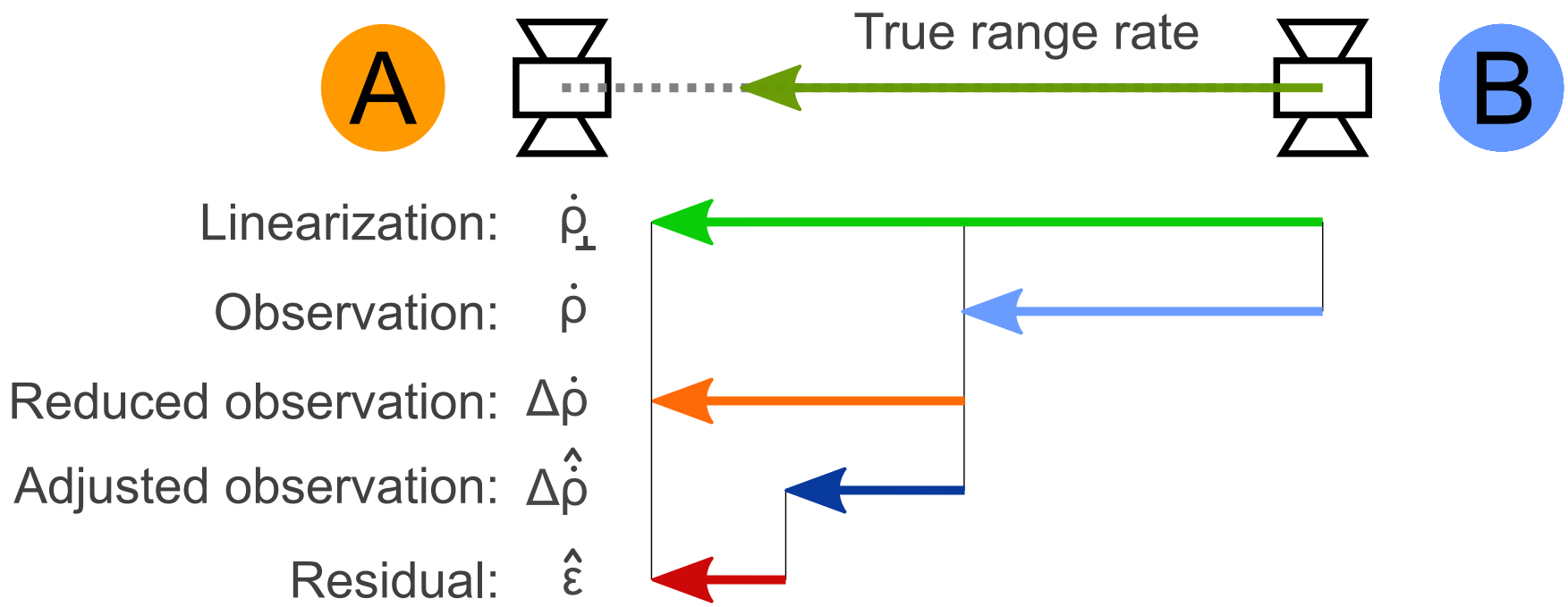
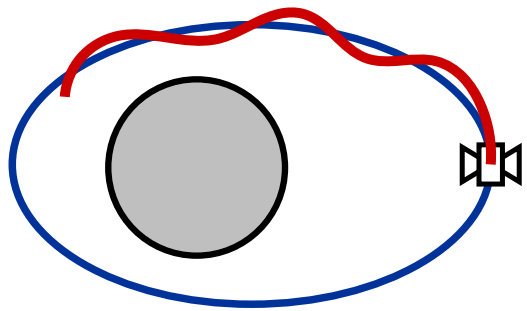


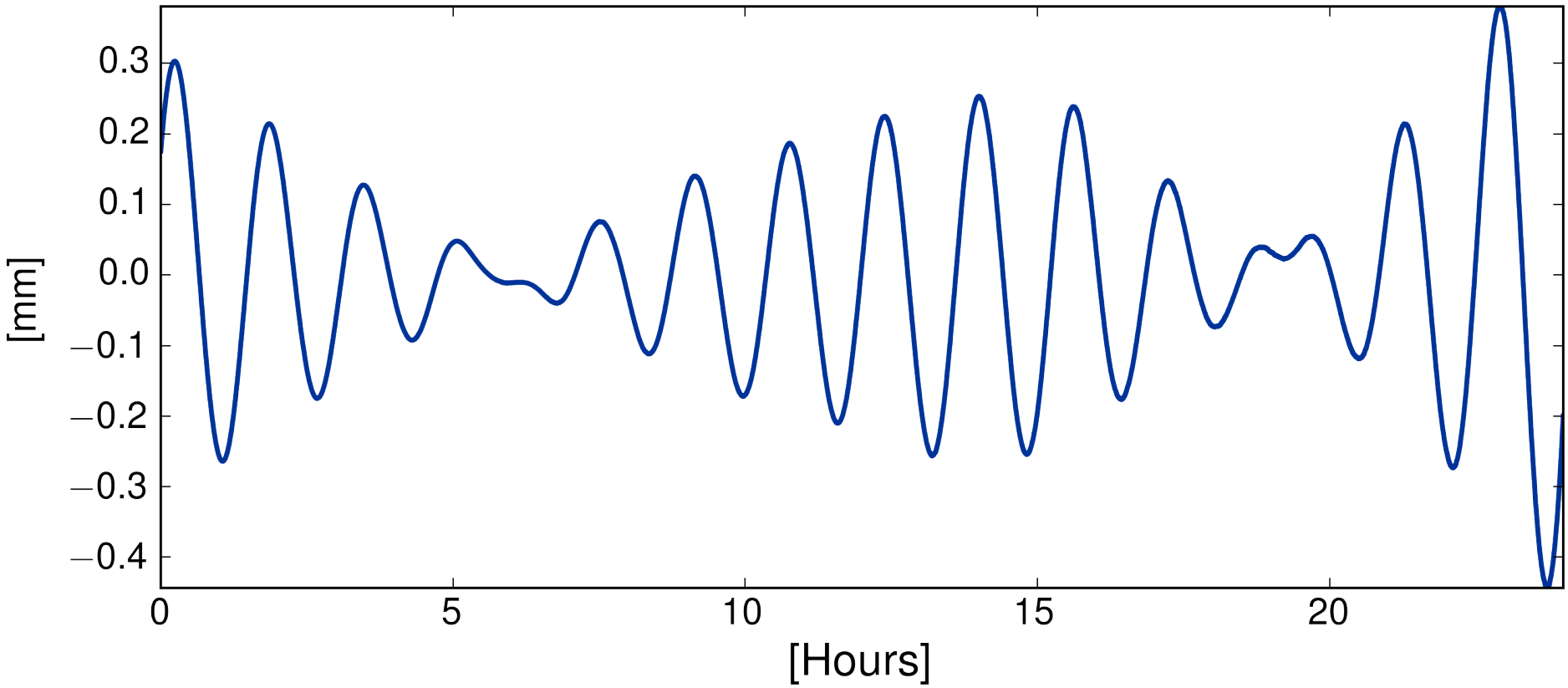
Linearization impacts reduced observation and residuals.

