Do we need a new definition of gravity anomalies?

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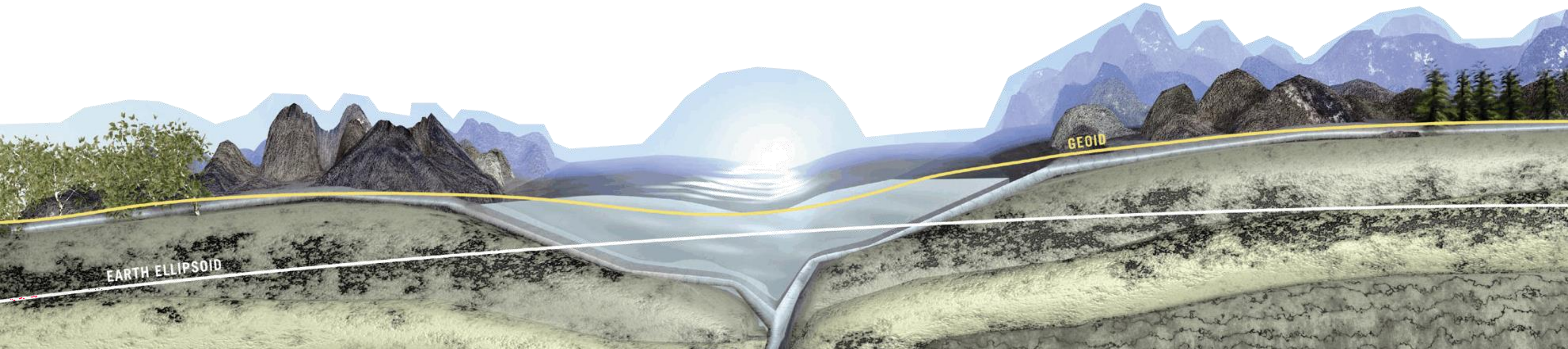
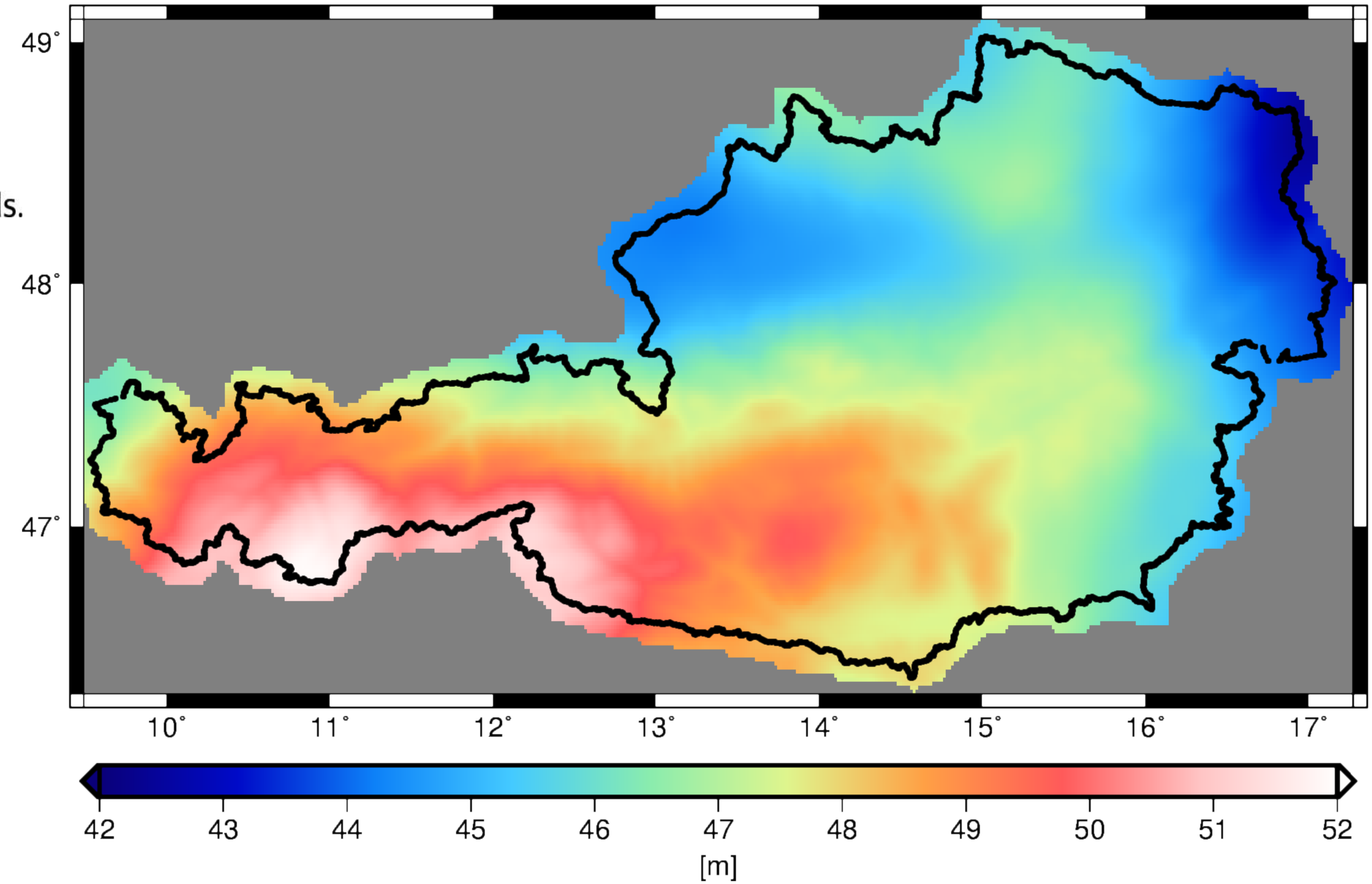


Goal: Geoid determination in Austria with cm accuracy

GARFIELD

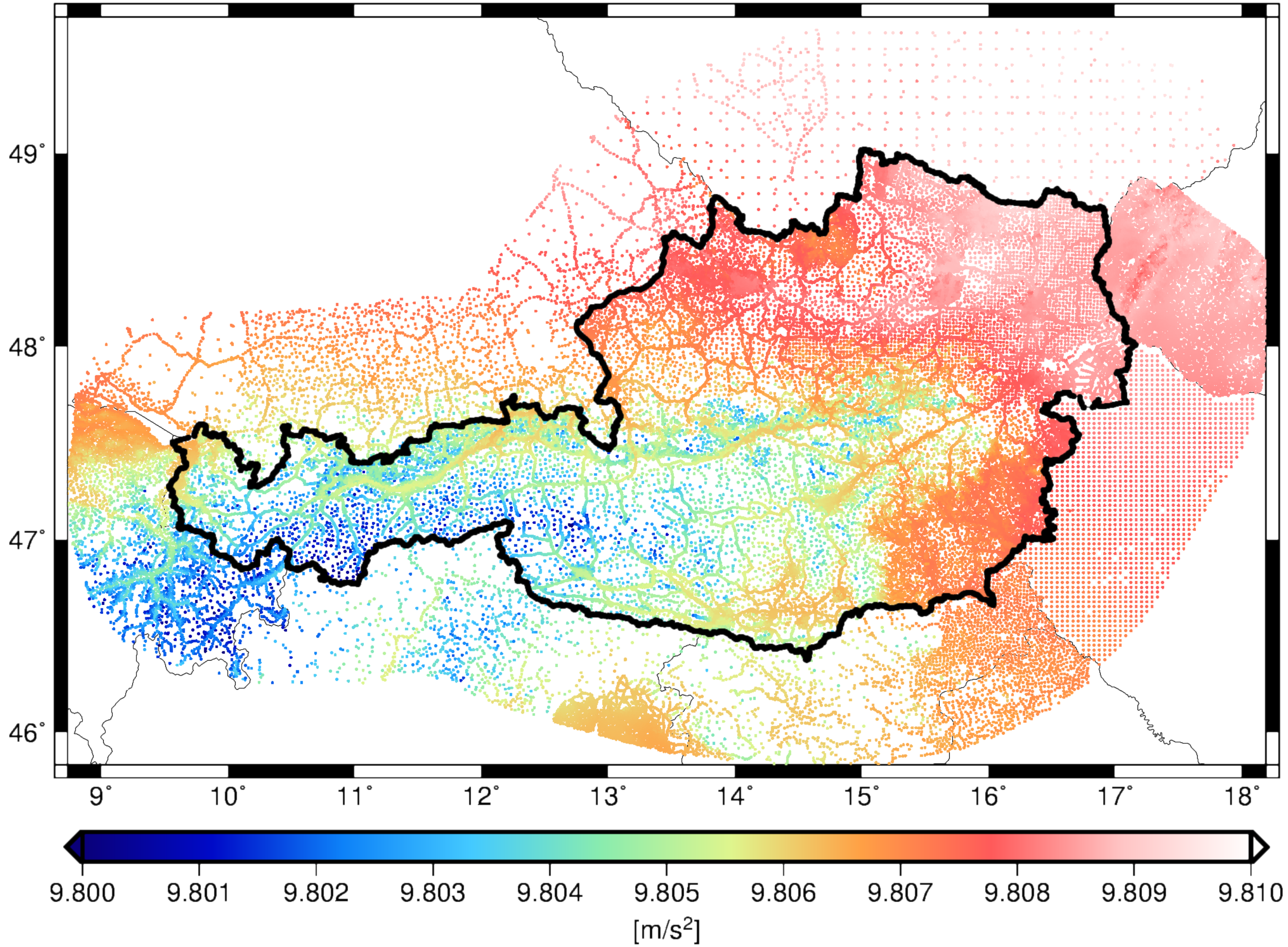
Der Wissenschaftsfonds.



