Institute of
Structural Analysis
Institut für Baustatik

Graz University of Technology

Iterative Coupling of Boundary and Discrete Element Method Using an Overlapping FEM Zone

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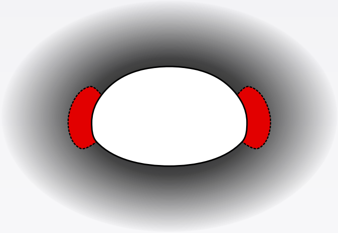
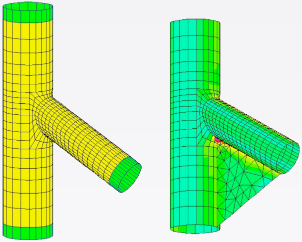




- Inelastic zones not known a priori
- May occurs in small portions of total domain
- Not all numerical methods suitable for modelling

Aim

Develop a Methodology

- Linear elastic analysis

  Detect zones with inelastic material behaviour

  Define new regions and remesh

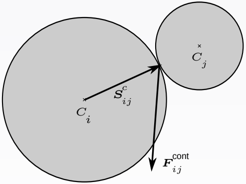
  Solve the coupled problem

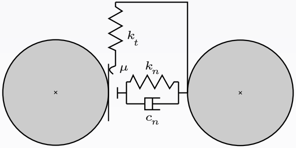
- Involve different methods where they work best
  - Boundary Element Method
  - Discrete Element Method

Contents

- Continuum theory
- Two basic fields:
  - Displacements: $\mathbf{u} = (u_x, u_y, u_z)$
  - Tractions: $\mathbf{t} = (t_x, t_y, t_z) = \boldsymbol{\sigma} \cdot \hat{\mathbf{n}}$
- $\mathbf{c} \cdot \mathbf{u} = \int\limits_S \mathbf{U}^* \cdot \mathbf{t} \, dS - \int\limits_S \mathbf{T}^* \cdot \mathbf{u} \, dS$
- Stresses on the boundary

- Material represented as collection of particles
- Particles interacts with contact forces
- Dynamic method – Newton's Second Law





- Equation of motion
  - $\sum_{i=1}^{N} \left[(m_i \ddot{\mathbf{u}}_i - \mathbf{F}_i) \, \delta u_i + (\mathbf{I}_i \dot{\boldsymbol{\omega}}_i - \mathbf{T}_i) \, \delta \phi_i \right] = 0$
  - Motion equations integrated in time with central difference scheme
- Fields:
  - Displacements: $\mathbf{u_i} = (u_x, u_y, u_z)$
  - Forces: $\mathbf{F_i} = (F_x, F_y, F_z) = F_i^{ext} + \sum_{j=1}^{n_i^{cont}} F_{ij}^{cont} + F_i^{damp}$
  - Angular velocities: $\boldsymbol{\omega}_\mathbf{i} = (\omega_x, \omega_y, \omega_z)$
  - Angular momentum: $\mathbf{T_i} = (T_x, T_y, T_z) = T_i^{ext} + \sum_{j=1}^{n_i^{cont}} S_{ij}^c \times F_{ij}^{cont} + T_i^{damp}$

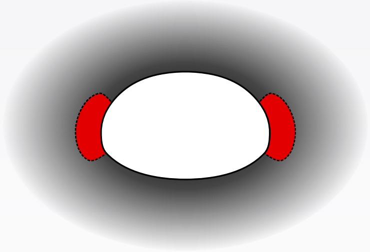
- Assumption:

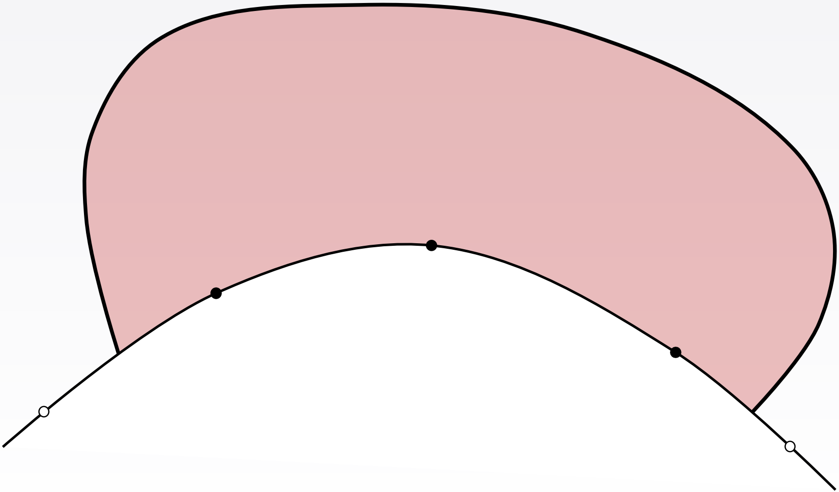
  Zone starts from the boundary

- Make an elastic analysis

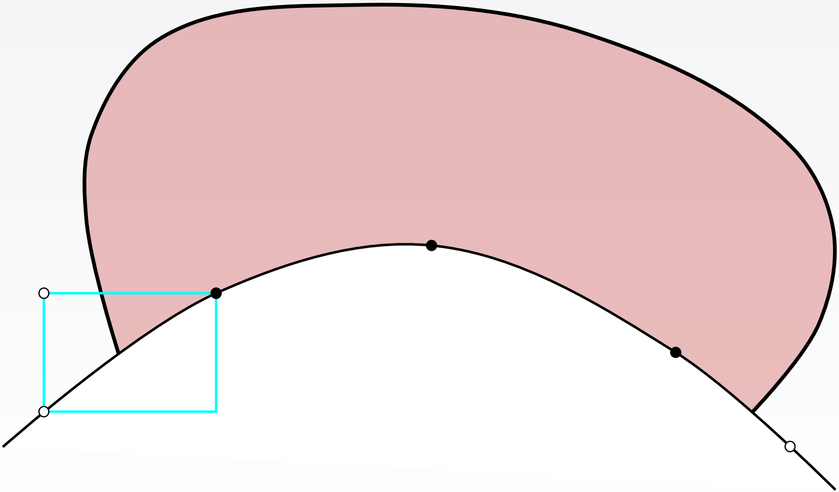
- Criteria expressed as a yield surface
  $F(\sigma) = \sigma_{max} - T$