- Keplerian orbit
- No noise on observables



Dynamic orbit error

Integrated dynamic orbit vs Keplerian orbit: X-coordinate difference



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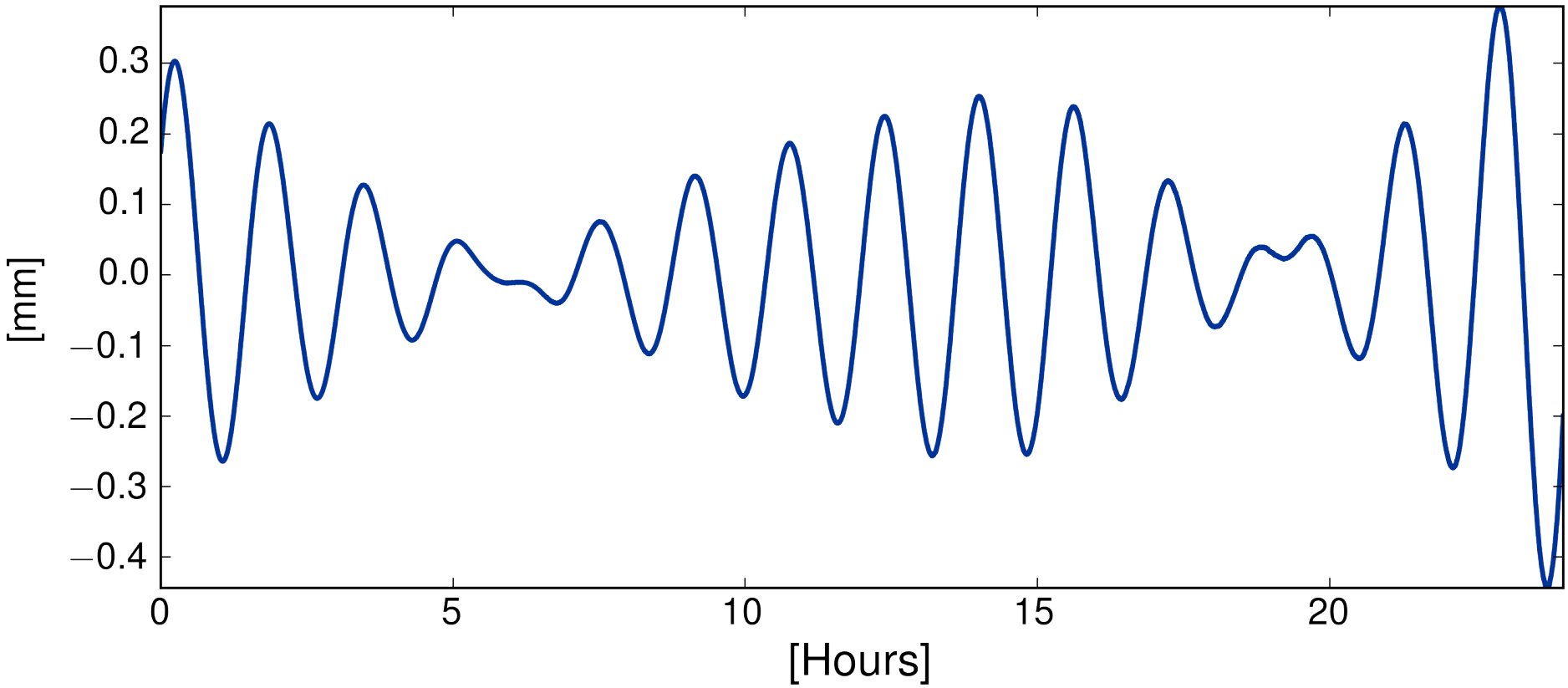
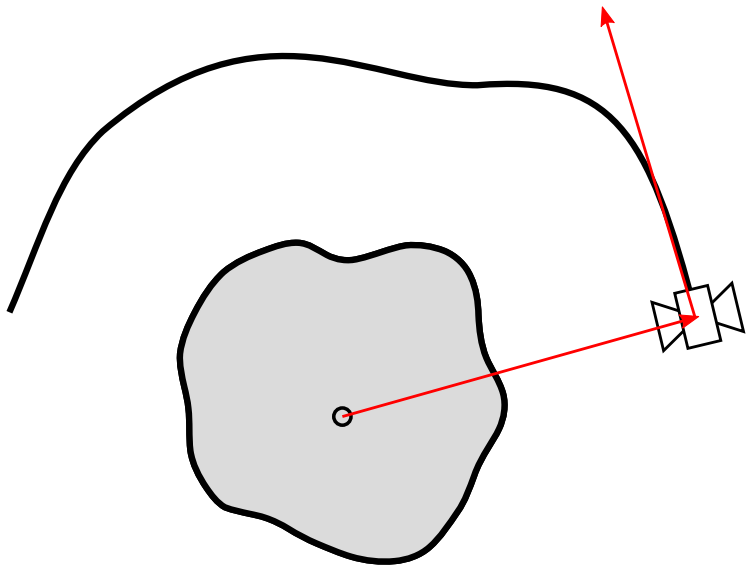


These errors result solely from the processing method!

Alternative approaches



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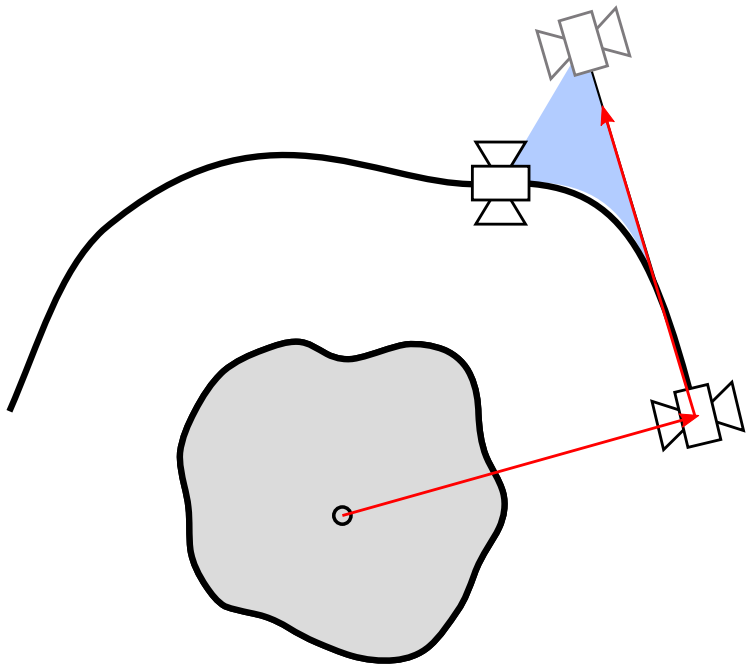

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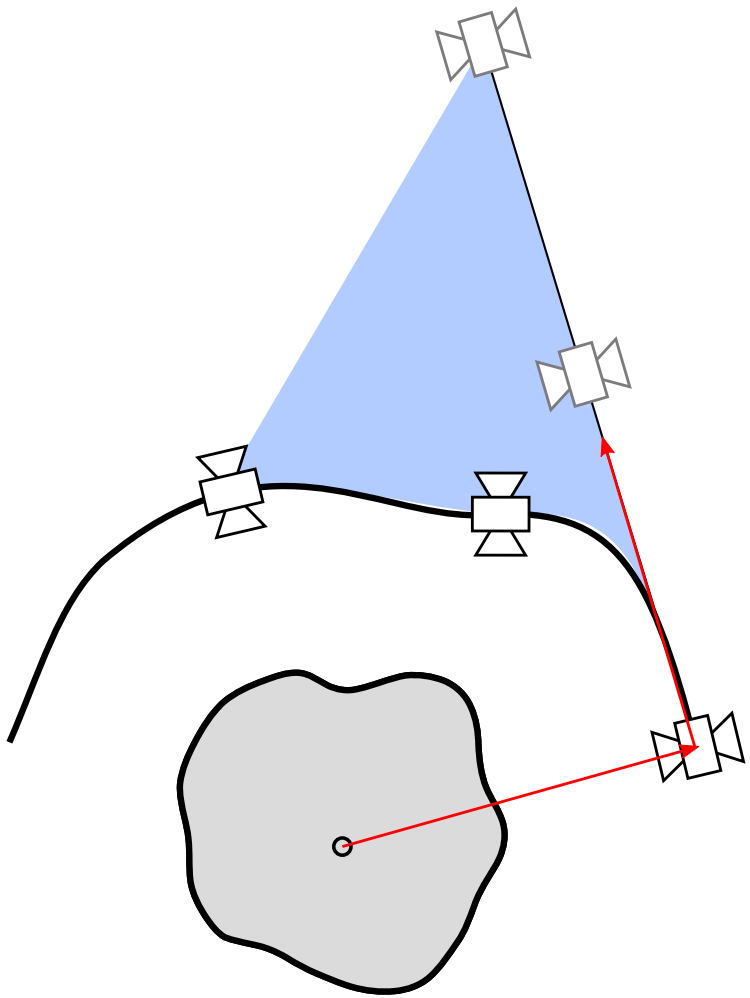

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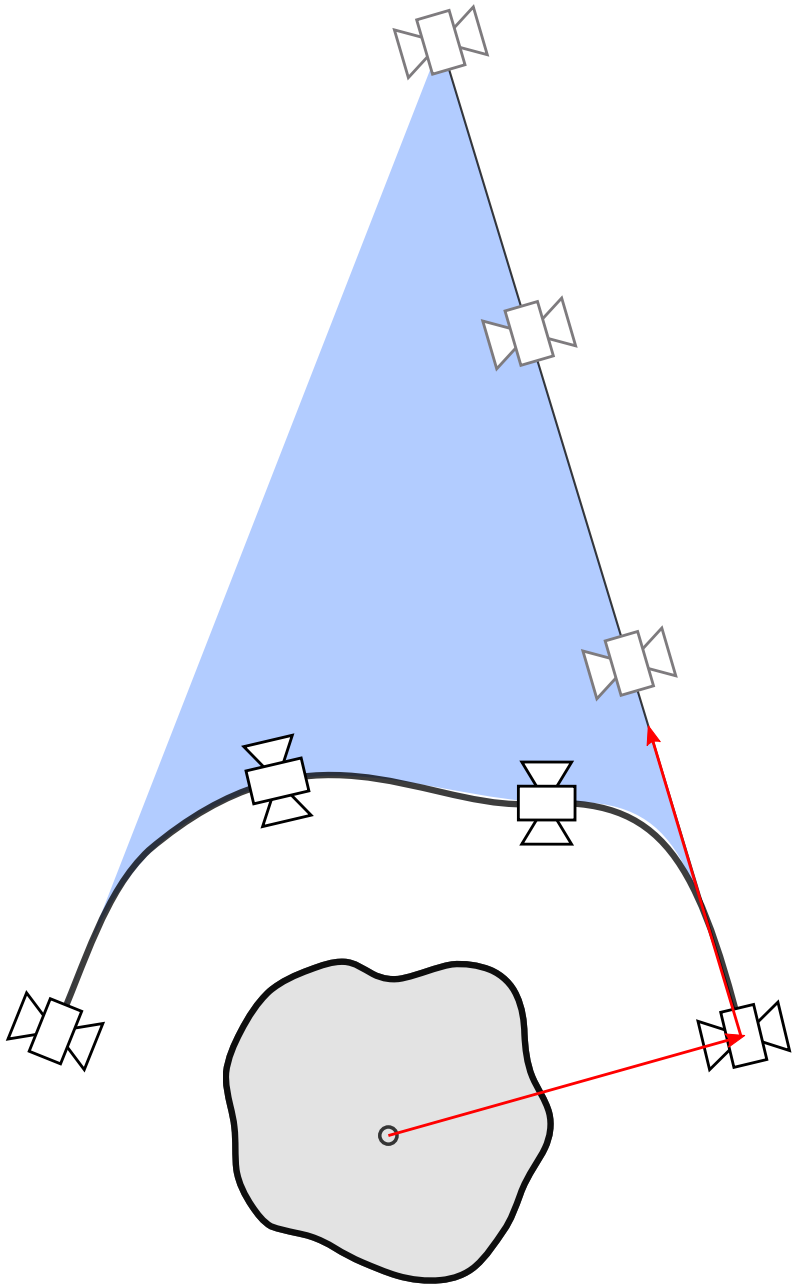