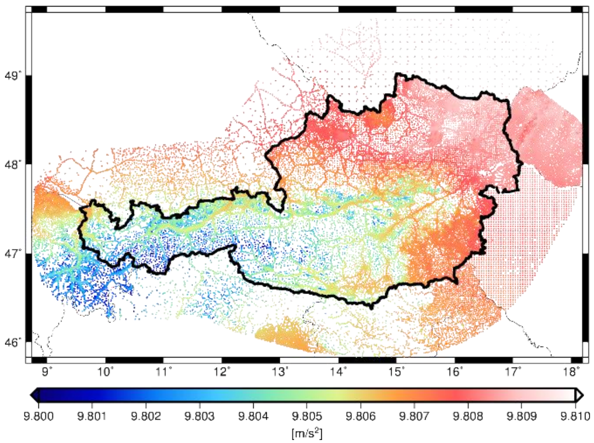
Input data: Gravimetric observations

72.723 observations in and around Austria

Accuracy for most data: << 0.1 mGal



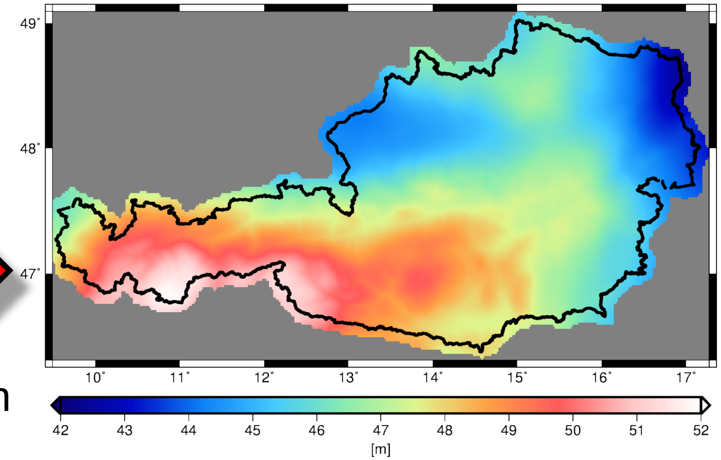
Fundamental equation of physical geodesy



Connection

$$\Delta g = -\frac{\partial T}{\partial r} - 2\frac{T}{r}$$

Linearized with
spherical approximation



Is the **fundamental equation of physical geodesy** in linearized form (with spherical approximation) **accurate enough** to model the observations?

Helmut Moritz (1980), Advanced physical geodesy:
“This spherical approximation causes an error which is negligible in most practical applications.”

“This error is [on a global average] on the order of $\pm$20 cm in the geoidal height. This is one order of magnitude smaller than the accuracy implied by the present gravimetric and satellite data.”

Observation equations

Linearization of non-linear observation equations

$$f(x) = f(x_0) + \left.\frac{\partial f}{\partial x}\right|_{x_0} (x - x_0) + \cdots$$

Unknown parameter	$x = W(L,B,h)$	Gravity potential
Taylor point	$x_0 = U(L,B,h)$	Normal potential with $N_0 = 0$
Difference	$x - x_0 = T(L,B,h)$	Disturbance potential
Observed	$f(x) = g(L,B,h)$	Observed absolute gravity
Computed	$f(x_0) = \gamma(L,B,h)$	Normal gravity

ellipsoidal height =
orthometric height + geoid height

$$h = H + N(W)$$

Observation equations

Linearization of non-linear observation equations

$$f(x) = f(x_0) + \left.\frac{\partial f}{\partial x}\right|_{x_0}(x - x_0) + \cdots \quad \Rightarrow \quad \Delta g = -\frac{\partial T}{\partial r} - 2\frac{T}{r}$$

Unknown parameter	$x = W(L,B,h)$	Gravity potential
Taylor point	$x_0 = U(L,B,h)$	Normal potential with $N_0 = 0$
Difference	$x - x_0 = T(L,B,h)$	Disturbance potential
Observed	$f(x) = g(L,B,h)$	Observed absolute gravity
Computed	$f(x_0) = \gamma(L,B,h_0)$	Normal gravity
Reduced observations	$f(x) - f(x_0) = g(L,B,H+N) - \gamma(L,B,H+N_0) = \Delta g$	(Free air) gravity anomalies
Linear model	$\left.\frac{\partial f}{\partial x}\right|_{x_0}(x - x_0) = -\frac{\partial T}{\partial r} - 2\frac{T}{r}$	Fundamental equation in physical geodesy (In spherical approximation)

Observation equations

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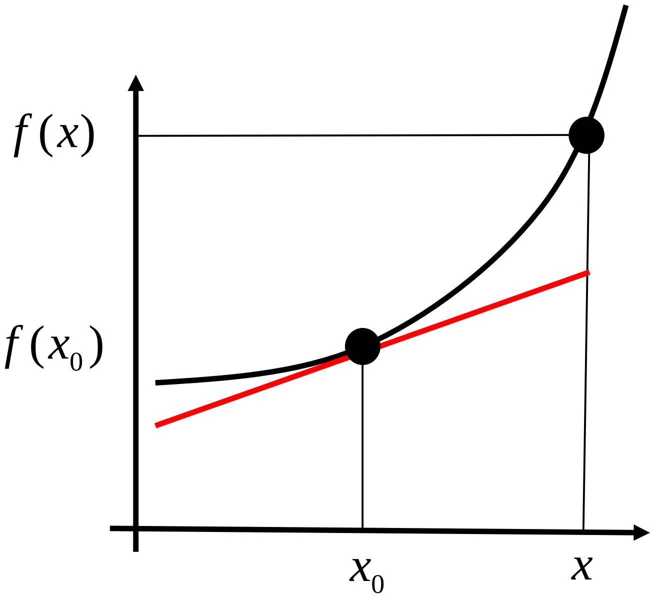

Possible improvements of accuracy:

1. Better linearization
   (without spherical approximation)

Observation equations

Linearization of non-linear observation equations

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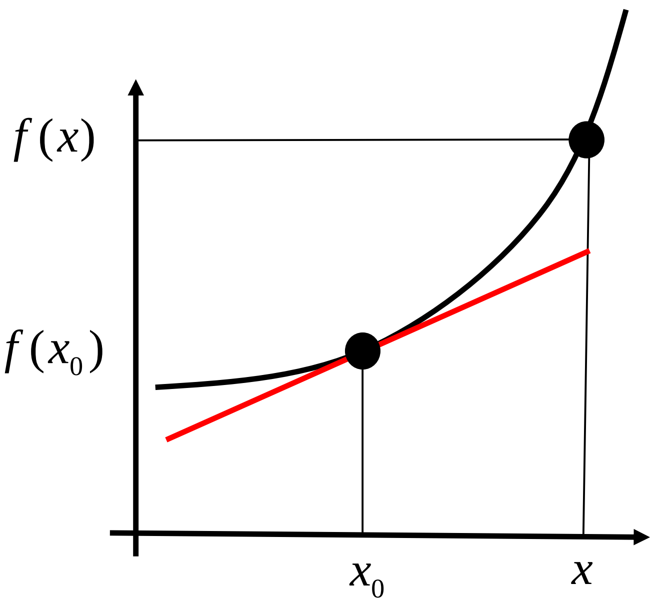

