Contributions of different gravity field quantities to the geoid computation

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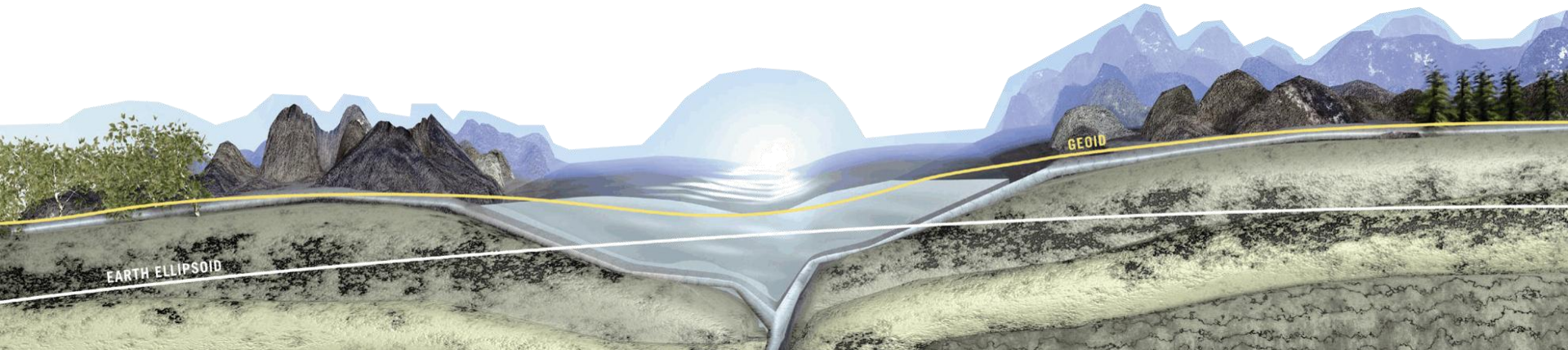
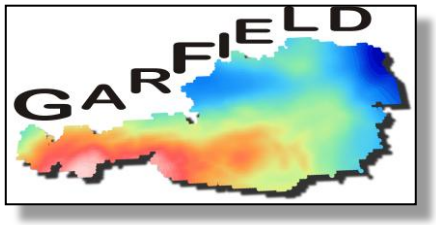


Introduction

- Current Austrian geoid initiative “Geoid for Austria - Regional gravity FIELD improved” (GARFIELD) - P25222-N29



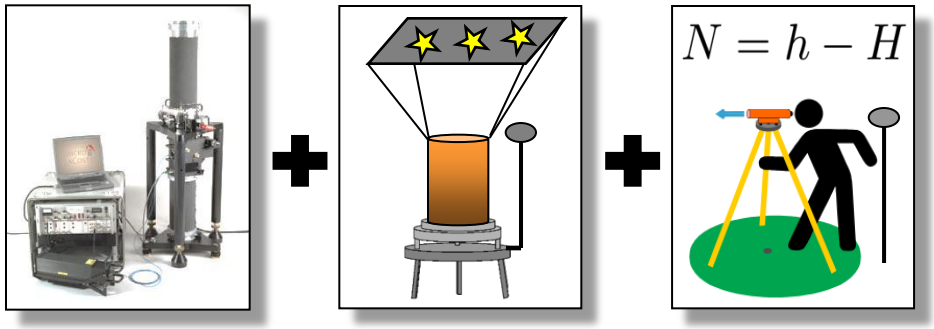
FWF Der Wissenschaftsfonds.

BEV Bundesamt für Eich- und Vermessungswesen

- Combination of global gravity field models with terrestrial gravity field observations
- Questions:
  - ***Which gravity field data is used?***
  - ***How is the data combined?***

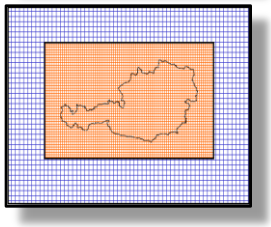
Computation Parameters

- **Remove-Compute-Restore** Technique
- Terrestrial input data
  - **41490** gravity measurements
  - **672** deflections of the vertical
  - **192** GPS/Leveling observations

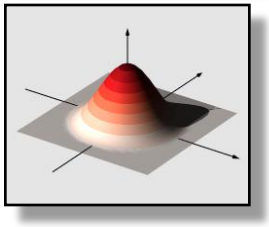


- Global gravity field model
  - **GOCO03s** [Mayer-Gürr T., et al. (2012)]

- Topographic reduction: **Prism formula**
  - Coarse & Dense digital terrain models
  - Standard crustal density of 2.670 kg/m^3



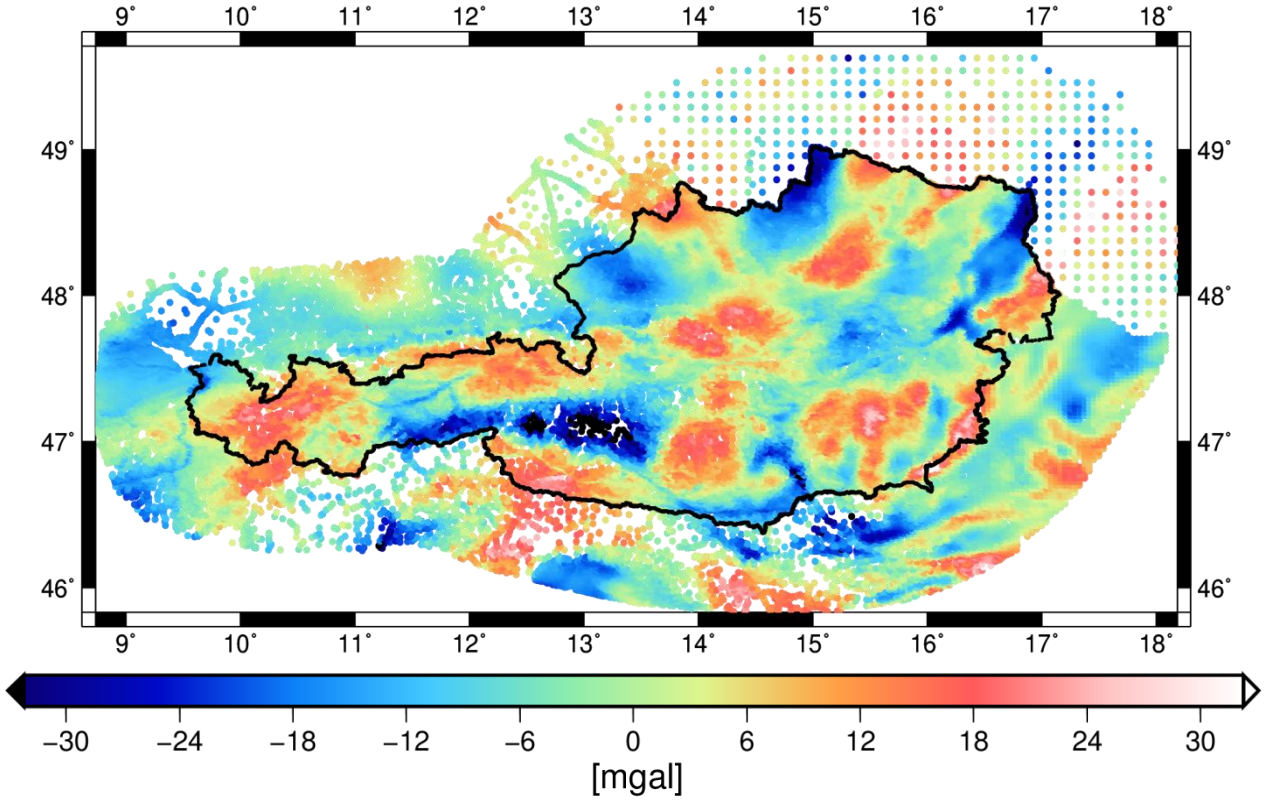
- Computation: **Least squares approach**
  - Radial Basis Function parametrization