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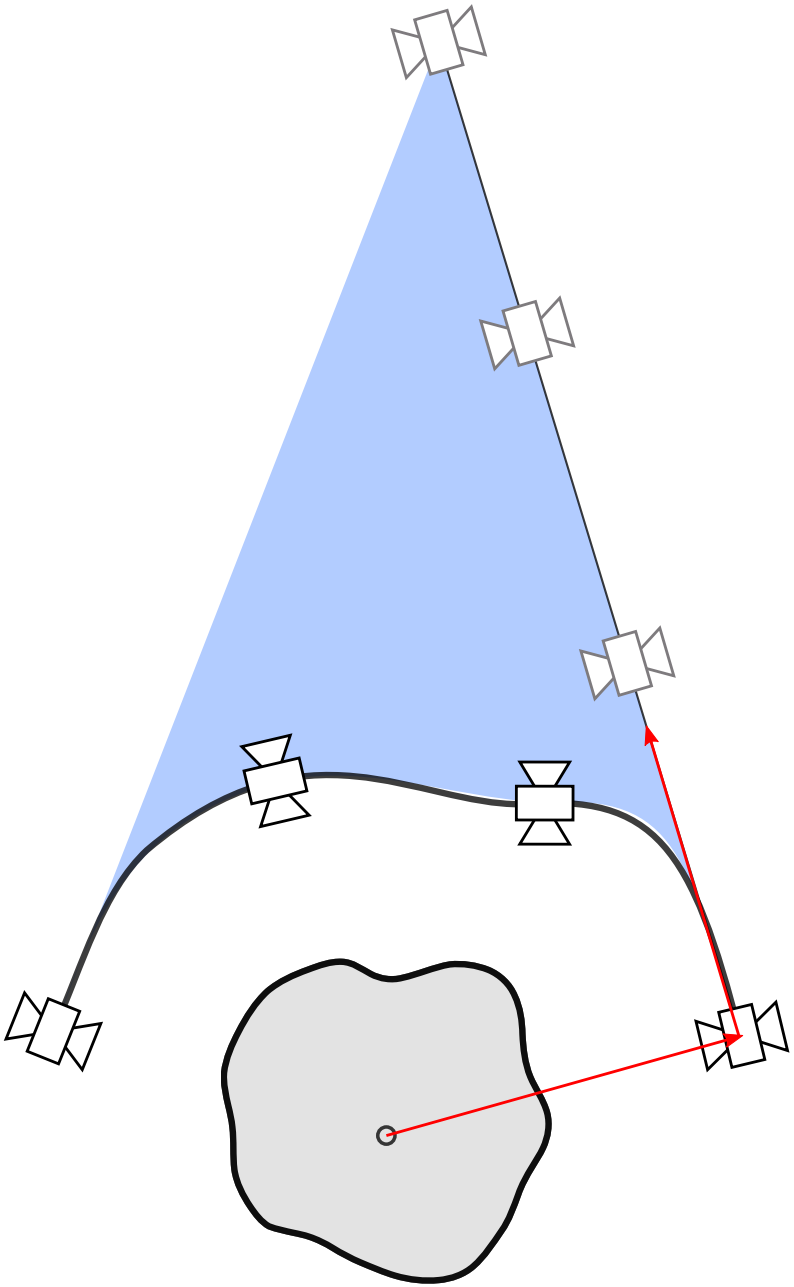
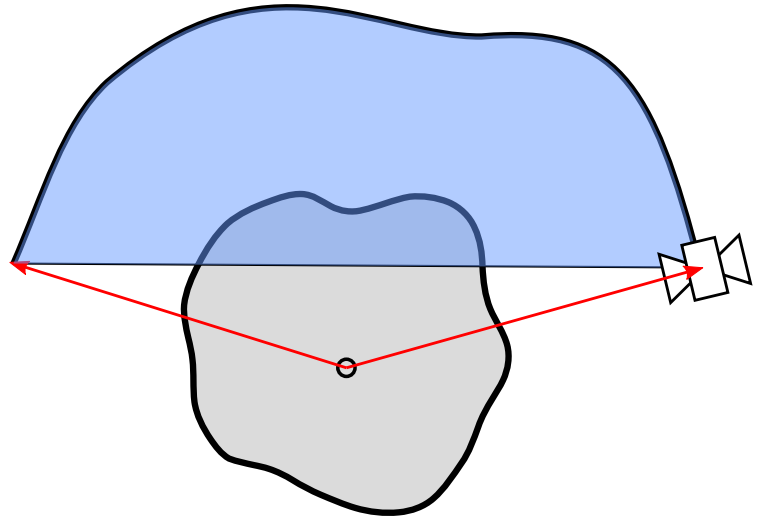
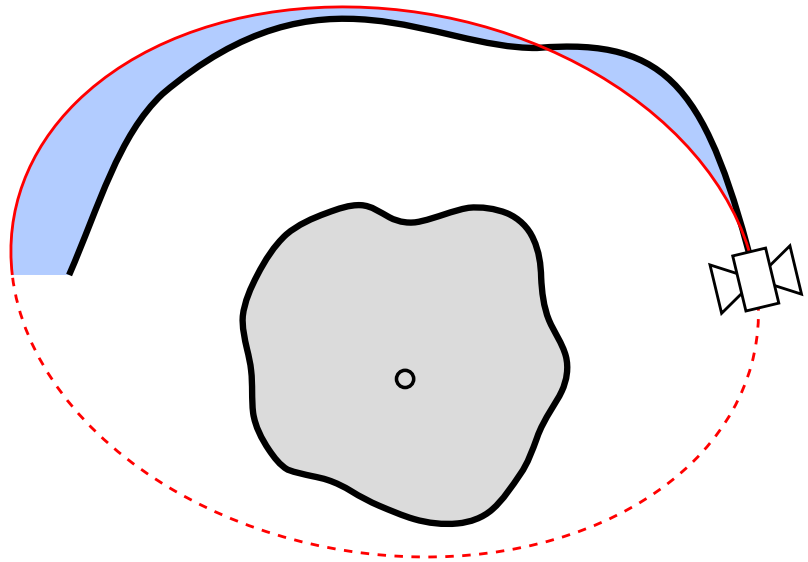
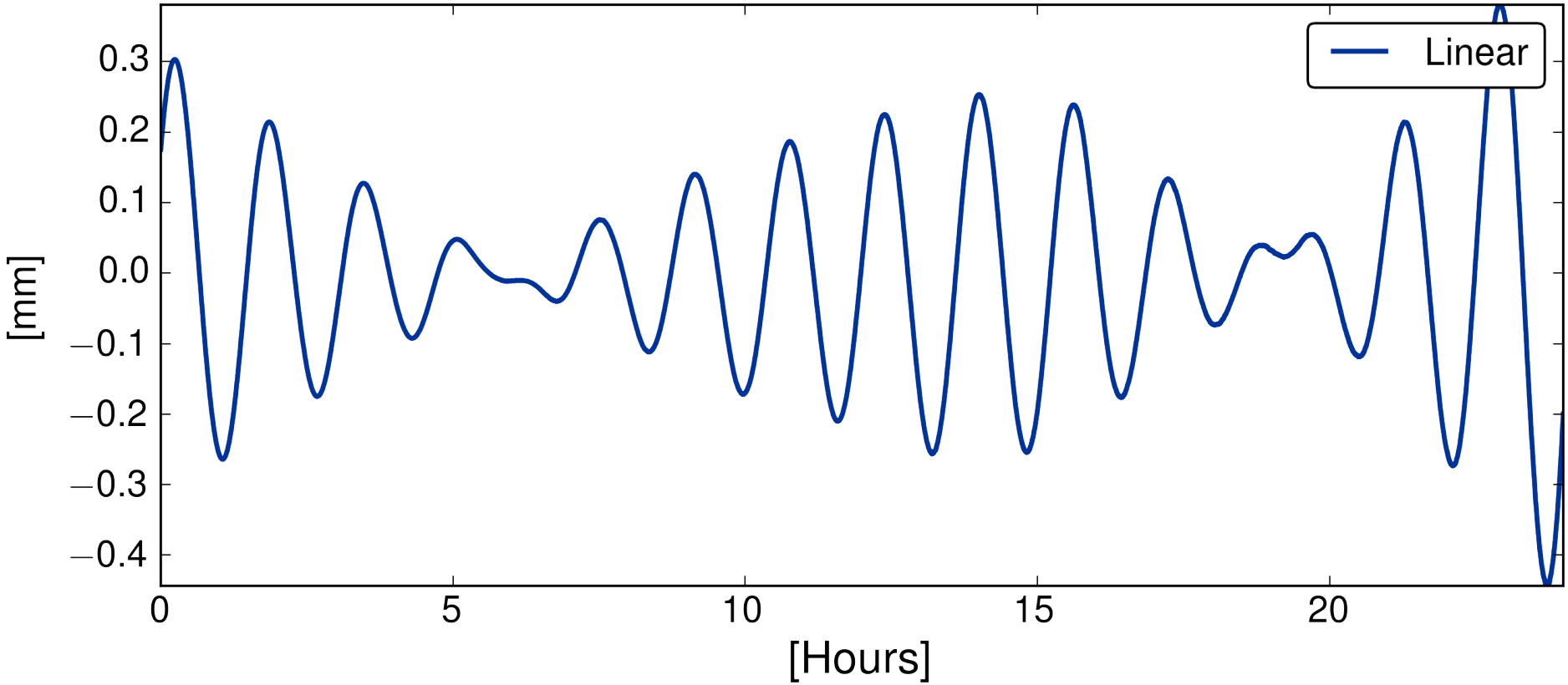