Possible improvements of accuracy:

1. Better linearization
   (without spherical approximation)

2. Inlcude quadratic terms

Observation equations

Linearization of non-linear observation equations

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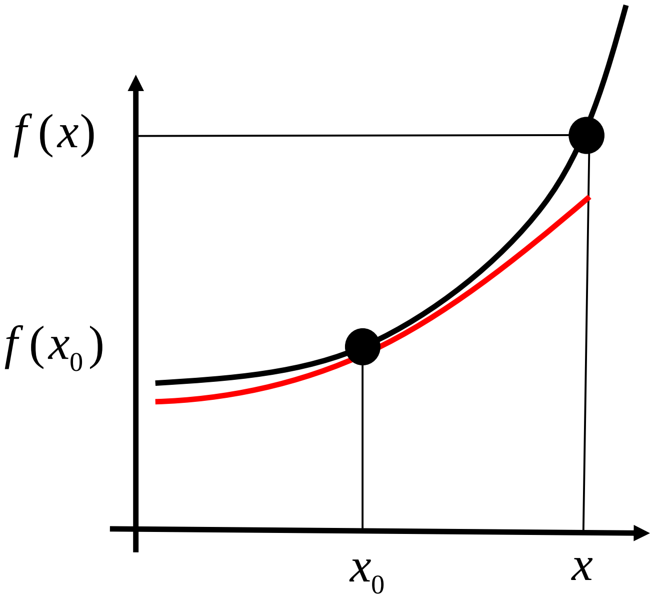

Possible improvements of accuracy:

1. Better linearization
   (without spherical approximation)
2. Include quadratic terms

Observation equations

Linearization of non-linear observation equations

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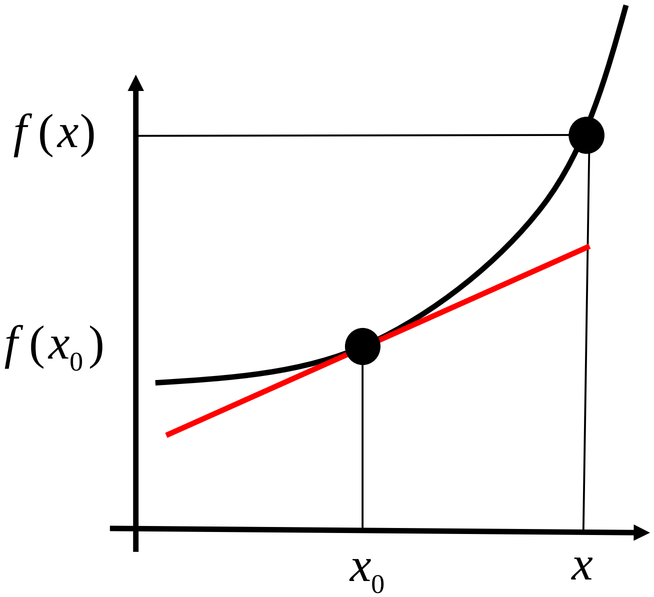

Possible improvements of accuracy:

1. Better linearization
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2. Include quadratic terms
3. Better Taylor point

Observation equations

Linearization of non-linear observation equations

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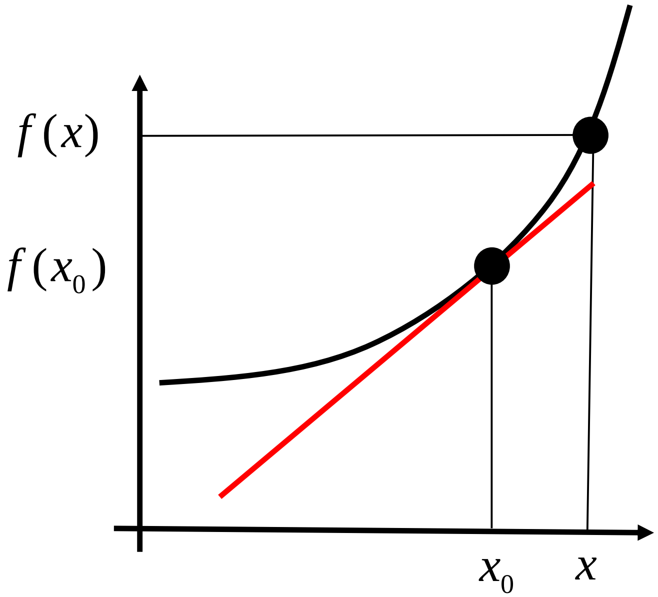


Possible improvements of accuracy:

1. Better linearization
   (without spherical approximation)

2. Include quadratic terms

3. Better Taylor point

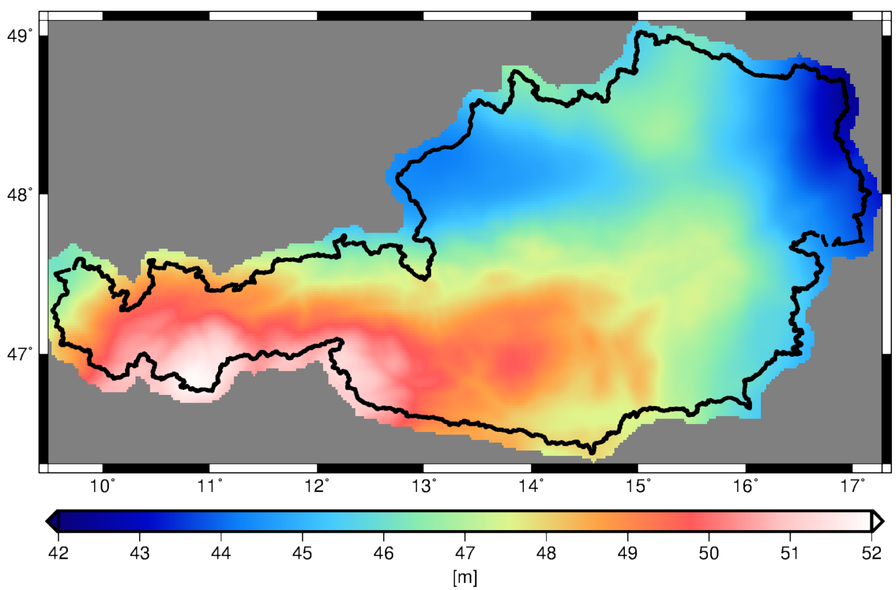
Taylor point

Approximate values (Taylor point)

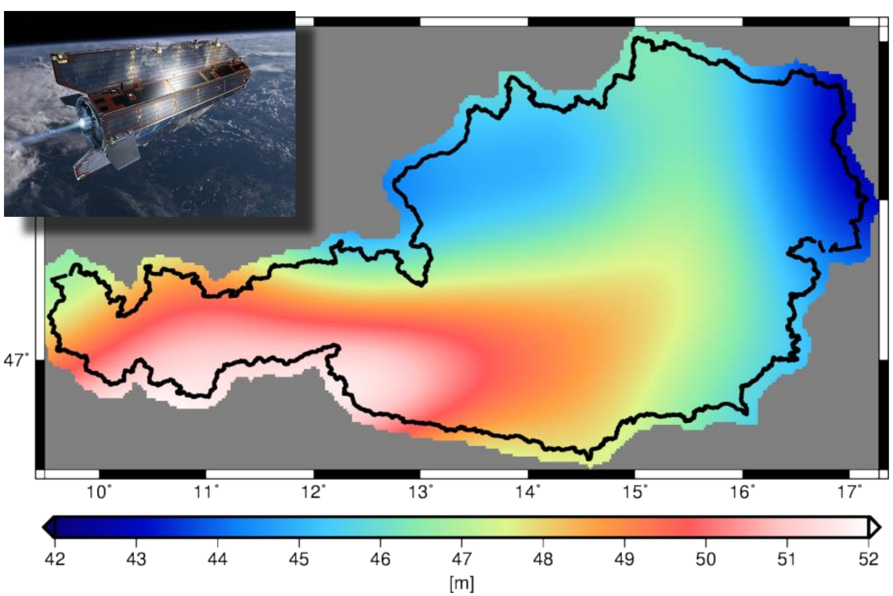
Classic $$f(x_0) = \gamma(L, B, H + N_0) \quad \text{with} \quad N_0 = 0$$

New approach $$f(x_0) = g_{sat}(L, B, H + N_{sat})$$

Geoid



Satellite model GOCO05s



Taylor point

Approximate values (Taylor point)