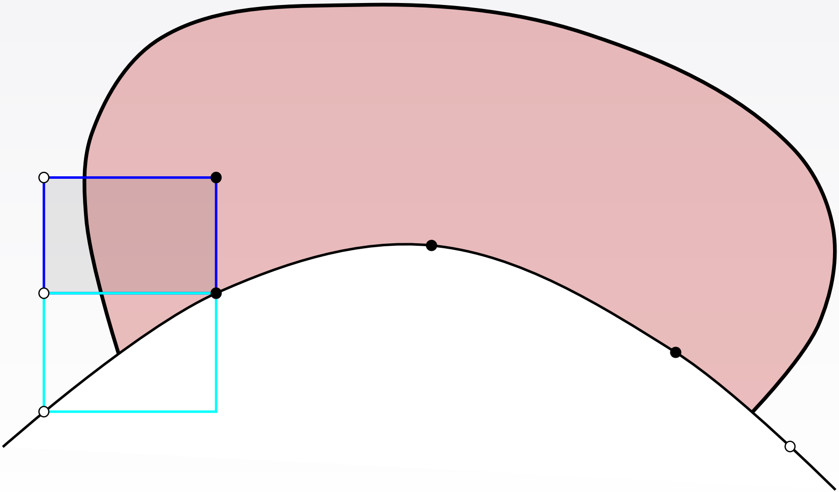
- Follow the isocurve



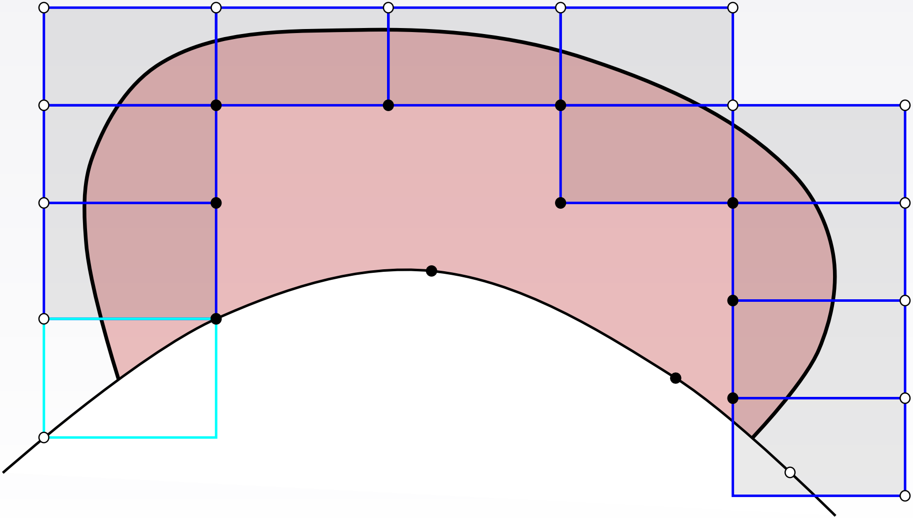


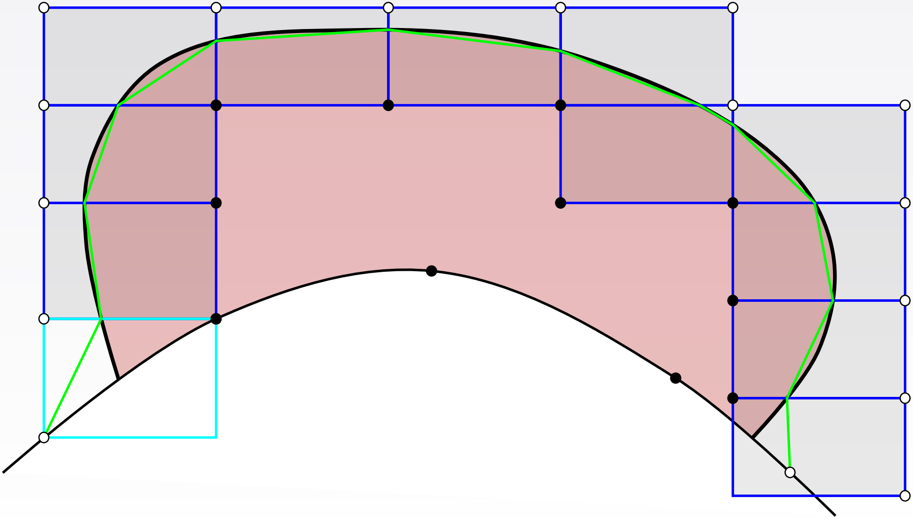
Modified Marching Squares





Modified Marching Squares





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Iterative Coupling

- No global stiffness matrix available
- Static BEM – dynamic DEM: a quasi-static approach
- Each method retains its advantages
- Different programming languages
- Add a FEM overlapping zone for smooth transition between DEM and BEM

FEM formulation

- From Hamilton's principle

$$\mathbf{M}_F \ddot{\mathbf{u}}_F = \mathbf{F}_F^{ext} - \mathbf{F}_F^{int}$$

- Assembled from elemental matrices and vectors:

$$\mathbf{m}_e = \int_{\Omega_e} \rho \mathbf{N}^T \mathbf{N} d\Omega_e$$

$$\mathbf{f}_e^{int} = \int_{\Omega_e} \mathbf{B}^T \boldsymbol{\sigma} d\Omega_e$$

$$\mathbf{f}_e^{ext} = \int_{\Omega_e} \mathbf{N}^T \rho \mathbf{b} d\Omega_e + \int_{\Gamma_e} \mathbf{N}^T \mathbf{t} d\Gamma_e$$

- Integrated in time with central difference scheme

Overlapping FEM Zone

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Transition Zone

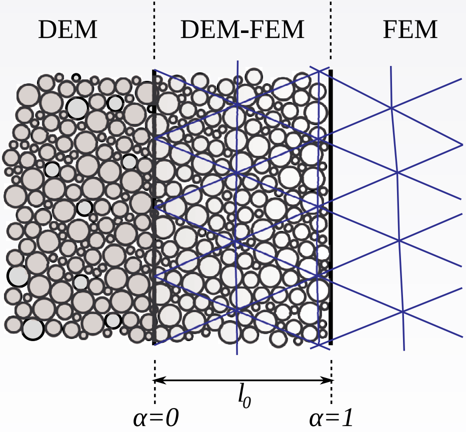
- Total energy as a linear combination
  $\partial W = \alpha \partial W_F + (1 - \alpha)\, \partial W_D$
- Kinematic constraints:

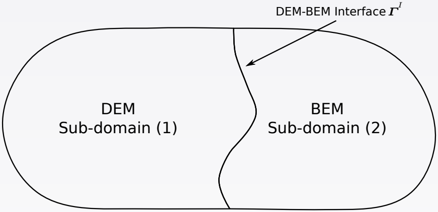
$$\mathbf{u}_{d_i} = \mathbf{N}\mathbf{r}_F^e$$

$$\dot{\mathbf{u}}_{d_i} = \mathbf{N}\dot{\mathbf{r}}_F^e$$

$$\ddot{\mathbf{u}}_{d_i} = \mathbf{N}\ddot{\mathbf{r}}_F^e$$

- Penalty method for imposing the above constraints

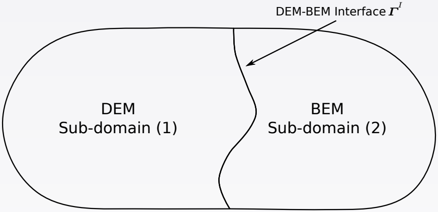


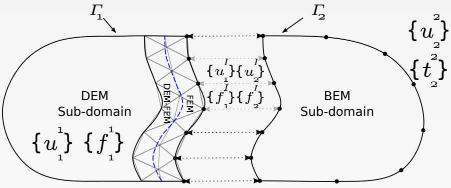


- sub-domain (1):
  $\mathbf{u}_1 = \{\mathbf{u}_1^1, \mathbf{u}_1^I\}^T$
  $\mathbf{f}_1 = \{\mathbf{f}_1^1, \mathbf{f}_1^I\}^T$
- sub-domain (2):
  $\mathbf{u}_2 = \{\mathbf{u}_2^2, \mathbf{u}_2^I\}^T$
  $\mathbf{t}_2 = \{\mathbf{t}_2^2, \mathbf{t}_2^I\}^T$

Compatibility and equilibrium conditions

- $\mathbf{u}_1^I = \mathbf{u}_2^I$
- $\mathbf{f}_1^I = -\mathbf{f}_2^I$





- sub-domain (1):
  $\mathbf{u}_1 = \{\mathbf{u}_1^1, \mathbf{u}_1^I\}^T$
  $\mathbf{f}_1 = \{\mathbf{f}_1^1, \mathbf{f}_1^I\}^T$
- sub-domain (2):
  $\mathbf{u}_2 = \{\mathbf{u}_2^2, \mathbf{u}_2^I\}^T$
  $\mathbf{t}_2 = \{\mathbf{t}_2^2, \mathbf{t}_2^I\}^T$

Compatibility and equilibrium conditions

- $\mathbf{u}_1^I = \mathbf{u}_2^I$
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BEM – DEM/FEM Region Coupling

- Methods with different fields
- Nodal point forces to be calculated on the BEM side on the interface
- BEM Region: Calculate stiffness matrix for the interface
  - $\{\mathbf{t}\}_c = \{\mathbf{t}\}_{c0} + \mathbf{K}_{BE}\{\mathbf{u}\}_c \quad \Rightarrow \quad \{\mathbf{F}\}_c = \{\mathbf{F}\}_{c0} + \mathbf{K}\{\mathbf{u}\}_c$

- Exchange displacements and forces an the interface
- Iterative procedure requires relaxation to converge
- Applied BC is a linear combination
  - Previous step solution
  - Boundary condition for $n+1$ iteration step provided from adjacent region

$$\mathbf{u}_{n+1}^{1} = \alpha \mathbf{u}_{n}^{1} + (1-\alpha)\,\mathbf{u}_{n+1}^{2}$$
$$\mathbf{f}_{n+1}^{1} = \beta \mathbf{f}_{n}^{1} + (1-\beta)\,\mathbf{f}_{n+1}^{2}$$

- Bad news: Relaxation parameters α, β are problem dependent
- Good news: Relaxation can be automated

- We need to know when to stop
- Displacement based criterion
- User sets maximum relative error between two sequential steps
- Criterion used: $\frac{\left|\left|\mathbf{u}_{n+1}^{I} - \mathbf{u}_{n}^{I}\right|\right|}{\left|\left|\mathbf{u}_{n+1}^{I}\right|\right|} < \epsilon$

Iterative Coupling: Implementation

How?