Elliptical parametrization

Integrated dynamic orbit vs Keplerian orbit: X-coordinate difference



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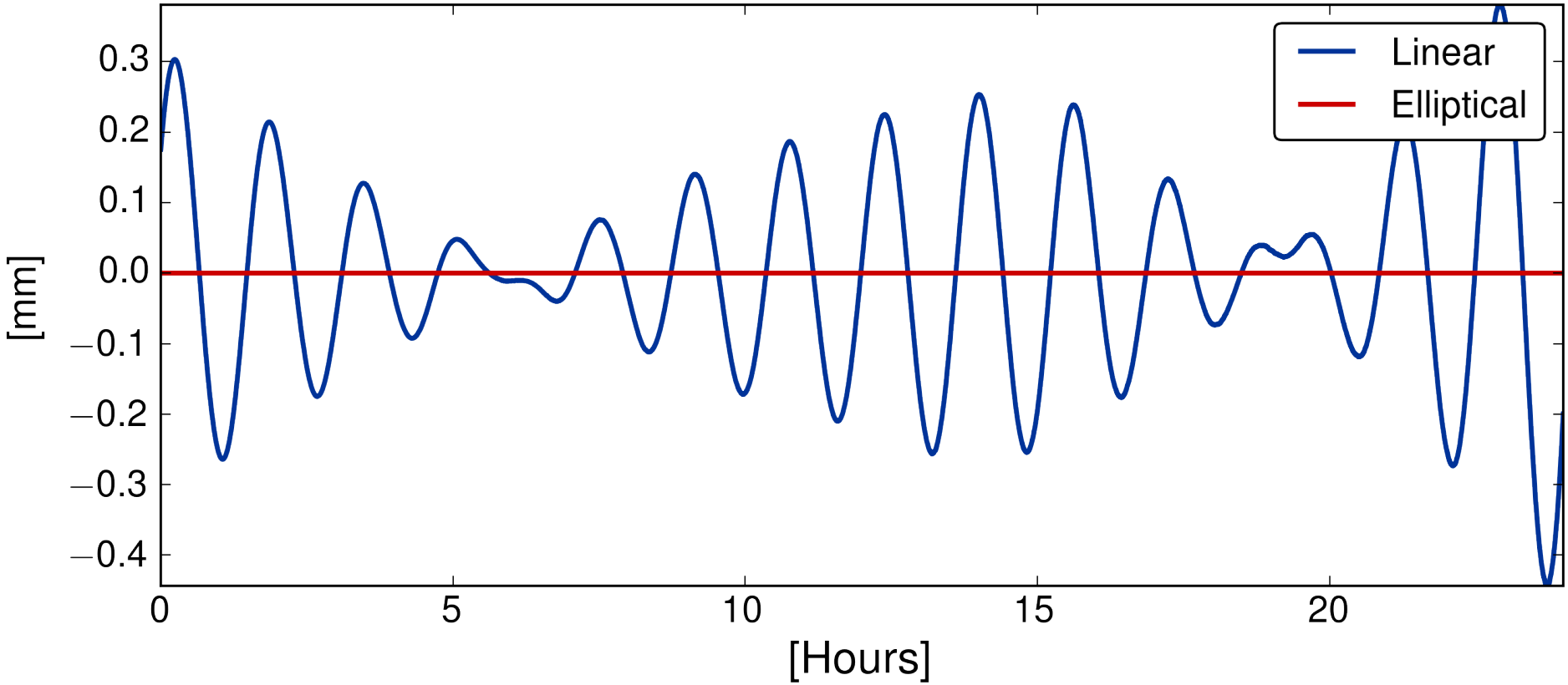

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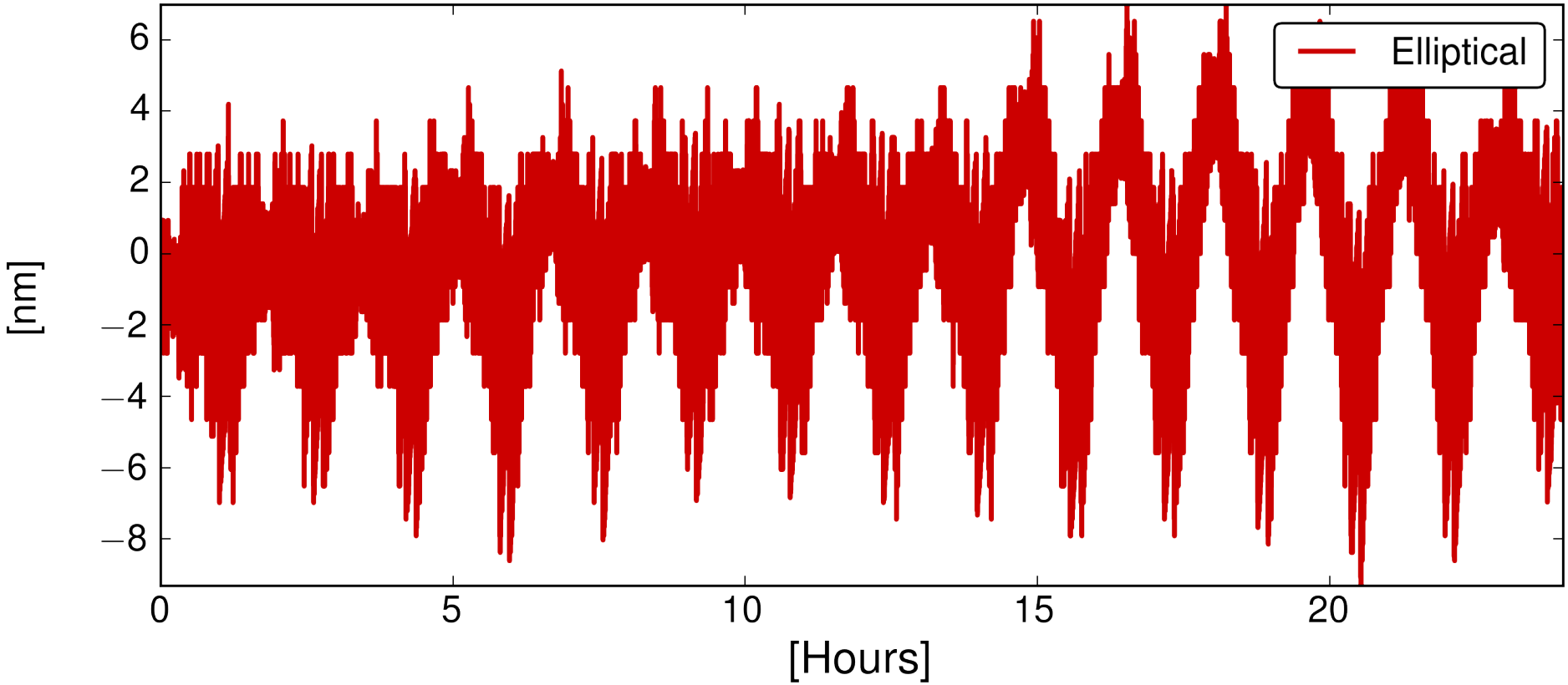