Realization - Consistent Reduction (1)

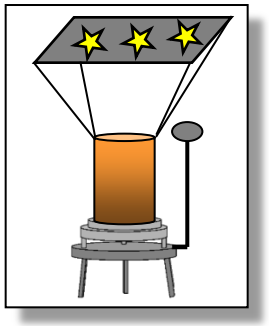
- Measured gravity reduced by:
  - Global gravity field model
  - Topographic effects

rms = 10.7 mgal

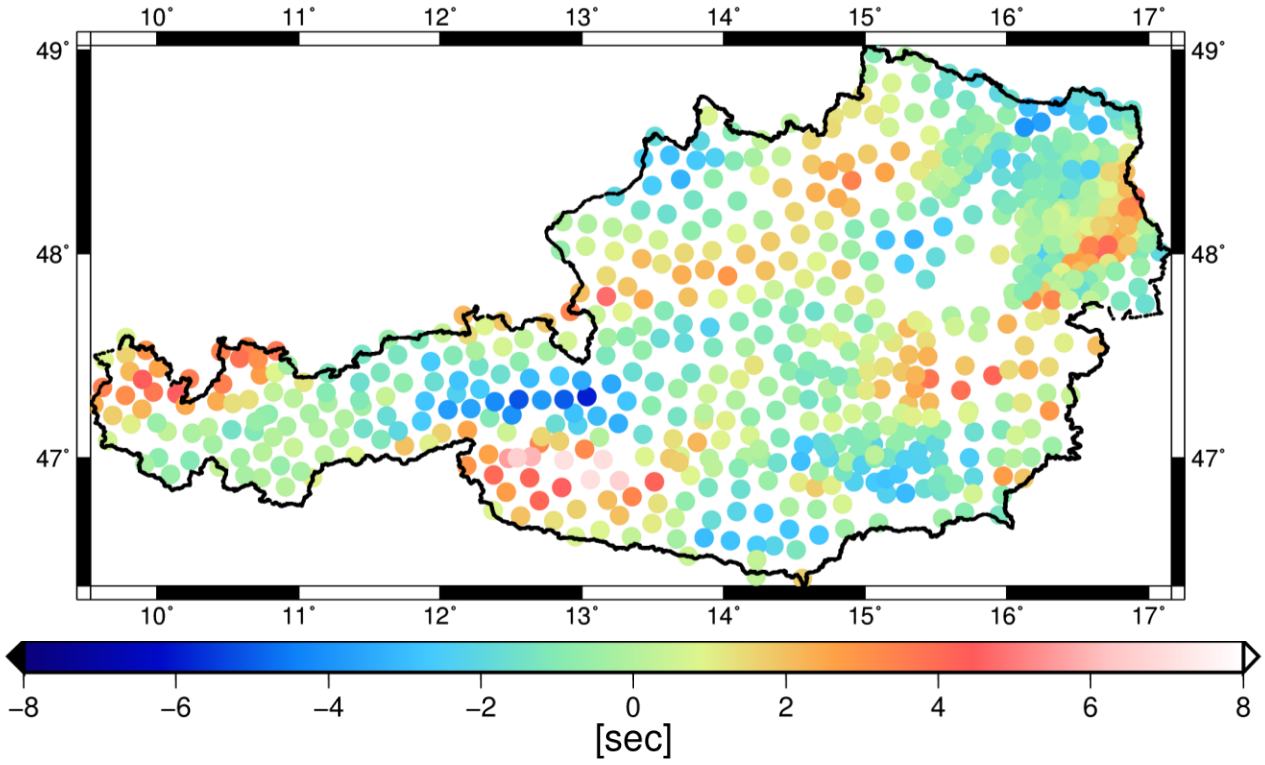


Realization - Consistent Reduction (2)

- Measured deflection of the vertical ξ (North-South component) reduced by:
  - Global gravity field model
  - Topographic effects