Classic $$f(x_0) = \gamma(L, B, H + N_0) \quad \text{with} \quad N_0 = 0$$

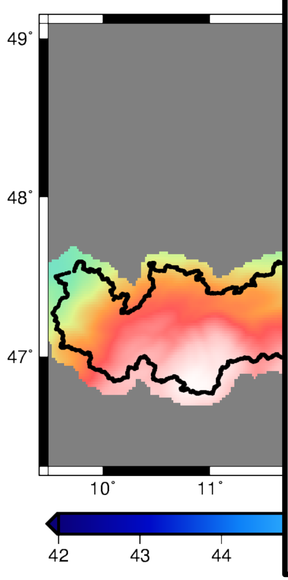
New approach $$f(x_0) = g_{sat}(L, B, H + N_{sat}) = \left\| \mathbf{g}_{sat} \right\| = \left\| \nabla W_{sat} \right\|$$

Gravity potential: $$W(\lambda, \vartheta, r) = V(\lambda, \vartheta, r) + Z(\lambda, \vartheta, r)$$

Gravitational potential: $$V(\lambda, \vartheta, r) = \frac{GM}{R} \sum_{n=0}^{n_{\max}} \left(\frac{R}{r} \right)^{n+1} \sum_{m=0}^{2n+1} c_{nm} C_{nm}(\lambda, \vartheta) + s_{nm} S_{nm}(\lambda, \vartheta)$$

Centrifugal potential: $$Z(\vartheta, r) = \frac{1}{2} \varpi^2 r \sin \vartheta$$

Gravity: $$g = \left\| \nabla W \right\| = \left\| \begin{pmatrix} \partial W / x \\ \partial W / x \\ \partial W / z \end{pmatrix} \right\| = \left\| \begin{pmatrix} \partial W / r \partial \vartheta \\ \partial W / r \sin \vartheta \, \partial \lambda \\ \partial W / \partial r \end{pmatrix} \right\|$$