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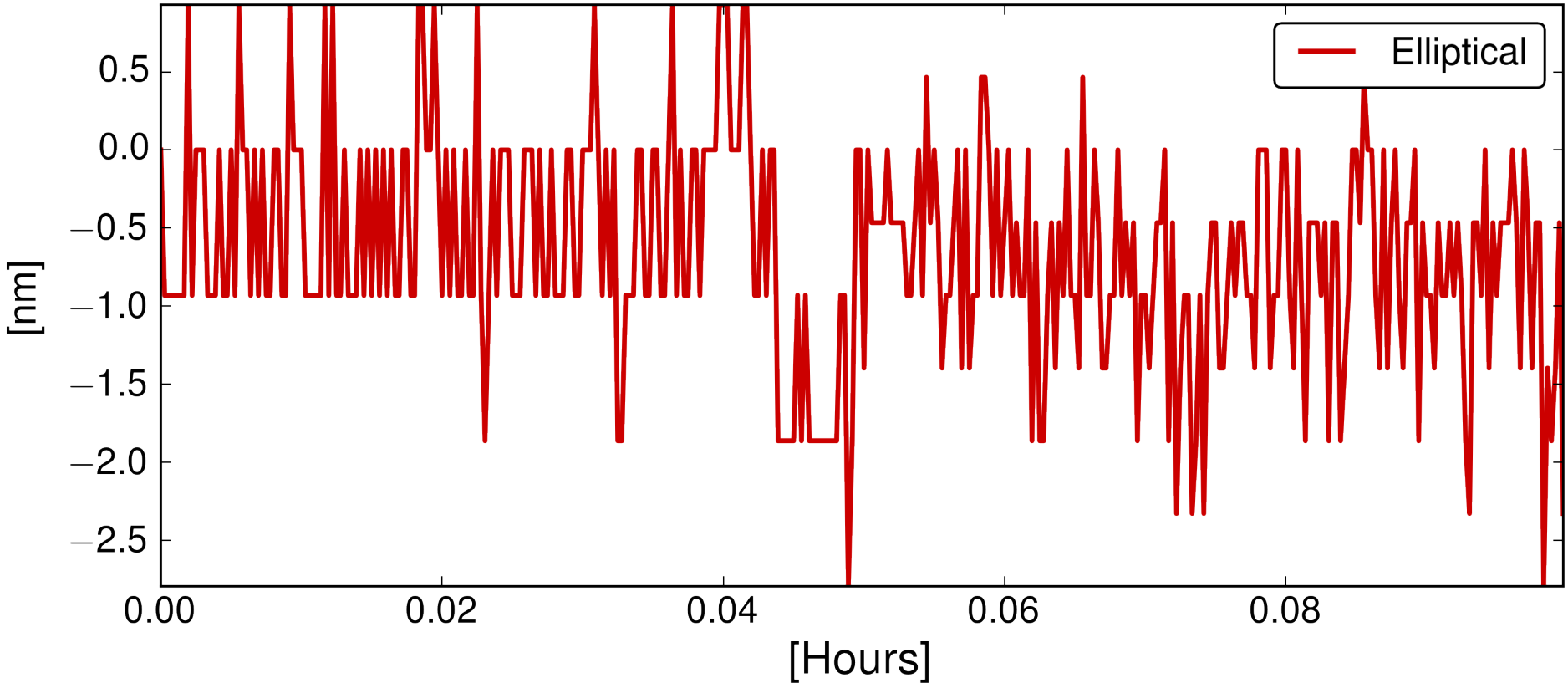

Example numeric values

Reference	6061682.767626133	
Elliptical	6061682.7676261332	$\sim$ 15 digits of precision
Linear	6061682.76761392 3	$\sim$ 11 digits of precision

Adding noise

Integrated dynamic orbit vs Keplerian orbit: X-coordinate difference

- $\sigma = 5\,\text{cm}$ white noise for orbit observations.
- $\sigma = 1 \times 10^{-10}\,\text{m/s}^2$ white noise for accelerometer observations.

Adding noise

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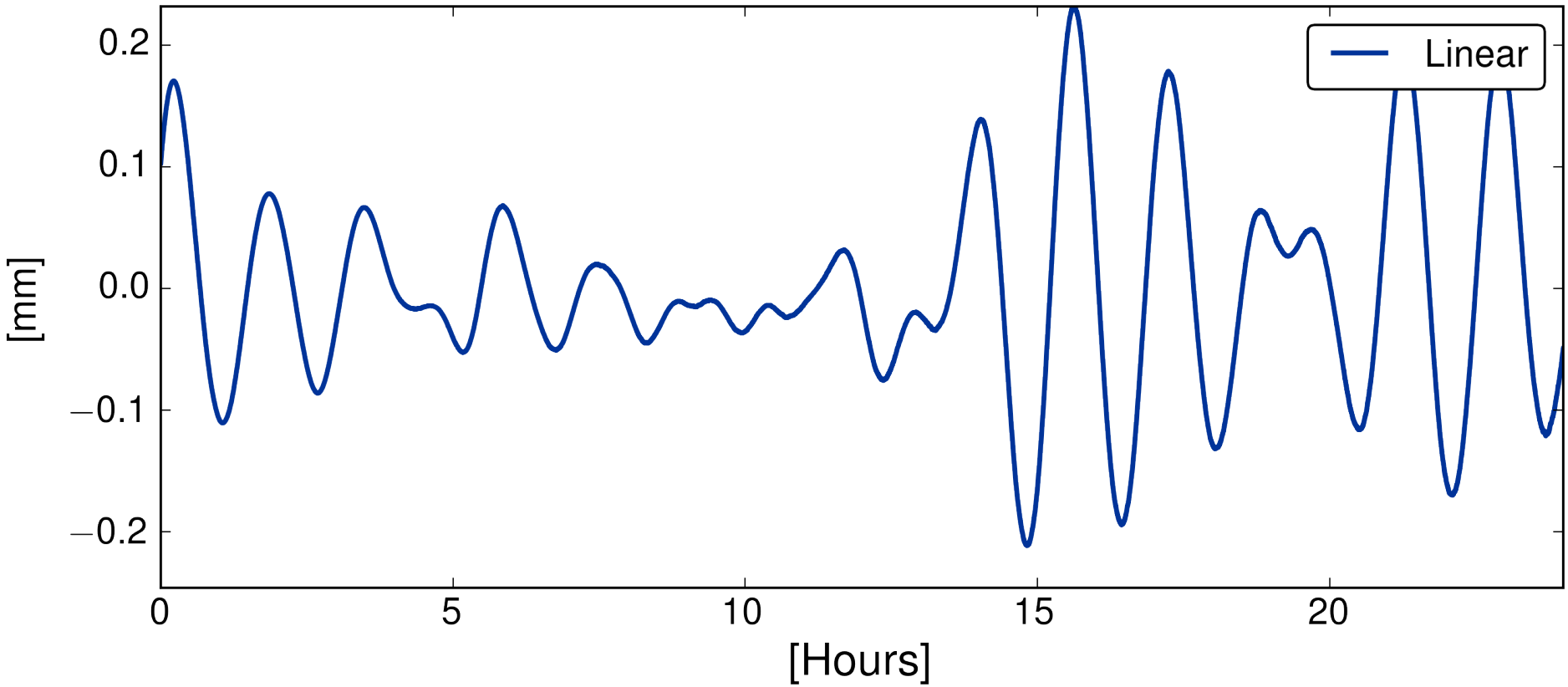