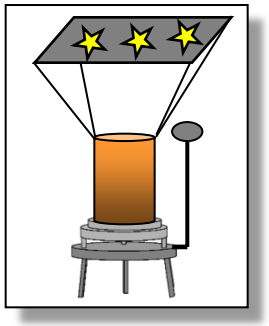
rms = 1.8 sec

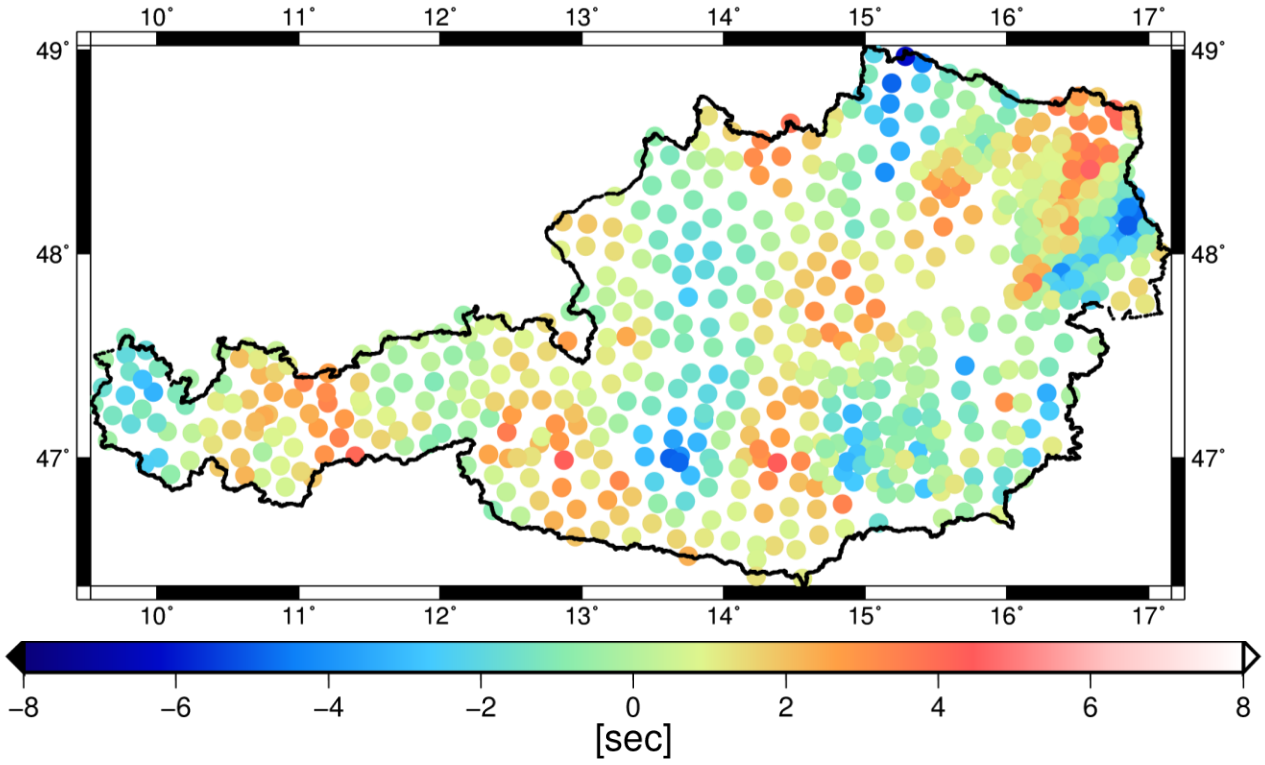


Realization - Consistent Reduction (3)

- Measured deflection of the vertical η (East-West component) reduced by:
  - Global gravity field model
  - Topographic effects

rms = 1.7 sec





Weighting (1)

- **Previous** Austrian geoid computation
  - Assumption about accuracy

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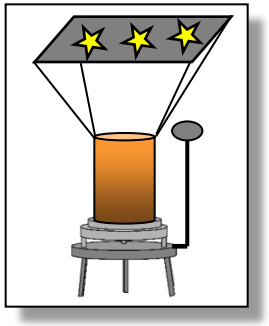
$\sigma_{AUT_{EMP}}$ [mgal]	$\sigma_{Neigh_{EMP}}$ [mgal]
1.00	1.00



=

$\sigma_{\xi,\eta_{EMP}}$ [“]
0.30

Question:

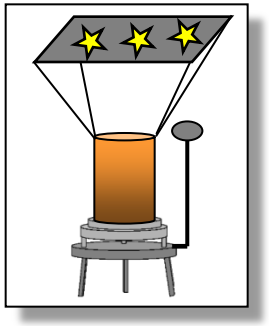
- ***Can these empirically determined a-priori accuracies be confirmed using VCE?***

Weighting (2)

- **Previous** Austrian geoid computation
  - Assumption about accuracy

=

$\sigma_{AUT_{EMP}}$ [mgal]	$\sigma_{Neigh_{EMP}}$ [mgal]
1.00	1.00



=

$\sigma_{\xi,\eta_{EMP}}$ [“]
0.30

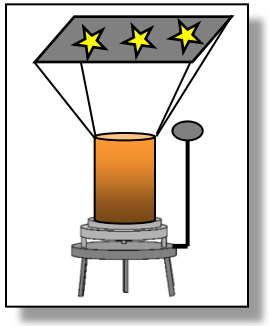
Question:

- ***Can these empirically determined a-priori accuracies be confirmed using VCE?***

- **New** solution
  - Using Variance Component Estimation

=

σ_{AUT} [mgal]	σ_{Neigh} [mgal]
0.95	0.88



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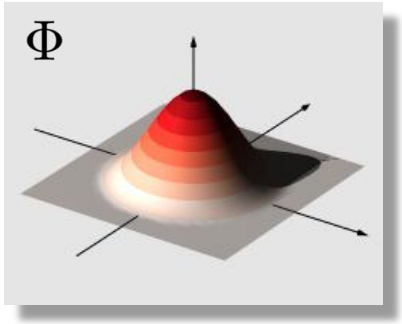
$\sigma_{\xi,\eta}$ [“]
0.52

Answer:

- ***Yes*** *for gravity data*
- ***No*** *for deflections of the vertical*

Radial Basis Functions (RBF)

- Used approach is based on [Eicker A. (2008)]



Gravity field signal:

$$s(x) = \sum_{i=1}^{N} a_i \Phi(x, x_i)$$

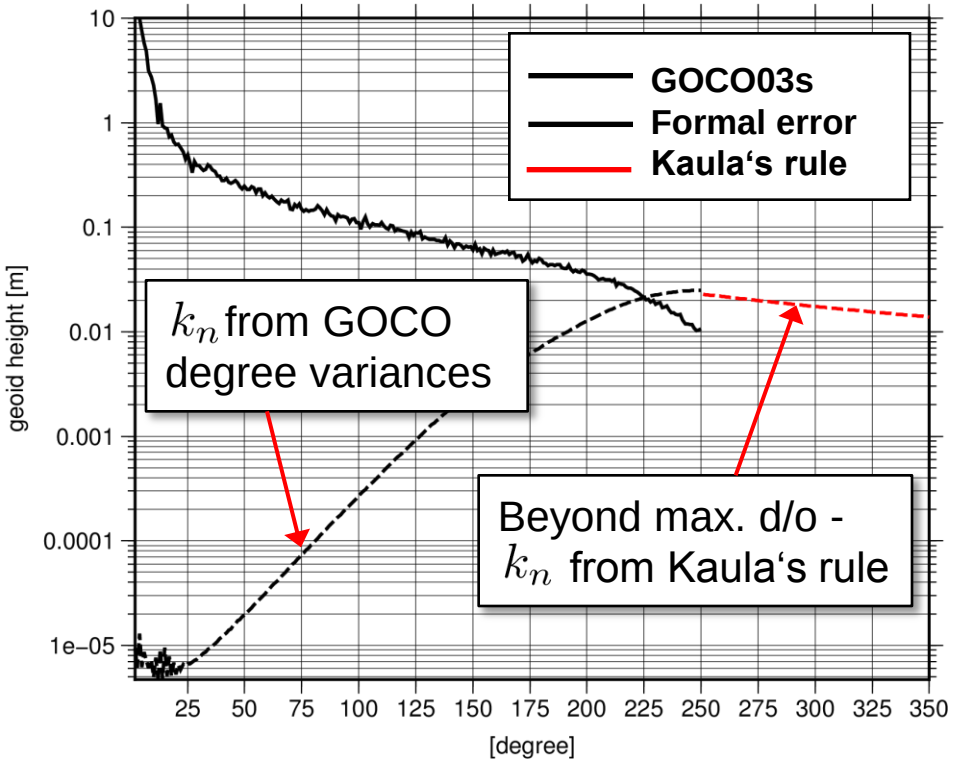
Unknown parameters a_i

RBF:

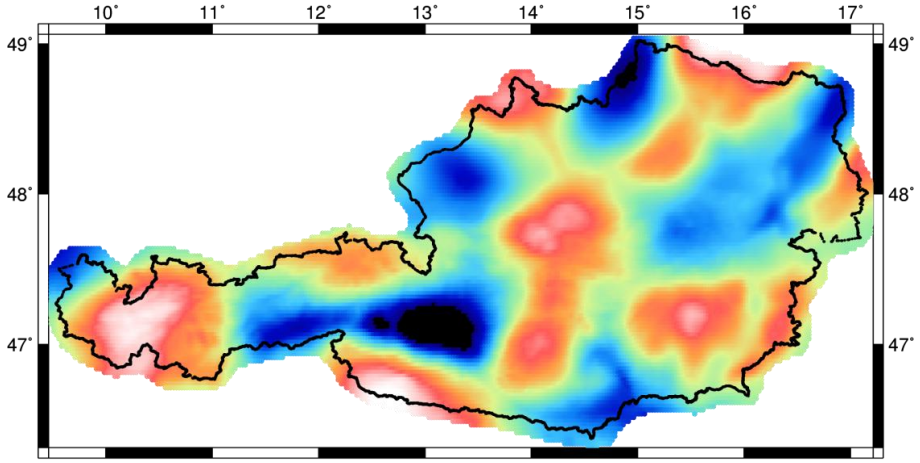
$$\Phi(\mathbf{x}, \mathbf{x}_i) = \sum_{n=2}^{\infty} \sum_{m=-n}^{n} k_n Y_{nm}(\mathbf{x}) Y_{nm}(\mathbf{x}_i)$$

Shape coefficients

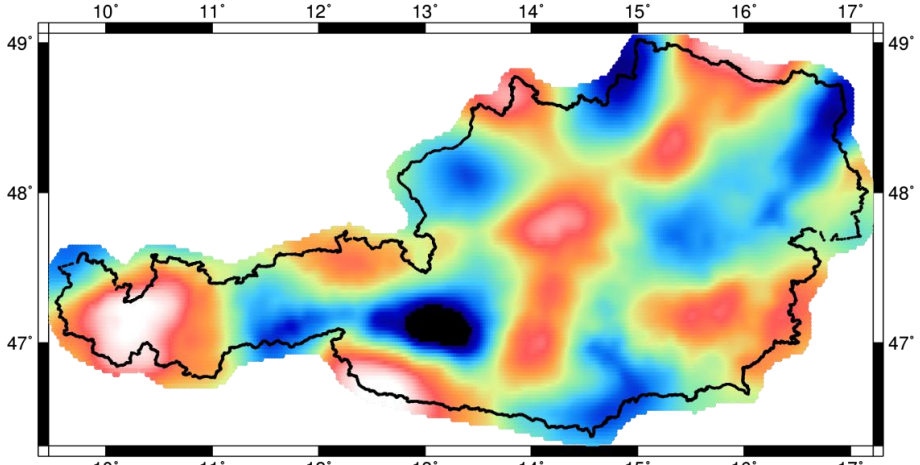
Spherical harmonics Y_{nm}
Grid points i



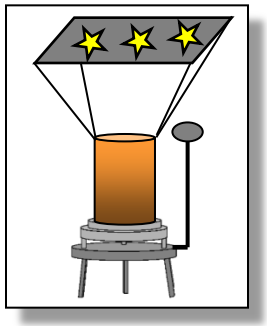
Computed Geoid N_{RBF} relative to GOCO03s

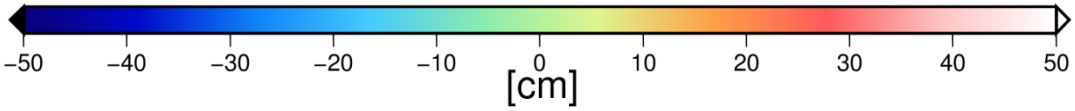


Gravimetric geoid
- 41491 gravity anomalies



Astrogeodetic geoid
- 672 deflections of the vertical



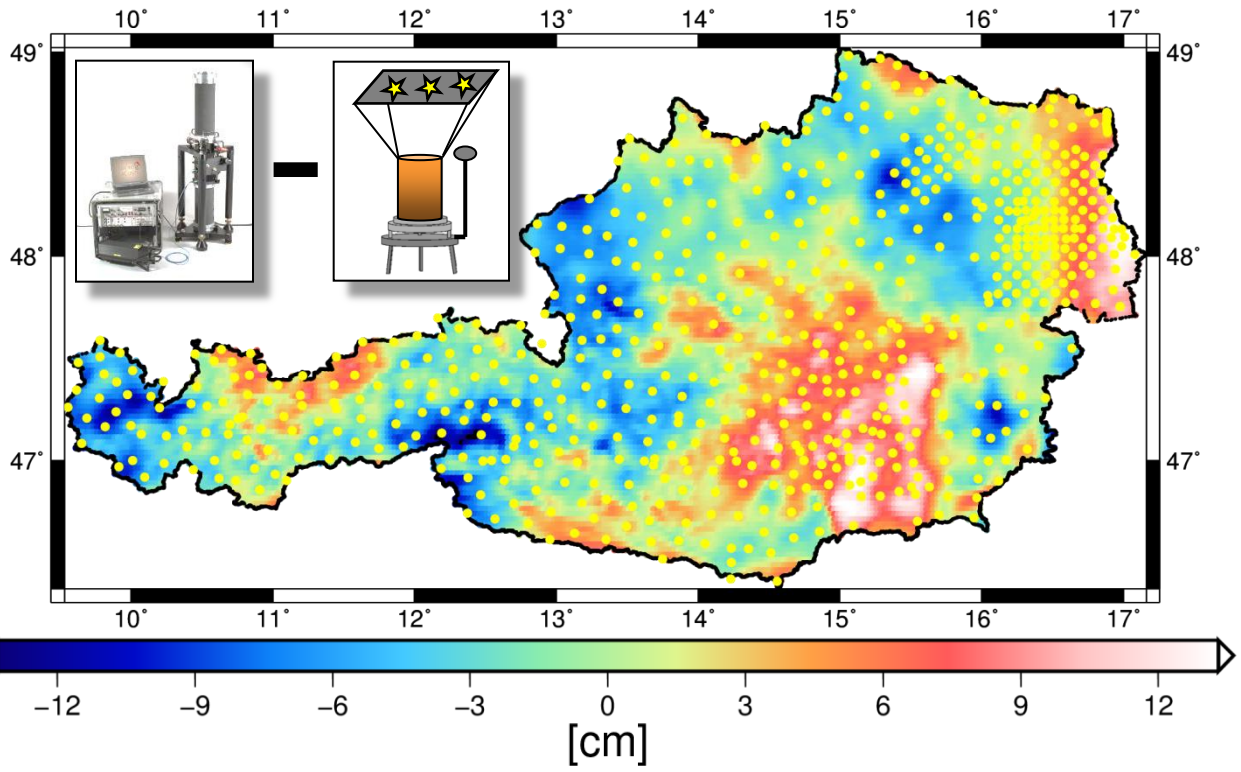


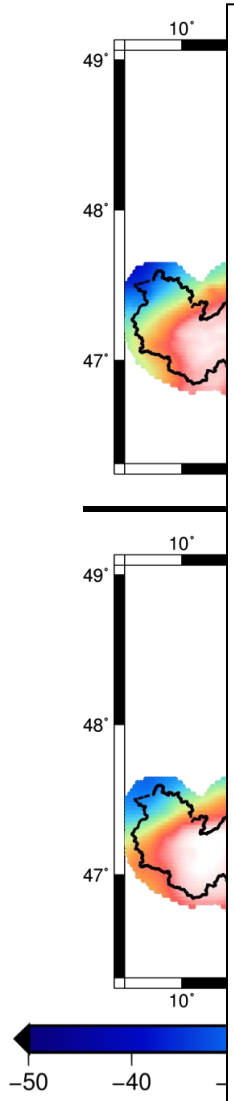
Only 3.2% of input data as compared to gravity anomalies!