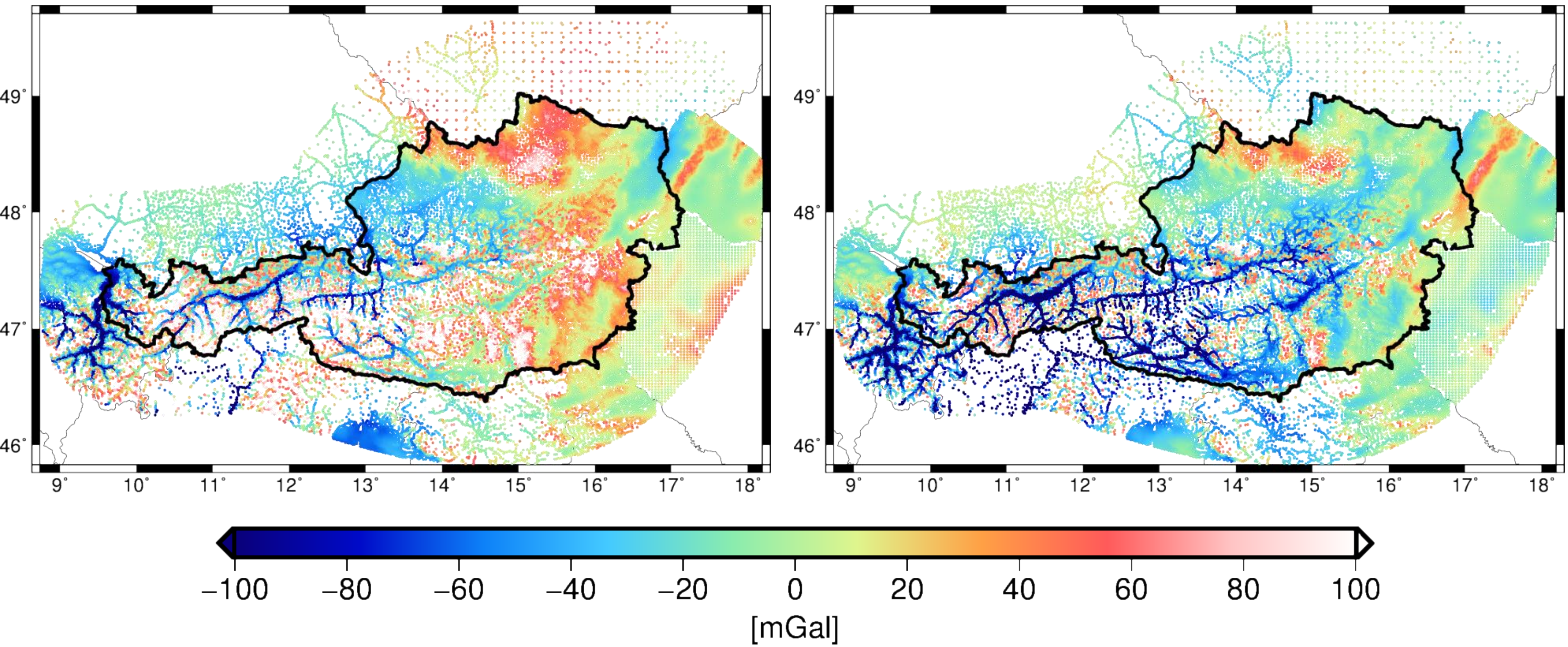
Reduced observations

Classic approach

$$\Delta g = g - \gamma$$

New approach

$$\Delta g = g - \left\| \mathbf{g}_{sat} \right\|$$



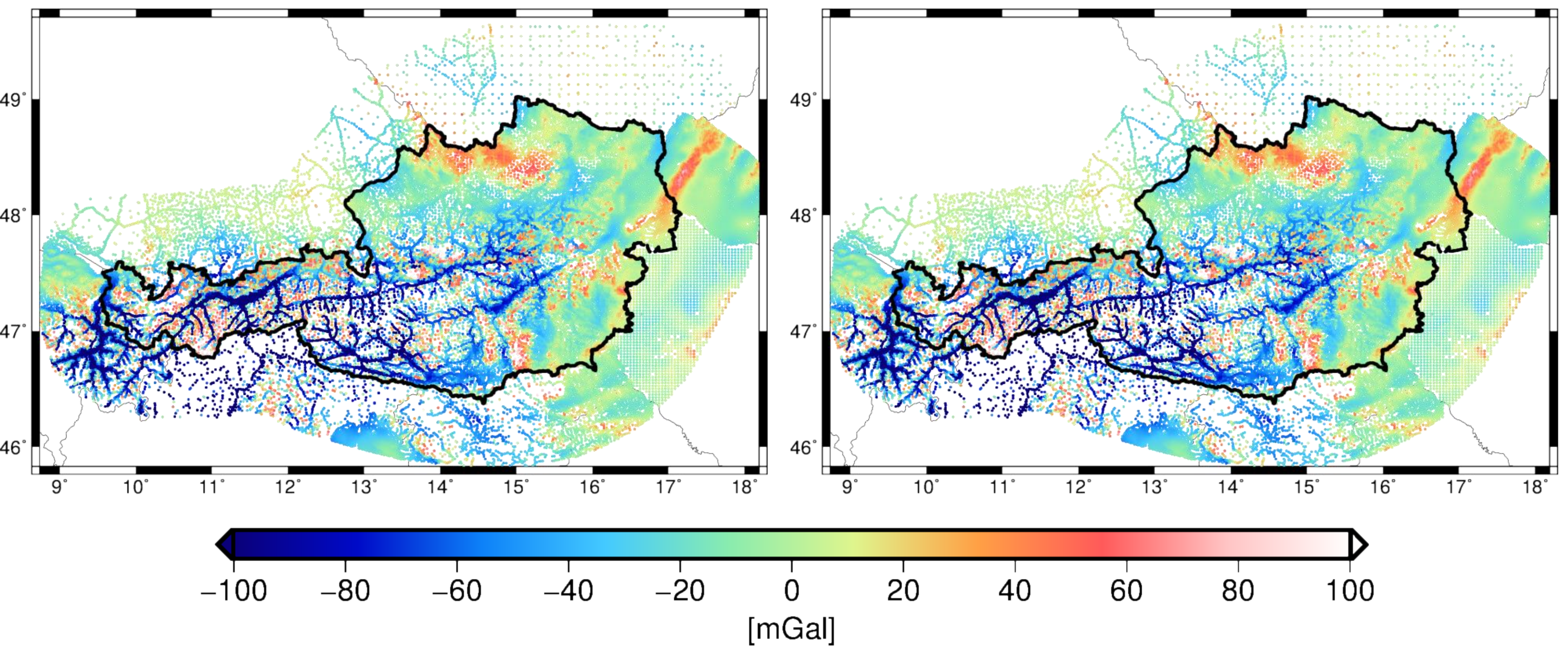
Reduced observations

Classic approach

$$\Delta g = g - \gamma - \Delta g_{sat}$$

New approach

$$\Delta g = g - \left\|\mathbf{g}_{sat}\right\|$$



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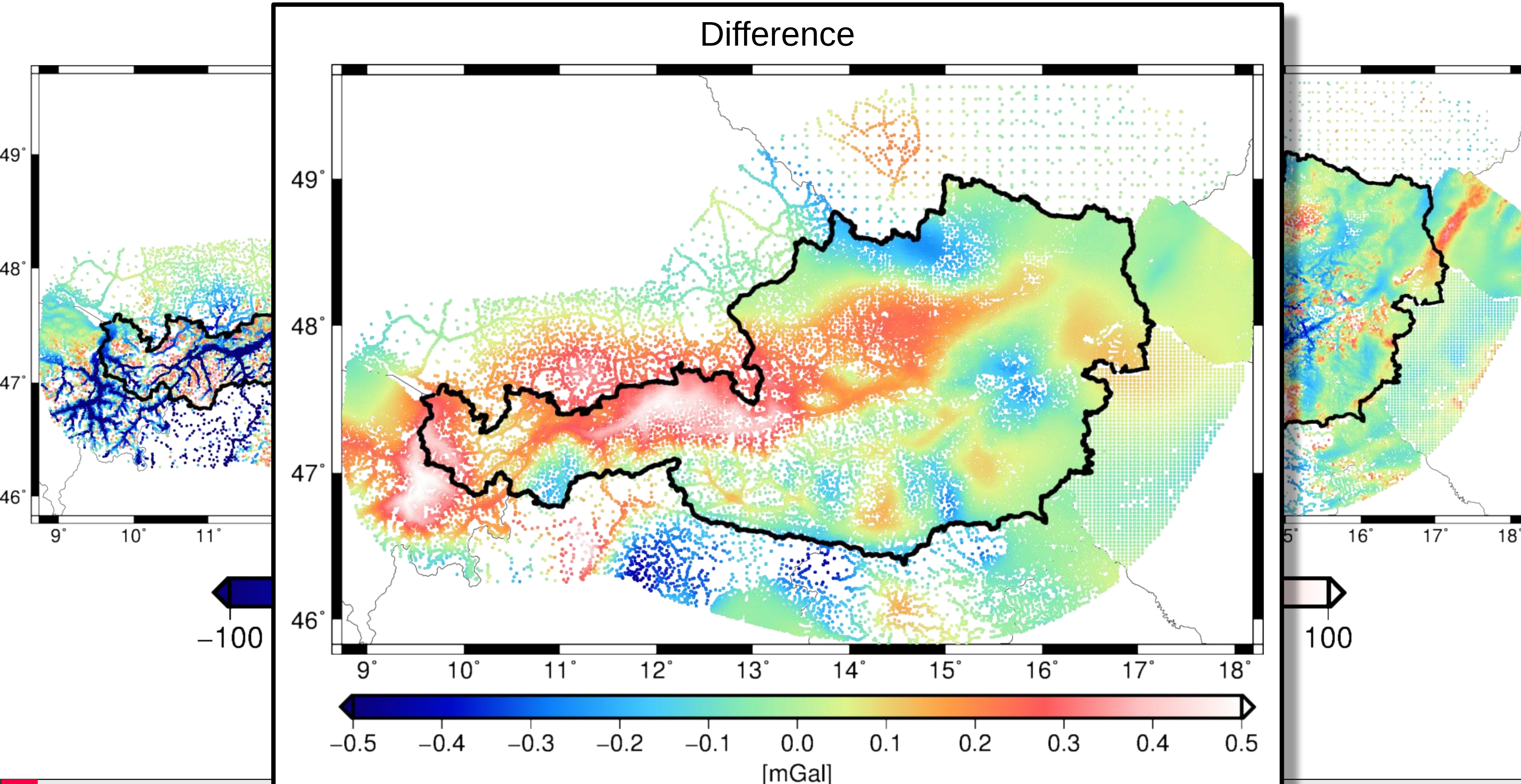

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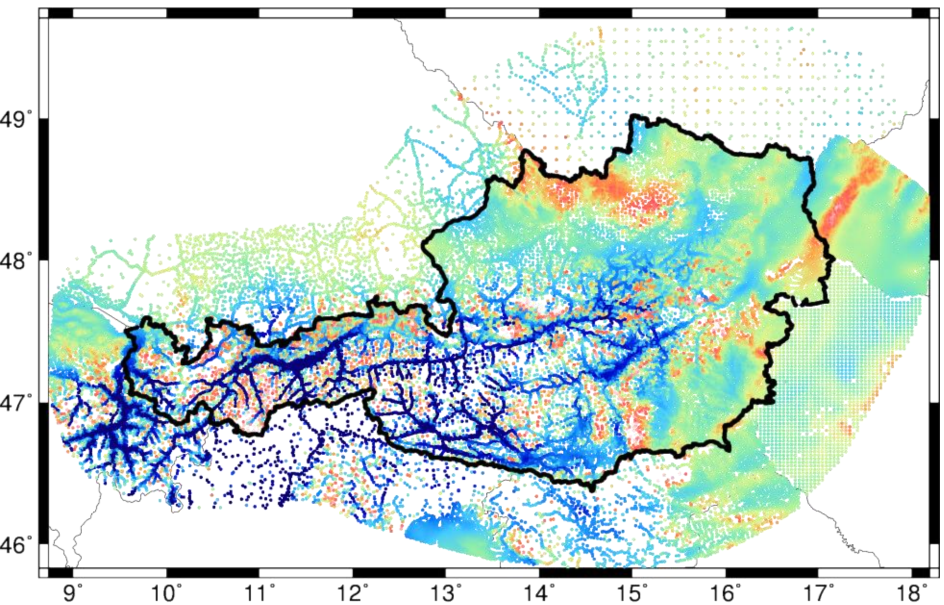
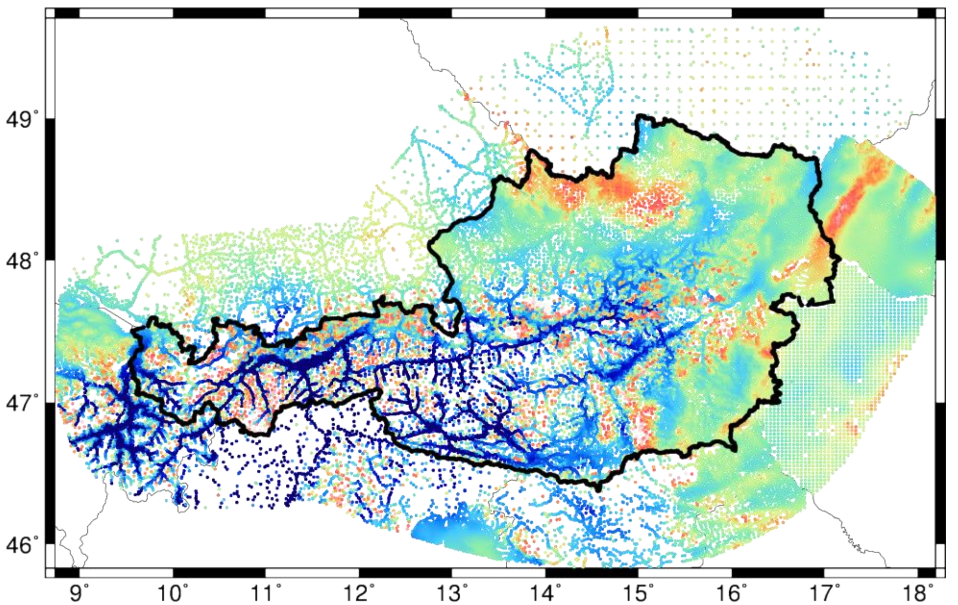
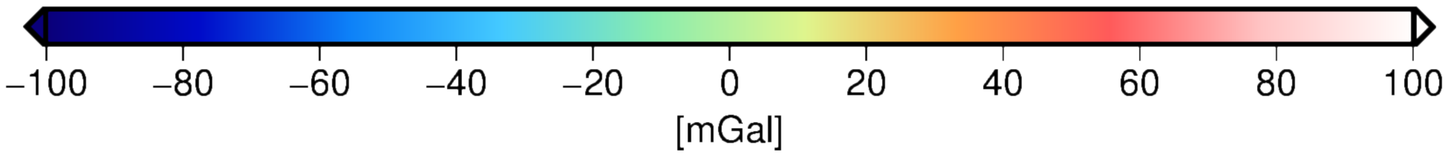