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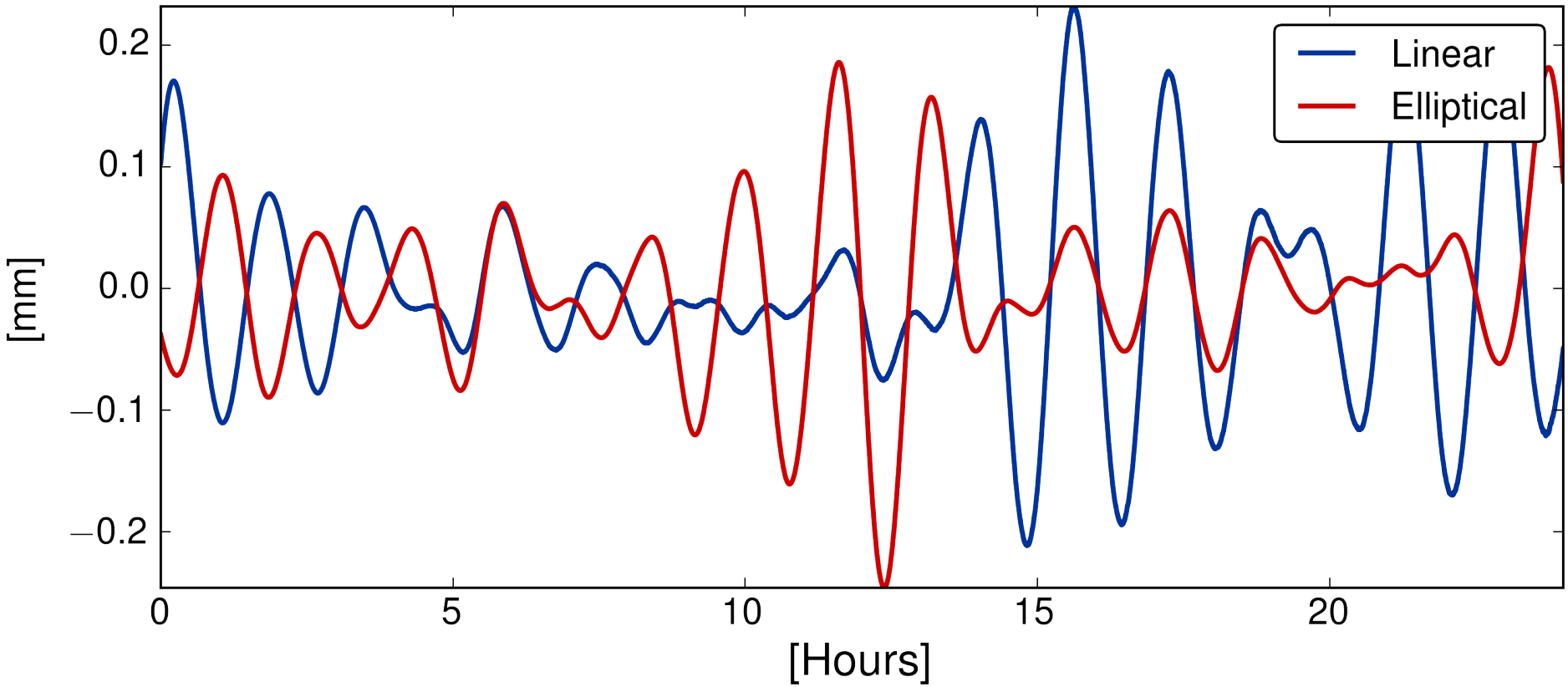

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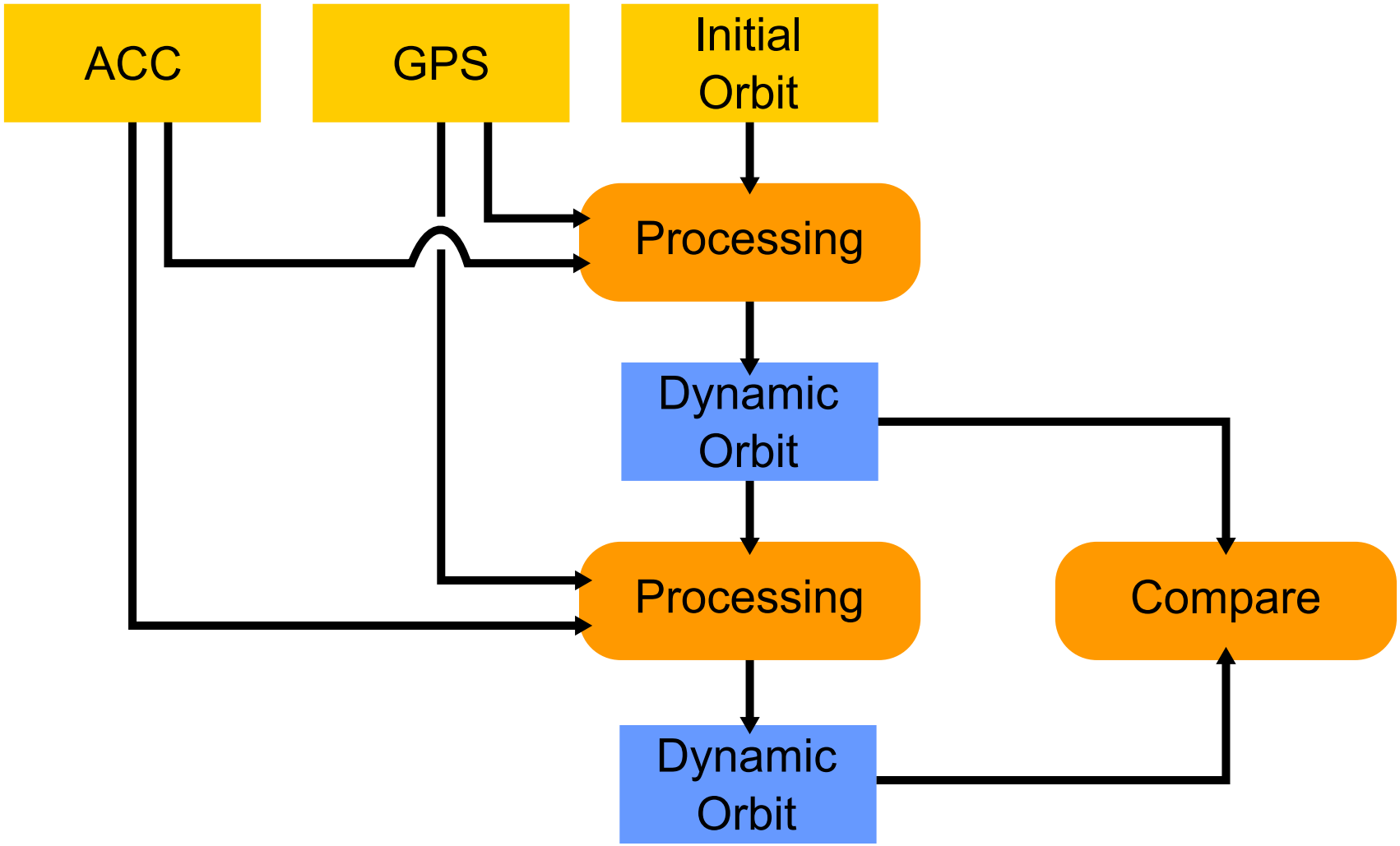
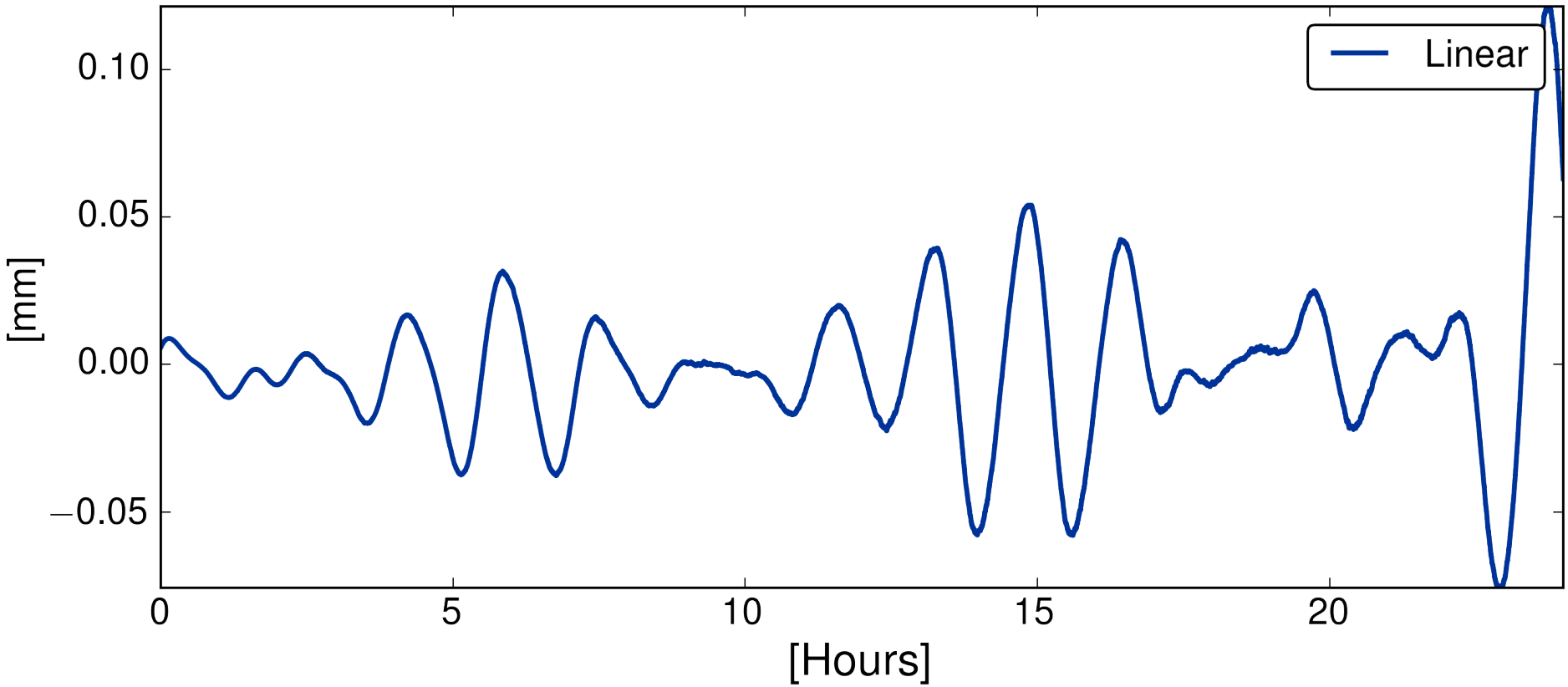
Repeated evaluation

Expectation: Dynamic orbit is invariant through multiple iterations.