Computed Geoid N_{RBF} relative to GOCO03s

- Differences between gravimetric and astrogeodetic geoid

[cm]	min	max	rms
	-14.0	15.6	4.8





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Contribution to a Combined Solution (1)

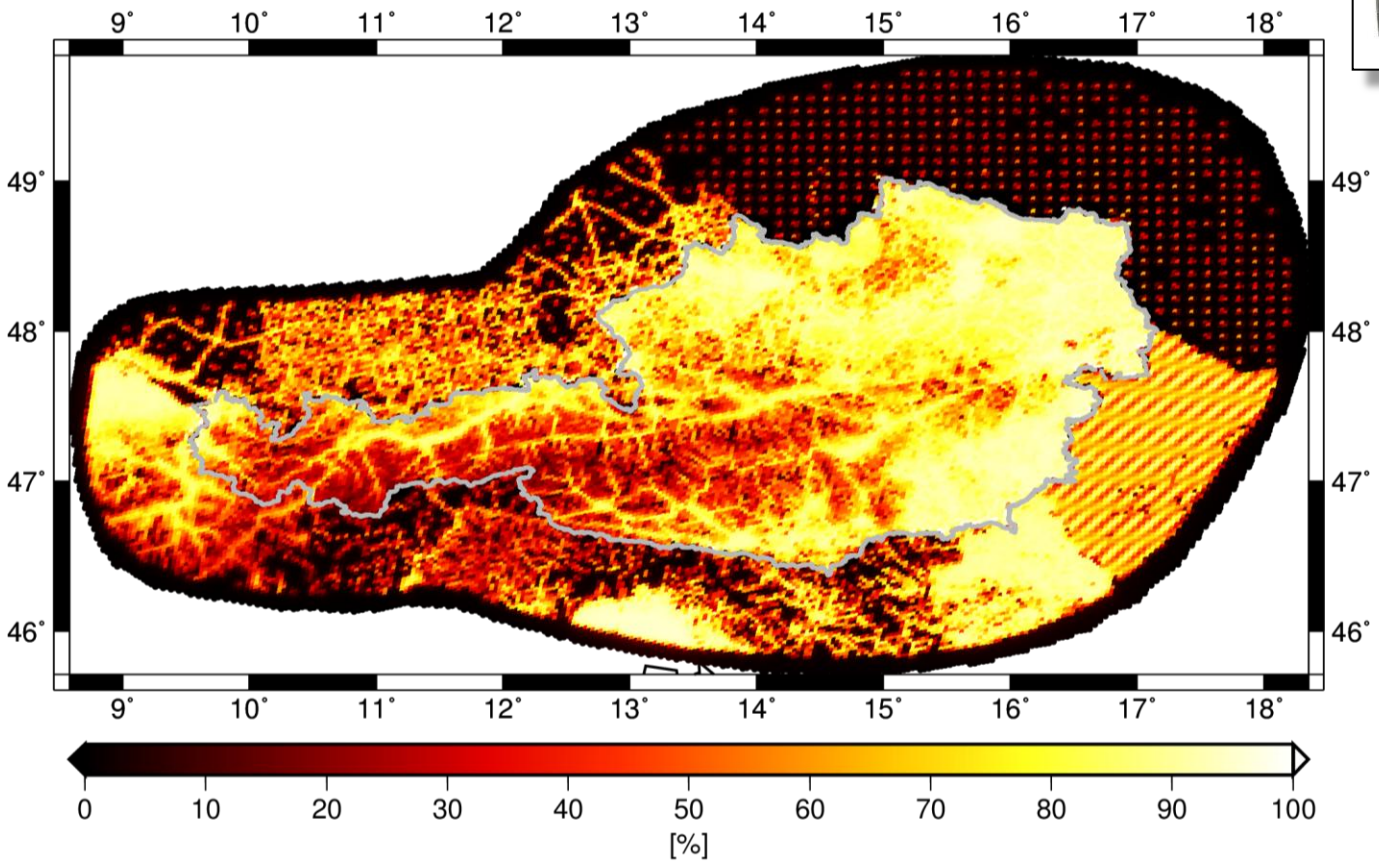
- Pure gravimetric and astrogeodetic geoids have been computed
- Variance component estimation provided a proper weighting between different observation groups
- Still to do
  - Compute combined geoid solution

Question:

- ***What is the contribution of each individual gravity field quantity to a combined geoid solution?***