Topography

Gravitational potential from topographic masses

$$T(\mathbf{r}_P) = G\iiint_\Omega \frac{1}{l(\mathbf{r}_P,\mathbf{r}_Q)}\,\rho(\mathbf{r}_Q)\,d\Omega(\mathbf{r}_Q)$$

Satellite model includes topographic effect

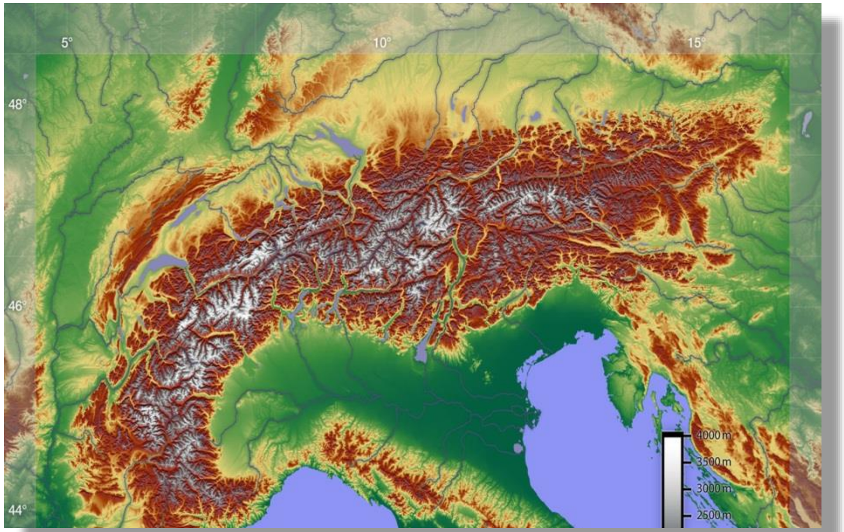
$\Rightarrow$ Spherical harmonic expansion of topography

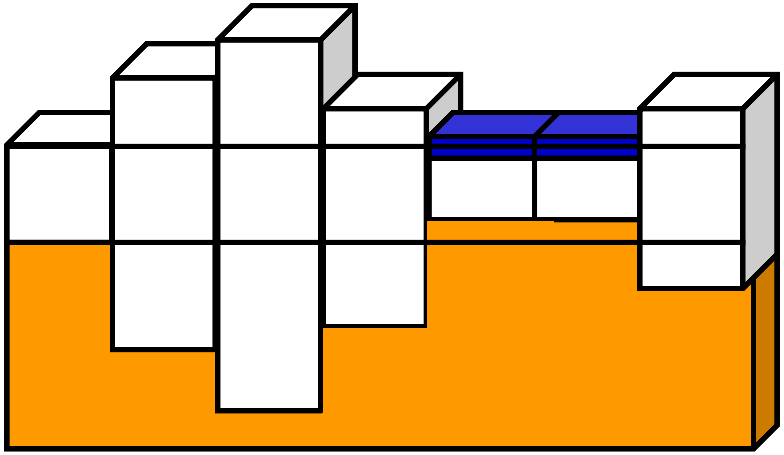
$$T_{topo}(\mathbf{r}_P) = \frac{GM}{R}\sum_{n=0}^{\infty}\left(\frac{R}{r}\right)^{n+1}\sum_{m=-n}^{n} a_{nm}^{topo} Y_{nm}(\mathbf{r}_P)$$

$$a_{nm}^{topo} = \frac{1}{M(2n+1)}\iiint_\Omega \left(\frac{r'}{R}\right)^n \rho(\mathbf{r}_Q)\,Y_{nm}(\mathbf{r}_Q)\,d\Omega(\mathbf{r}_Q)$$

Topography without satellite model part

$$\delta T_{topo} = T_{topo} - \sum_{n=0}^{N} T_n^{topo}$$





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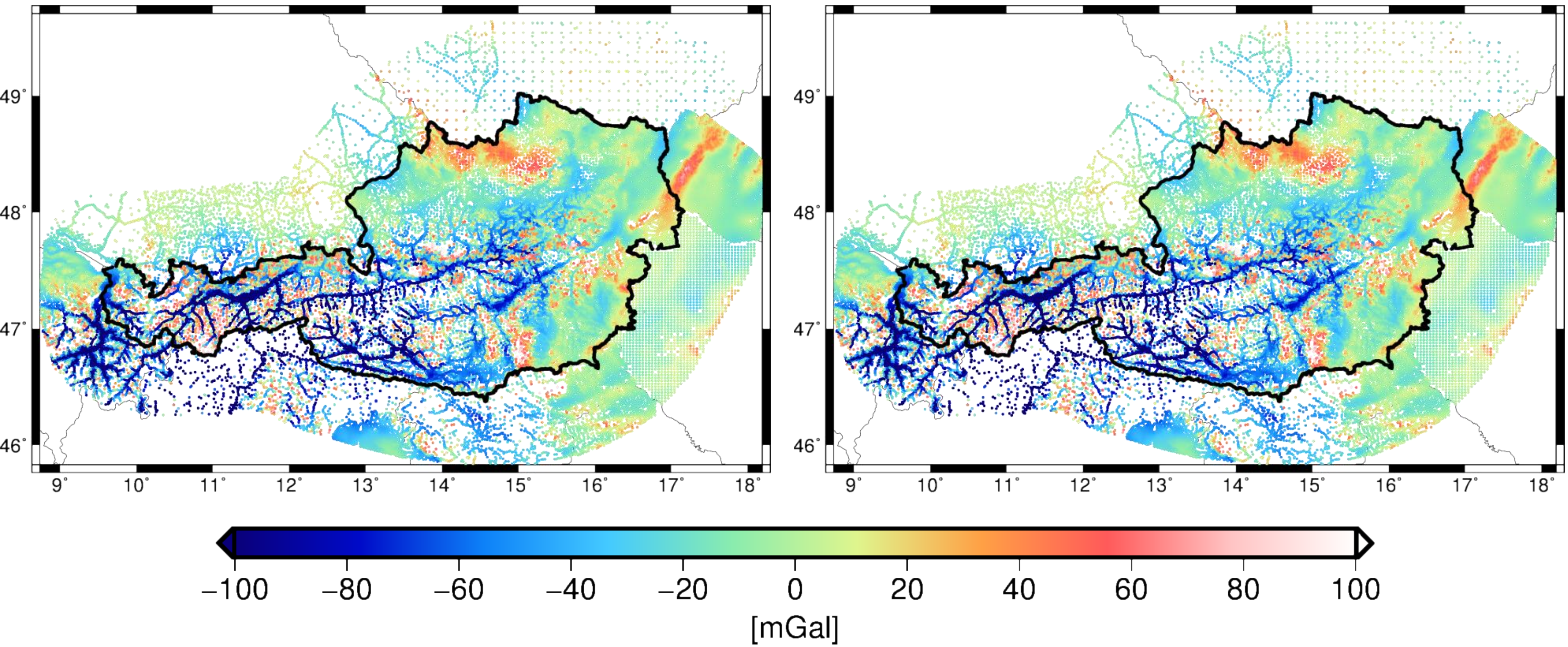