Repeated evaluation

Integrated dynamic orbit vs integrated dynamic orbit: X-coordinate difference



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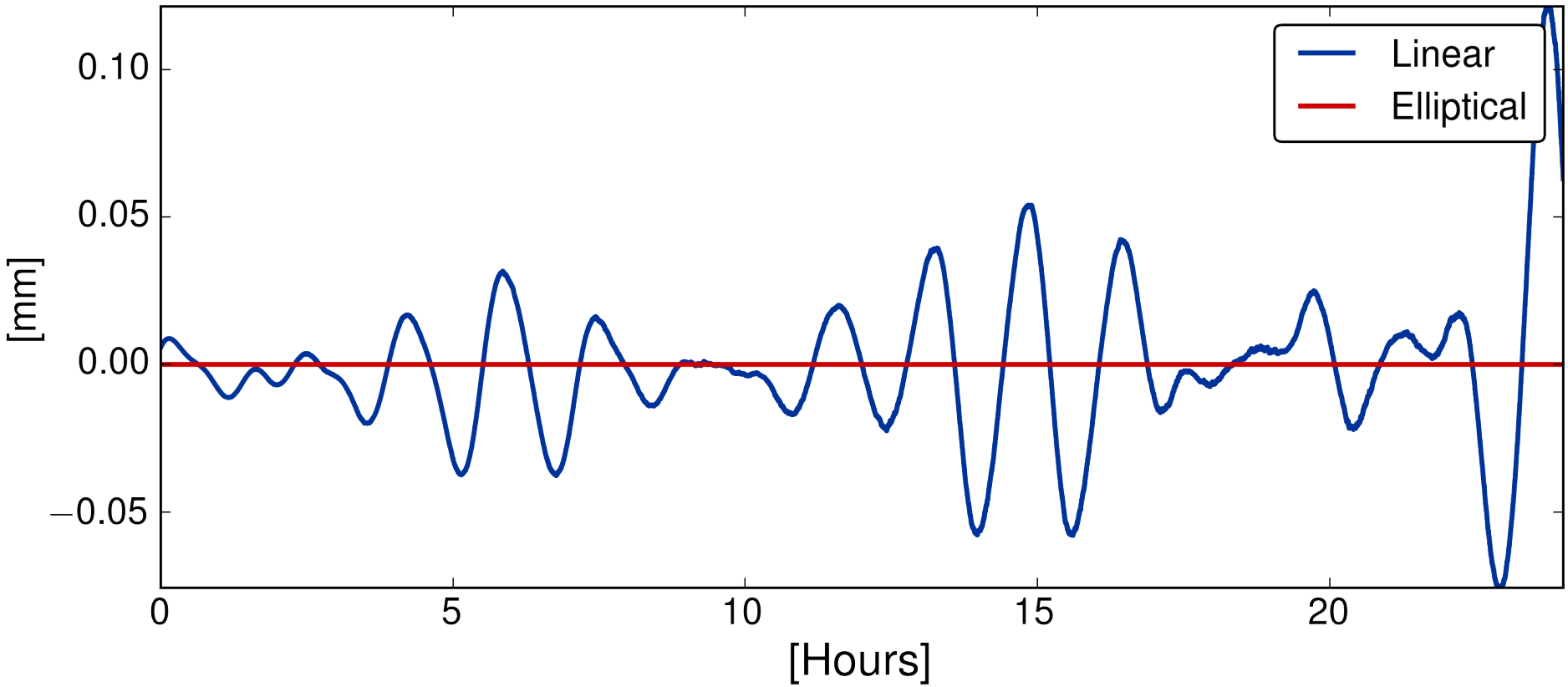

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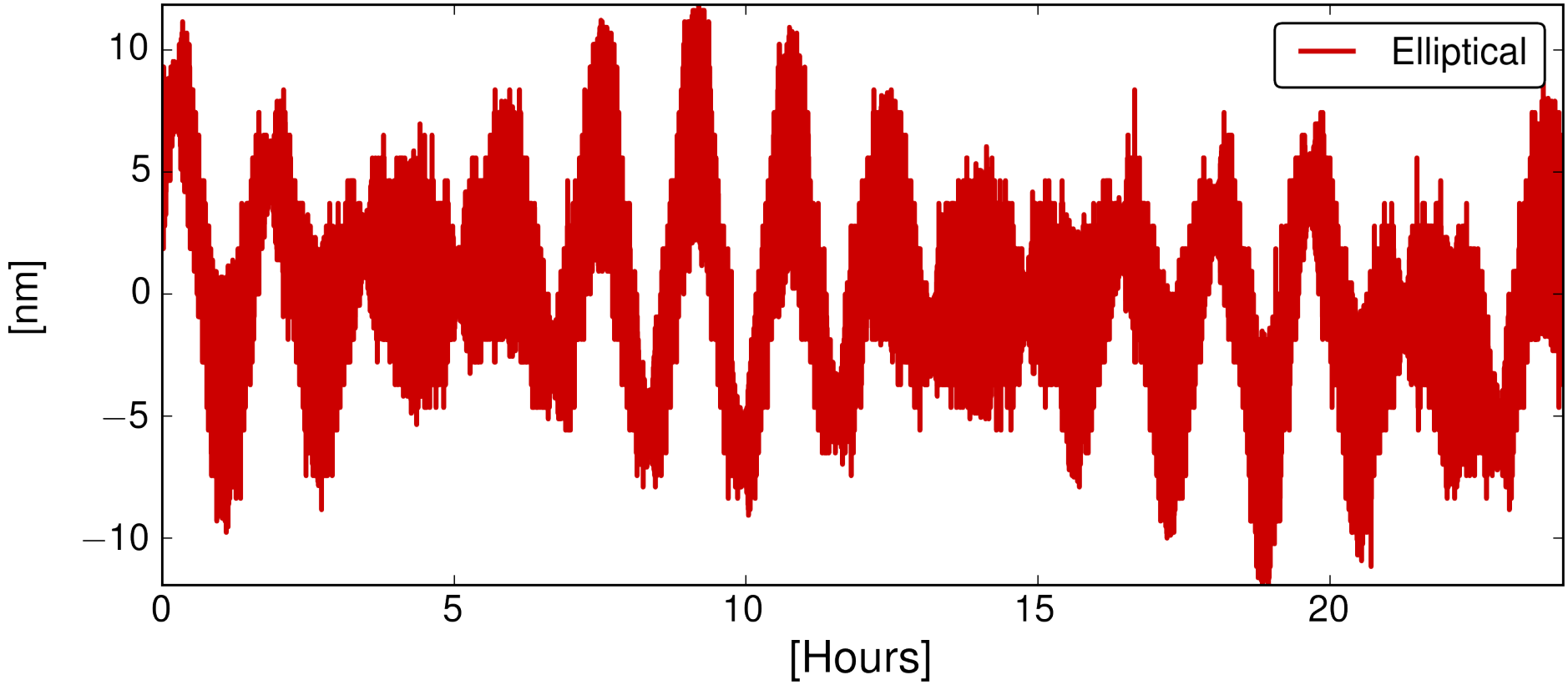

- New processing shows improved internal consistency.
- Better orbits $\rightarrow$ better linearization $\rightarrow$ reduced artifacts.

Conclusion

Take-aways

- We applied improved force model integration to dynamic orbit computation.
- Elliptical method shows promise for reducing processing artifacts in adjusted SST observations and residuals.

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Further investigations / To do

- Analyze impact on real dynamic orbits.
- Investigate effect on SST residuals.