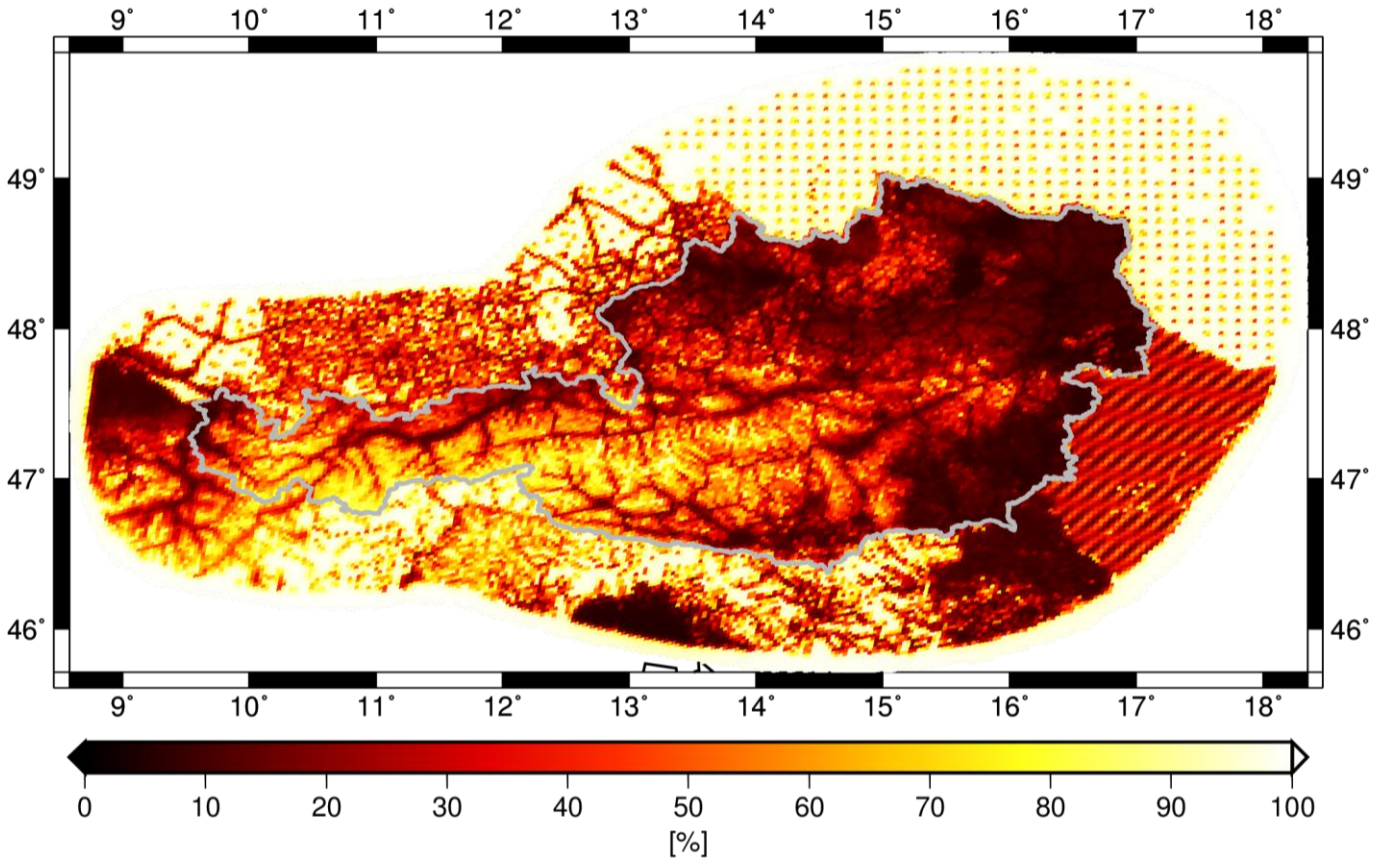
Contribution to a Combined Solution (2)

- Contribution by normal equations
  - **41490** gravity anomalies → 37891 RBF parameter
  - $[\mathbf{N}_{\triangle g}\ \mathbf{N}_{total}^{-1}]_{ii}$ with $\mathbf{N}_{total} = \mathbf{N}_{\triangle g} + \mathbf{N}_{\xi,\eta} + \mathbf{N}_{GOCO}$



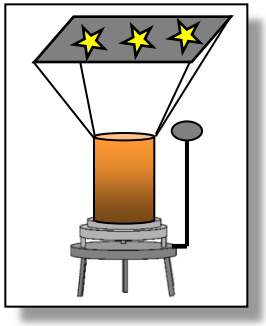
Contribution to a Combined Solution (3)

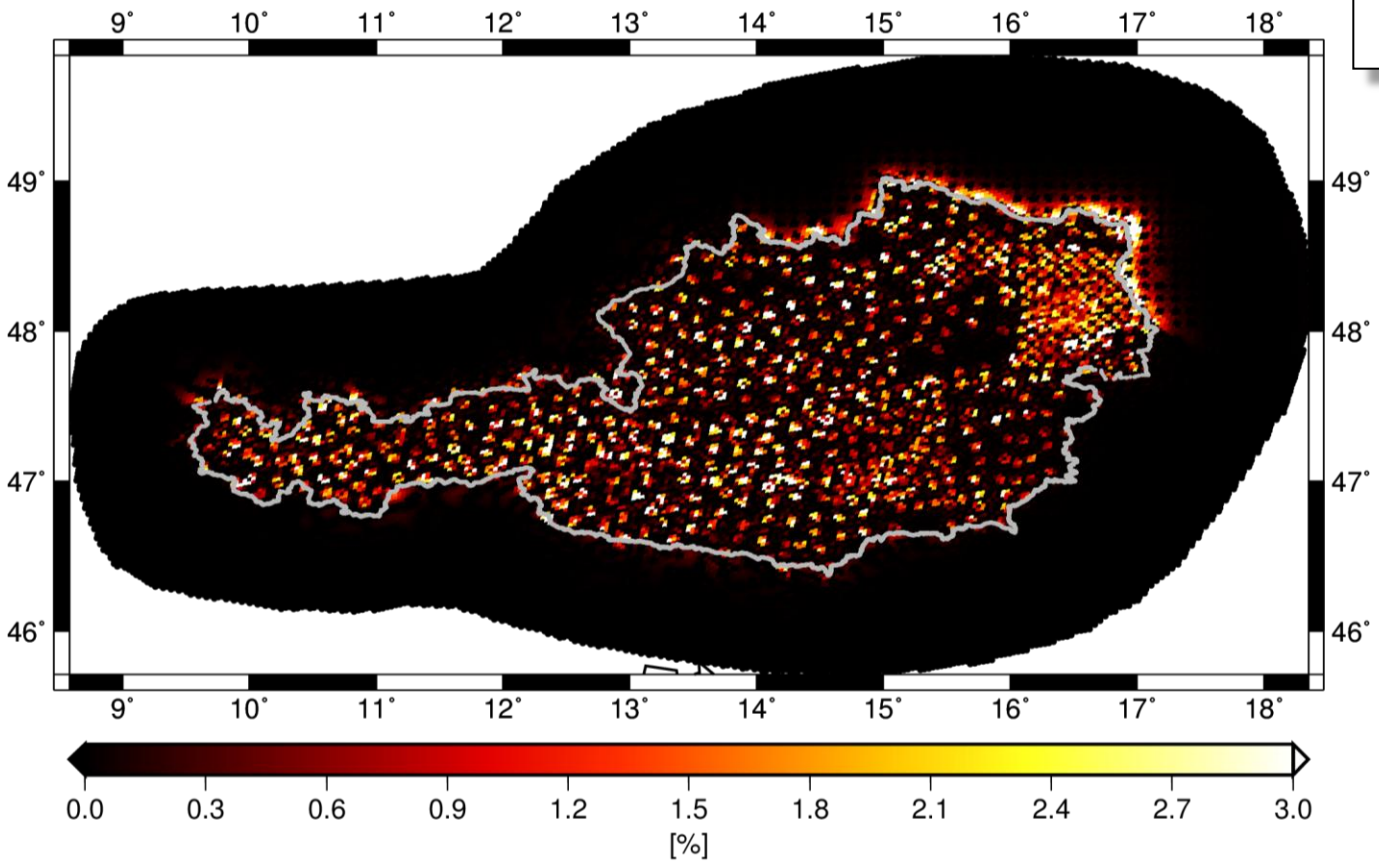
- Contribution by normal equations
  - **Regularization** based on GOCO model → 37891 RBF parameter
  - $[\mathbf{N}_{GOCO}\ \mathbf{N}_{total}^{-1}]_{ii}$ with $\mathbf{N}_{total} = \mathbf{N}_{\triangle g} + \mathbf{N}_{\xi,\eta} + \mathbf{N}_{GOCO}$