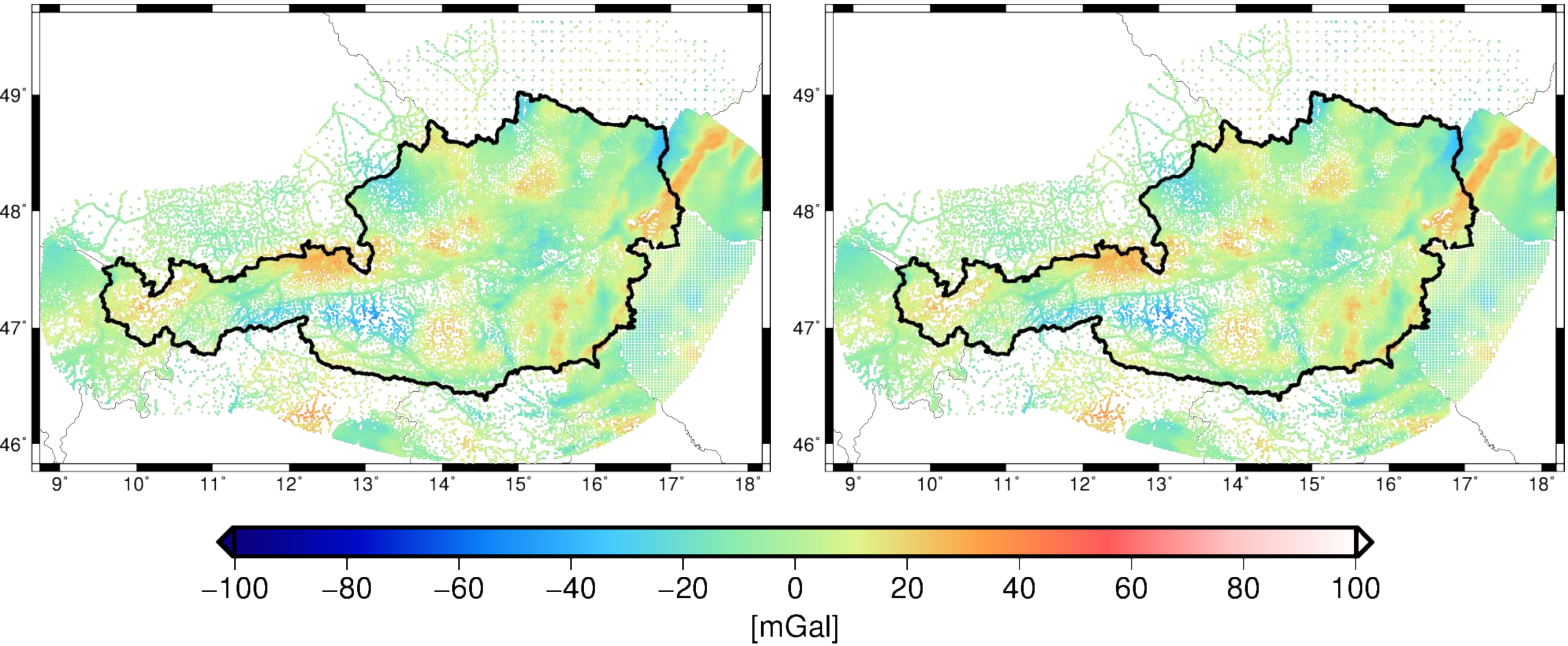
Reduced observations

Classic approach

$$\Delta g = g - \gamma - \Delta g_{sat} - \delta g_{topo}$$

New approach

$$\Delta g = g - \left\| \mathbf{g}_{sat} + \mathbf{g}_{topo} \right\|$$



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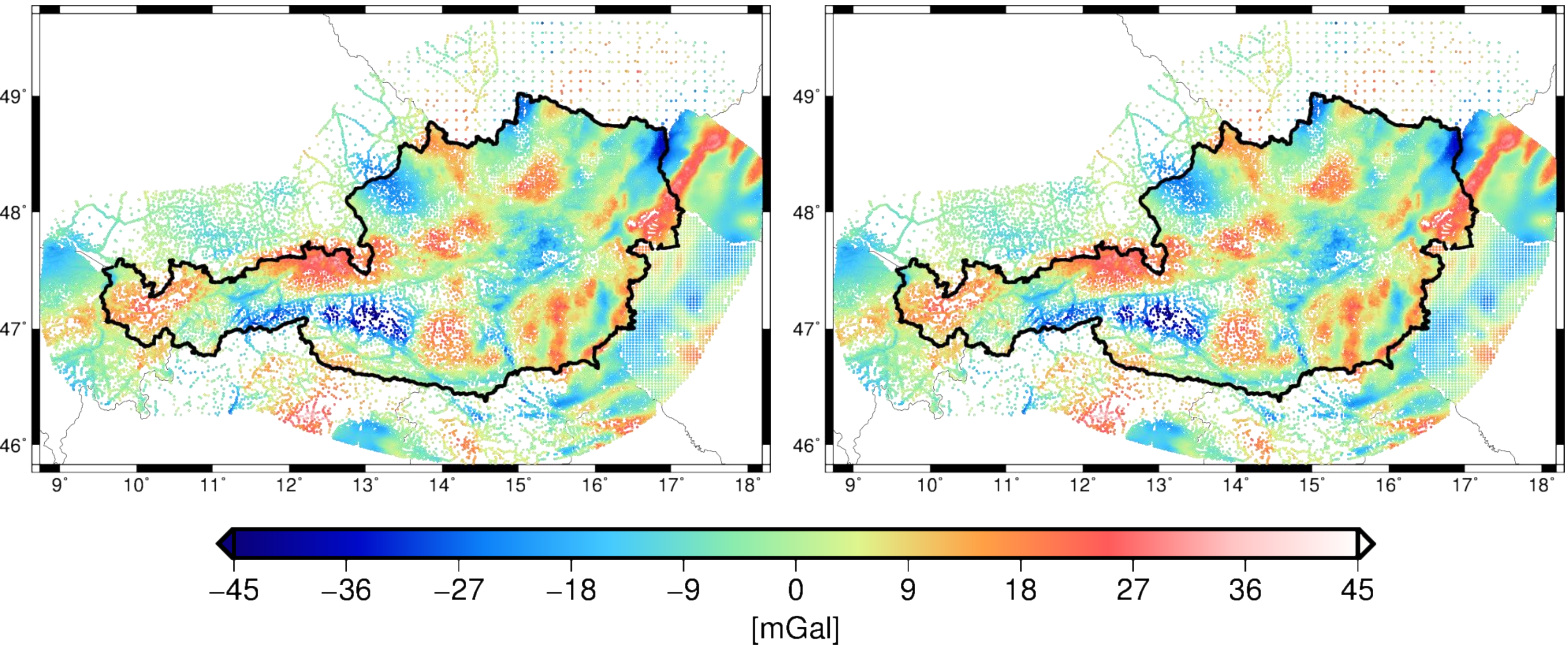

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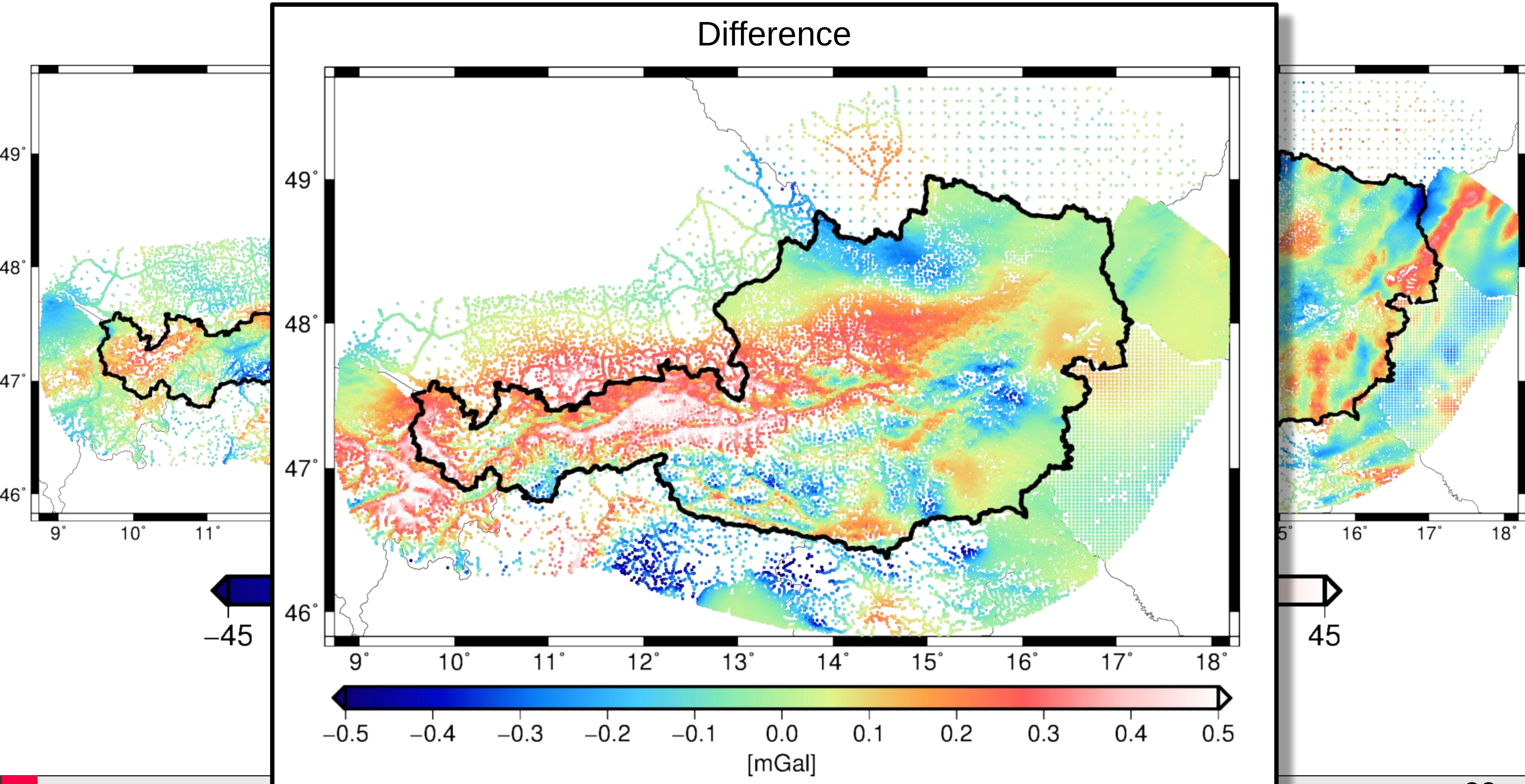