Two step process

Preprocessing:

- Type of boundary conditions on interface
- Subdomain solution sequence determined

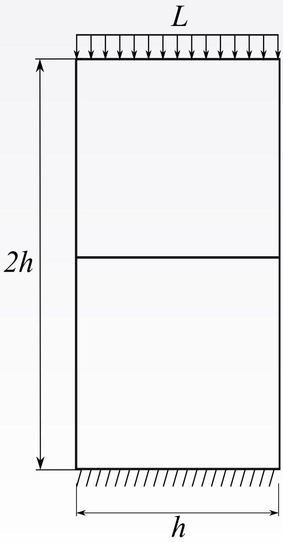
Actual iterative procedure:

- Solve each sub-domains separately
- Stop procedure after convergence criterion satisfied

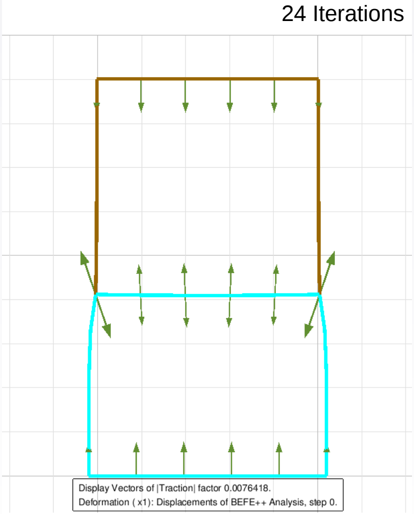
Contents

Parameters

- Two regions: BEM – BEM
- Fixed base
- Constant traction on top
- $E_1 = 500\text{MPa},\ \nu = 0.3$
- $E_2 = 5000\text{MPa},\ \nu = 0.3$
- $\alpha = \beta = 0.7$
- $\epsilon = 1.0e^{-4}$



DEM - BEM results



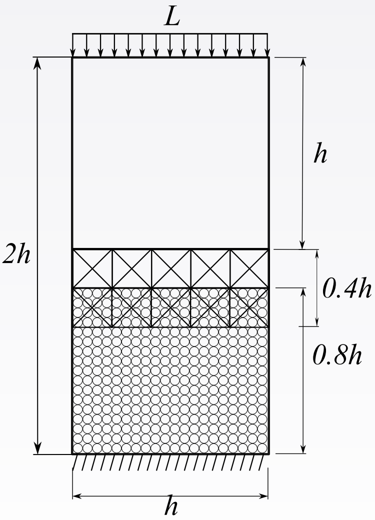
Parameters

- BEM region

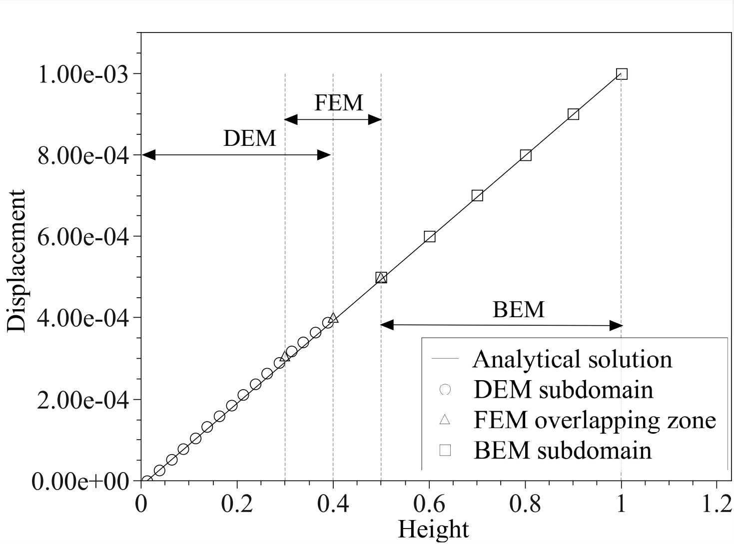
  $$E = 100\,GPa$$
  $$\nu = 0$$
  $$L = 0.05\,GPa$$

- DEM region

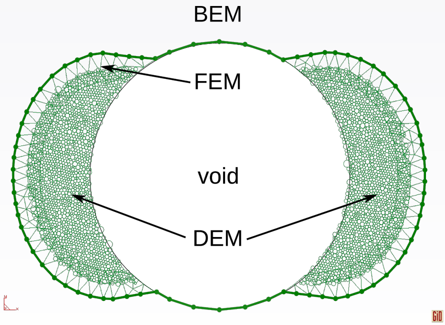
  Uniform mesh
  $$k_n = k_s = 94.6\,\frac{GN}{m}$$



DEM - BEM results



- Simulate a sequential excavation process
- Transfer stresses to DEM subdomain



Thank You!

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