Contribution to a Combined Solution (4)

- Contribution by normal equations
  - **672** deflections of the vertical → 37891 RBF parameter
  - $[\mathbf{N}_{\xi,\eta}\ \mathbf{N}_{total}^{-1}]_{ii}$ with $\mathbf{N}_{total} = \mathbf{N}_{\triangle g} + \mathbf{N}_{\xi,\eta} + \mathbf{N}_{GOCO}$



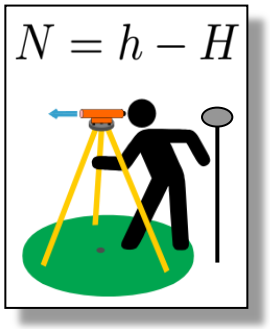


Validation (1)

- Full Restore step
- Different solutions
  - **Astrogeodetic** geoid based on 672 deflections of the vertical
  - **Gravimetric** geoid based on 41490 gravity anomalies
  - **Combined solution** (Astrogravimetric)

Questions:

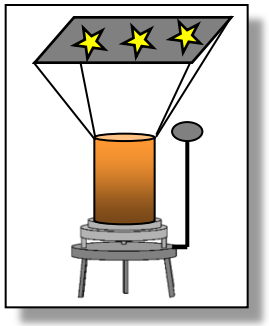
- ***Which is the best solution compared to GPS/Leveling?***
- ***Is there a significant impact on the combined solution caused by deflections of the vertical?***

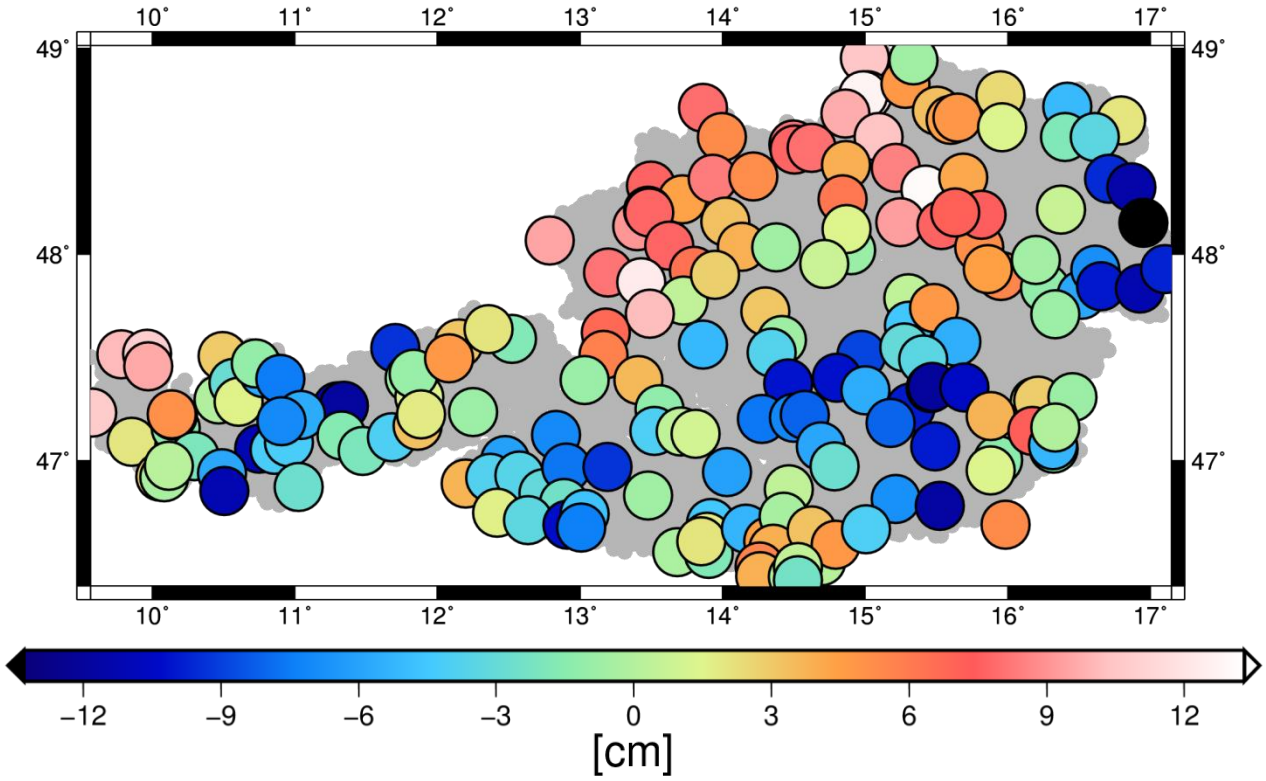


Validation (2)

- Estimated geoid heights based on RBF parametrization - full Restore step
  - 192 GPS/Leveling points compared to the **astrogeodetic geoid**