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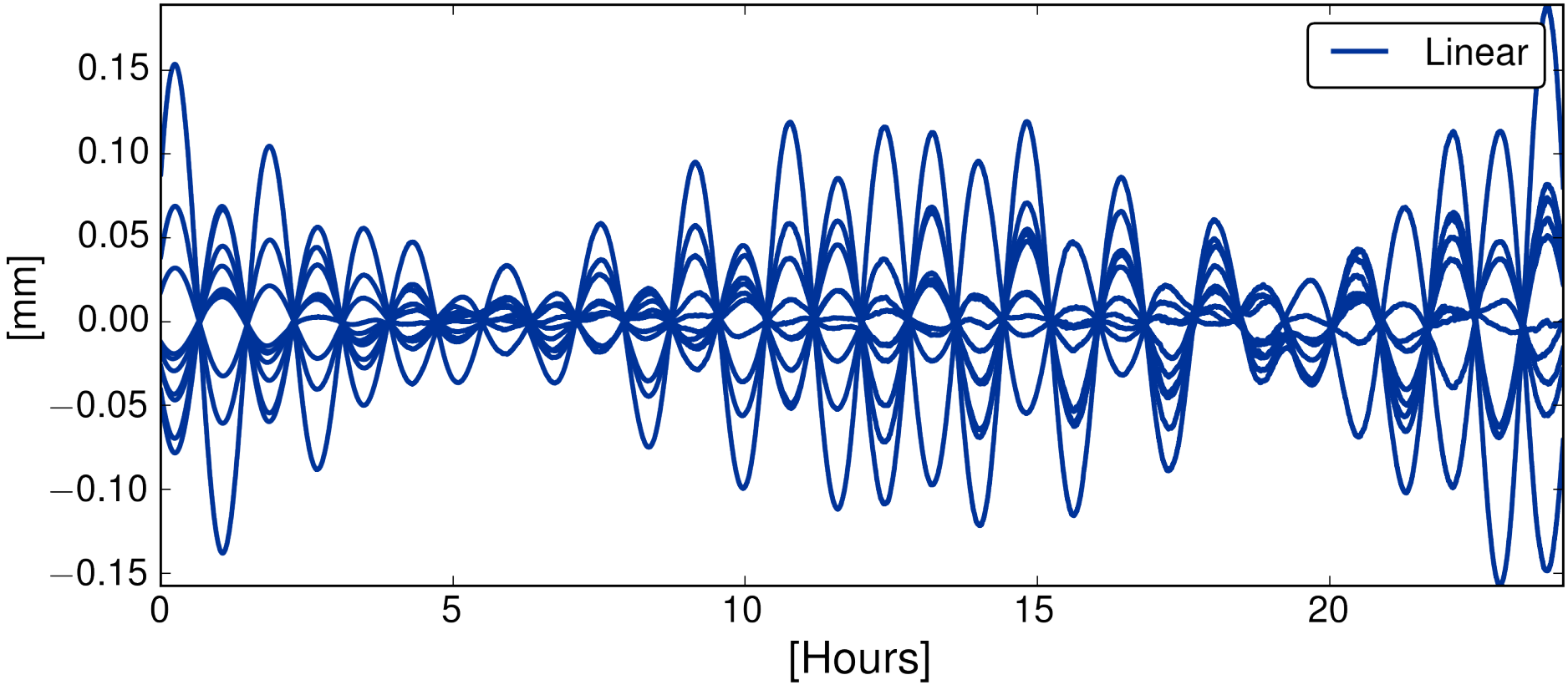
Thank you for your attention!

Backup

10 iterations of processing compared to the first: X-coordinate difference



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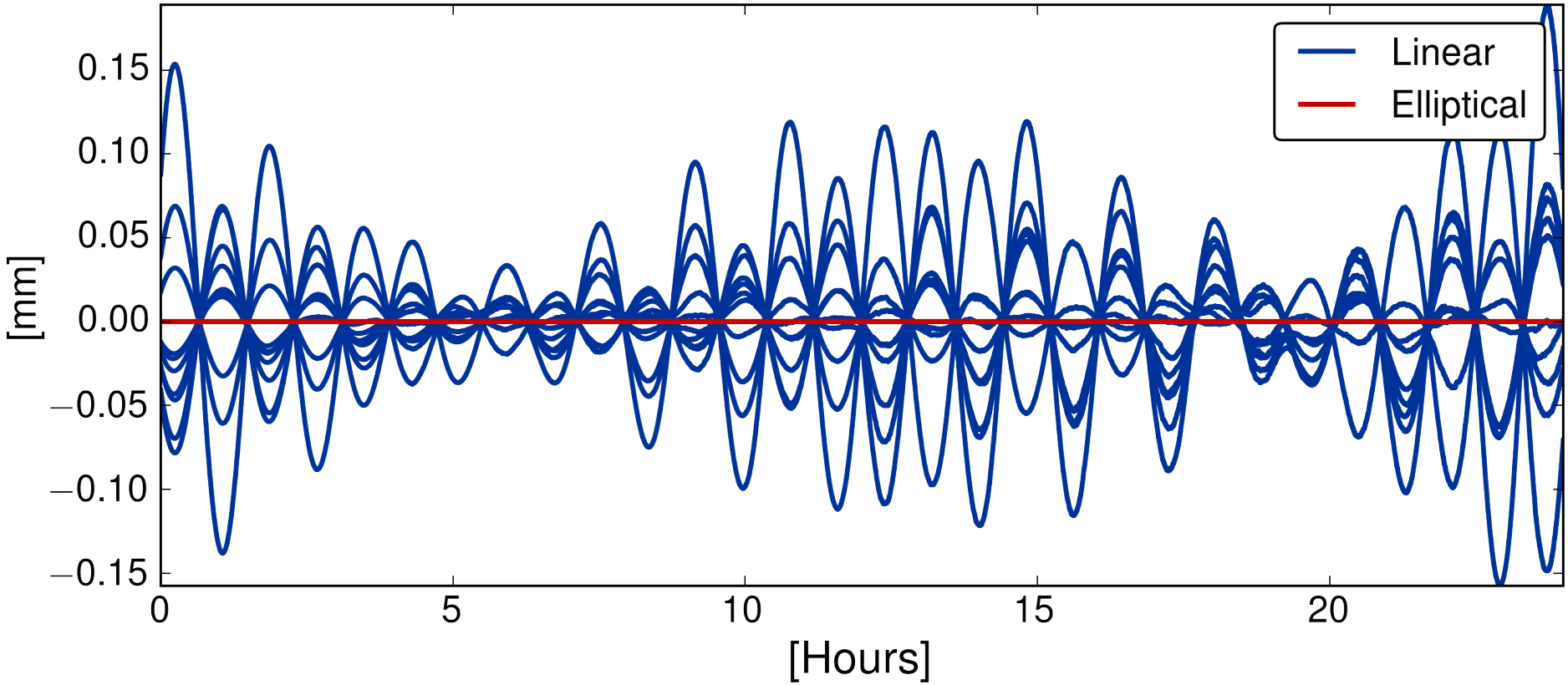

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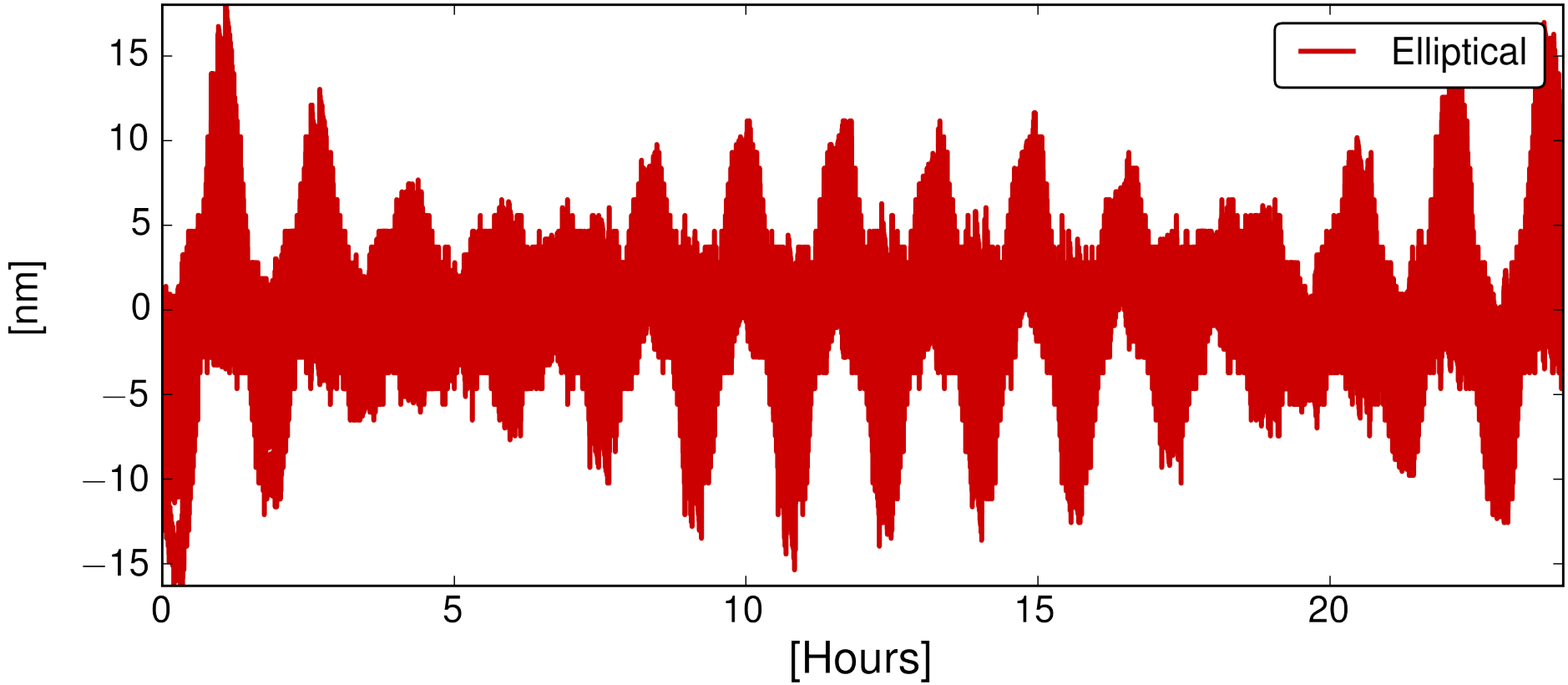

Real SST residuals