[cm]	min	max	rms
	-14.0	13.0	6.2

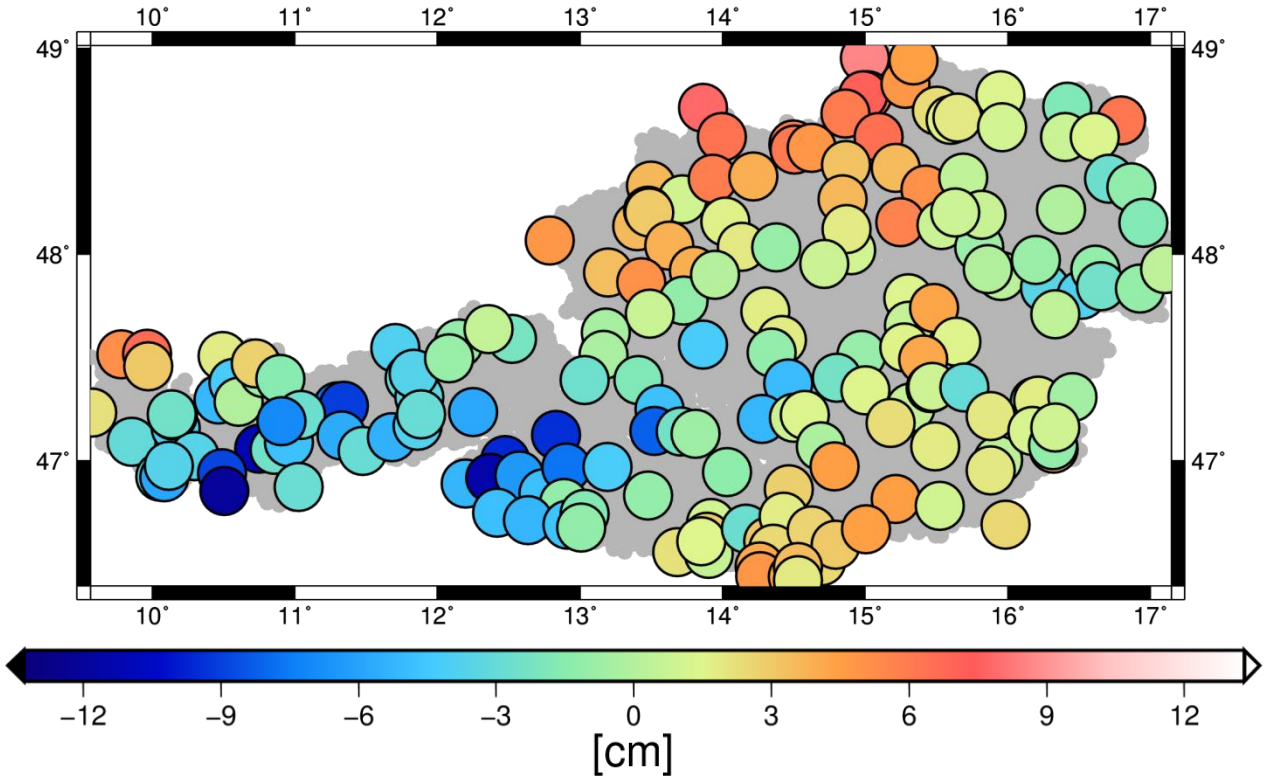




Validation (3)

- Estimated geoid heights based on RBF parametrization - full Restore step
  - 192 GPS/Leveling points compared to the **gravimetric geoid**

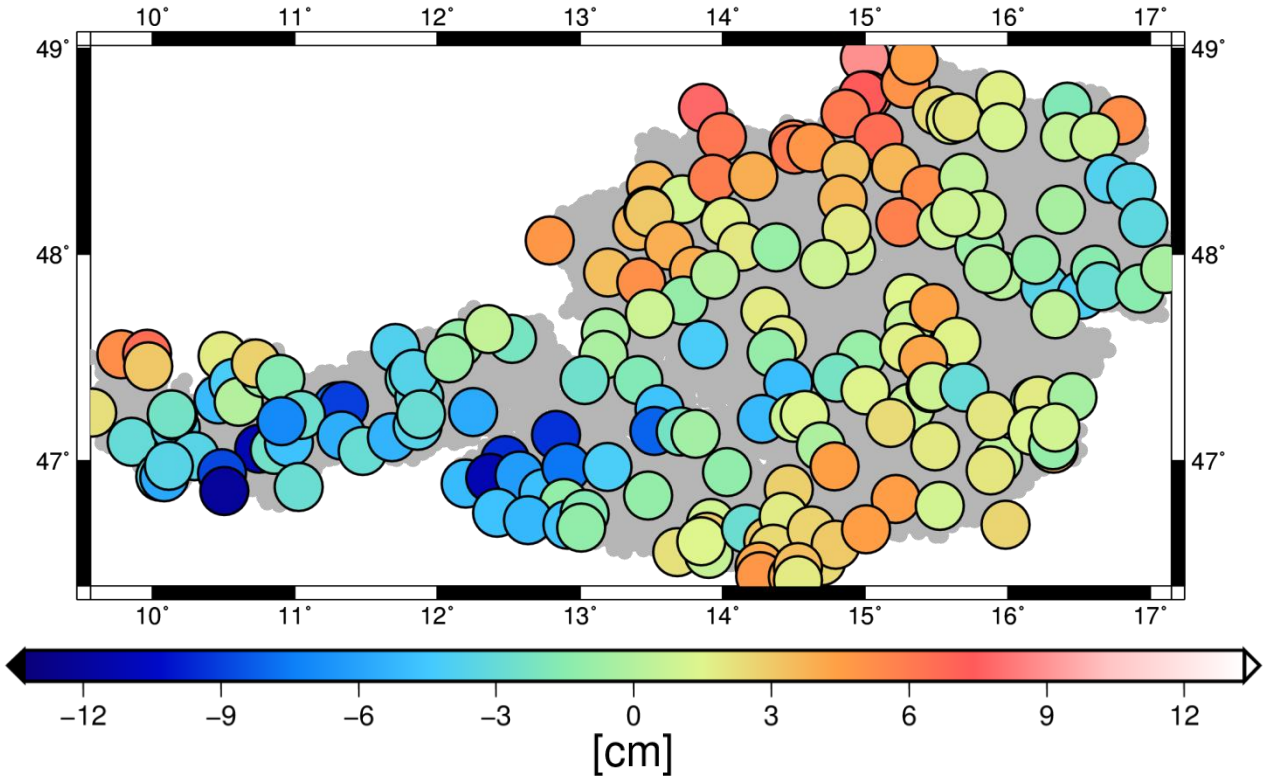
[cm]	min	max	rms
	-12.0	8.8	3.9

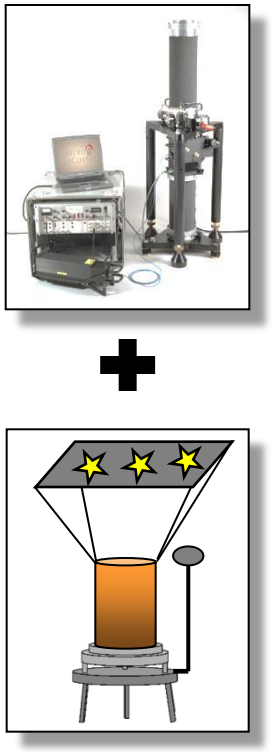


Validation (4)

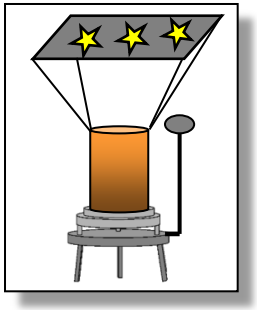
- Estimated geoid heights based on RBF parametrization - full Restore step
  - 192 GPS/Leveling points compared to the **combined solution**

[cm]	min	max	rms
	-12.1	8.8	3.9

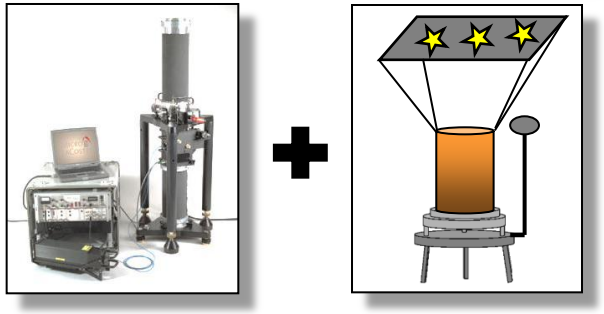




Summary



- **Astrogeodetic geoid**
  - Sparse observations provide a reasonable geoid
  - Solution is not competitive to the gravimetric geoid
- **Gravimetric geoid**
  - Huge number of gravity observations available
  - Results make us confident for further computation
- **Combined solution**
  - Solution is dominated by gravity observations
  - Number of deflections is not sufficient to provide significant contributions to a combined solution



Further investigations

- 1:1 ratio of input observations: *Deflections of the vertical perform better*
- 6x more gravity observations are needed to provide a solution of equal quality

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