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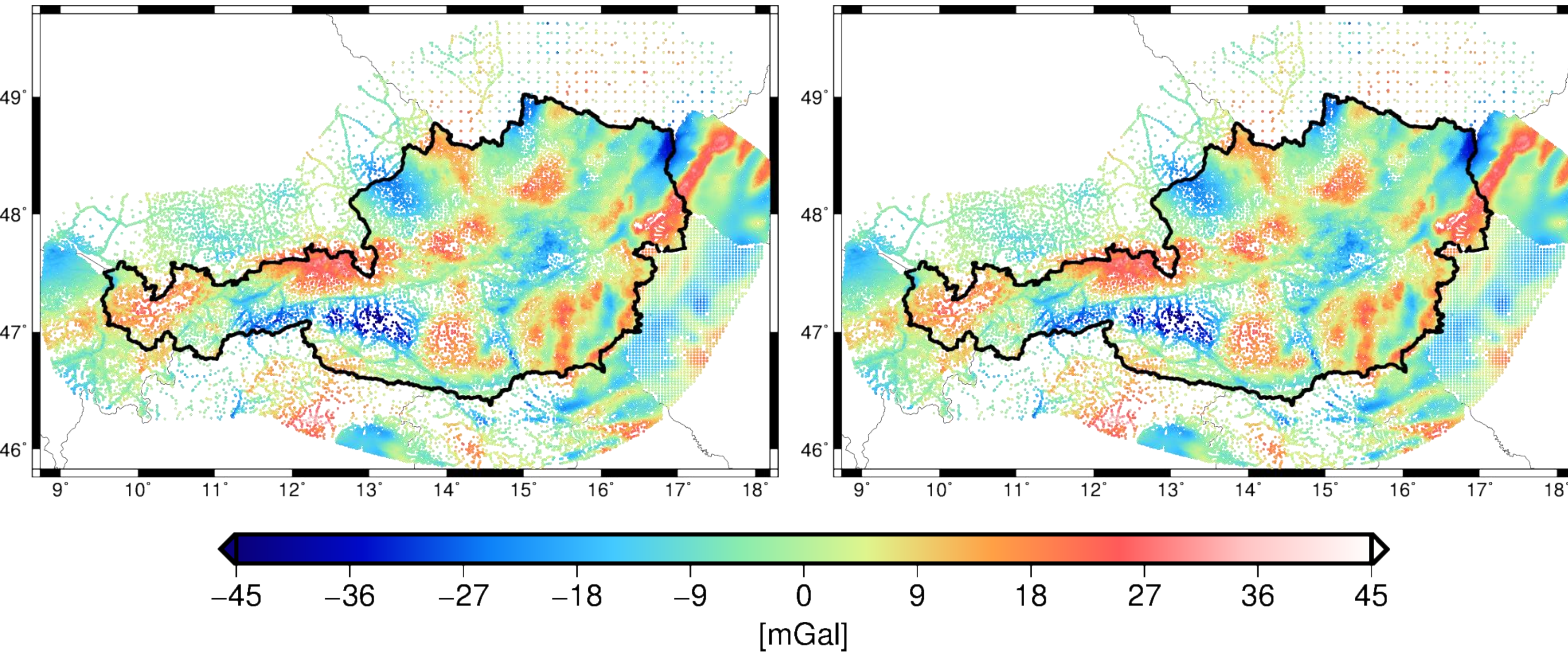


Estimated Geoid

Classic approach

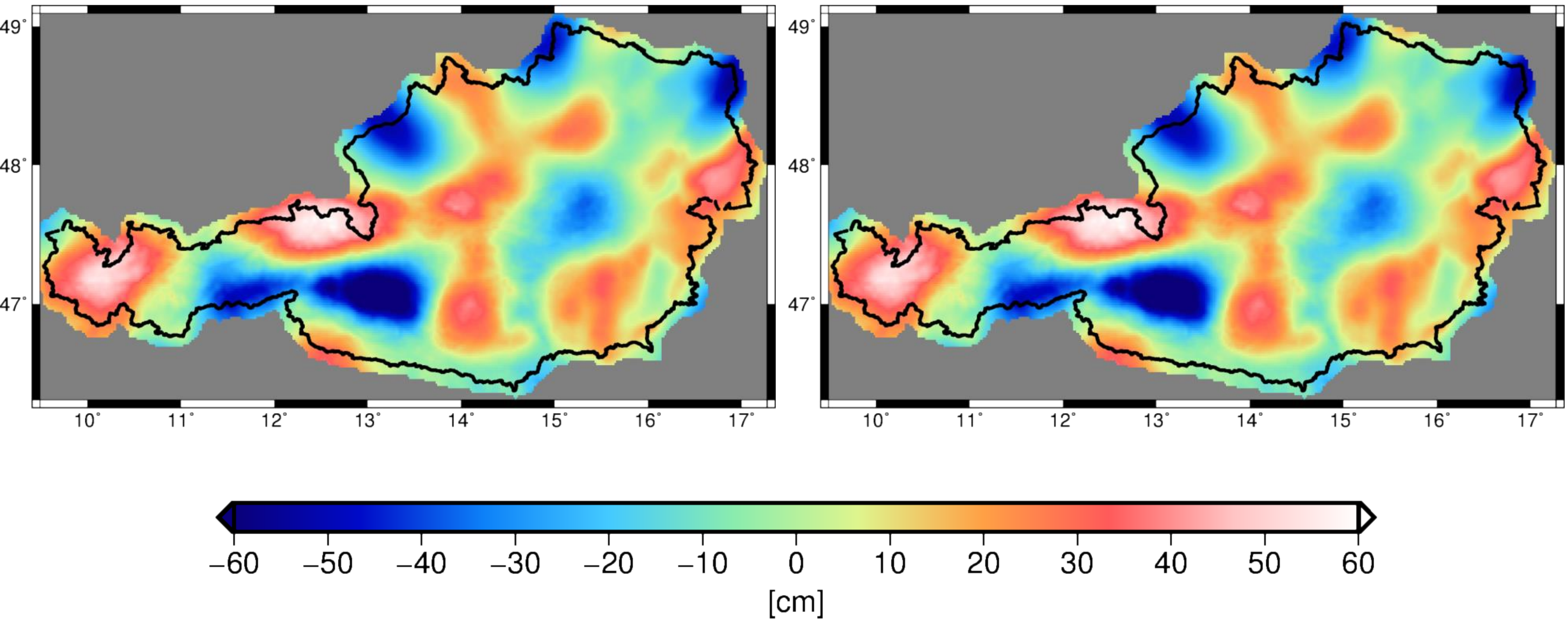
$$\Delta g = g - \gamma - \Delta g_{sat} - \delta g_{topo}$$

$\Rightarrow$ Residual geoid

New approach

$$\Delta g = g - \left\| \mathbf{g}_{sat} + \mathbf{g}_{topo} \right\|$$

$\Rightarrow$ Residual geoid



Estimated Geoid

Classic approach

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New approach

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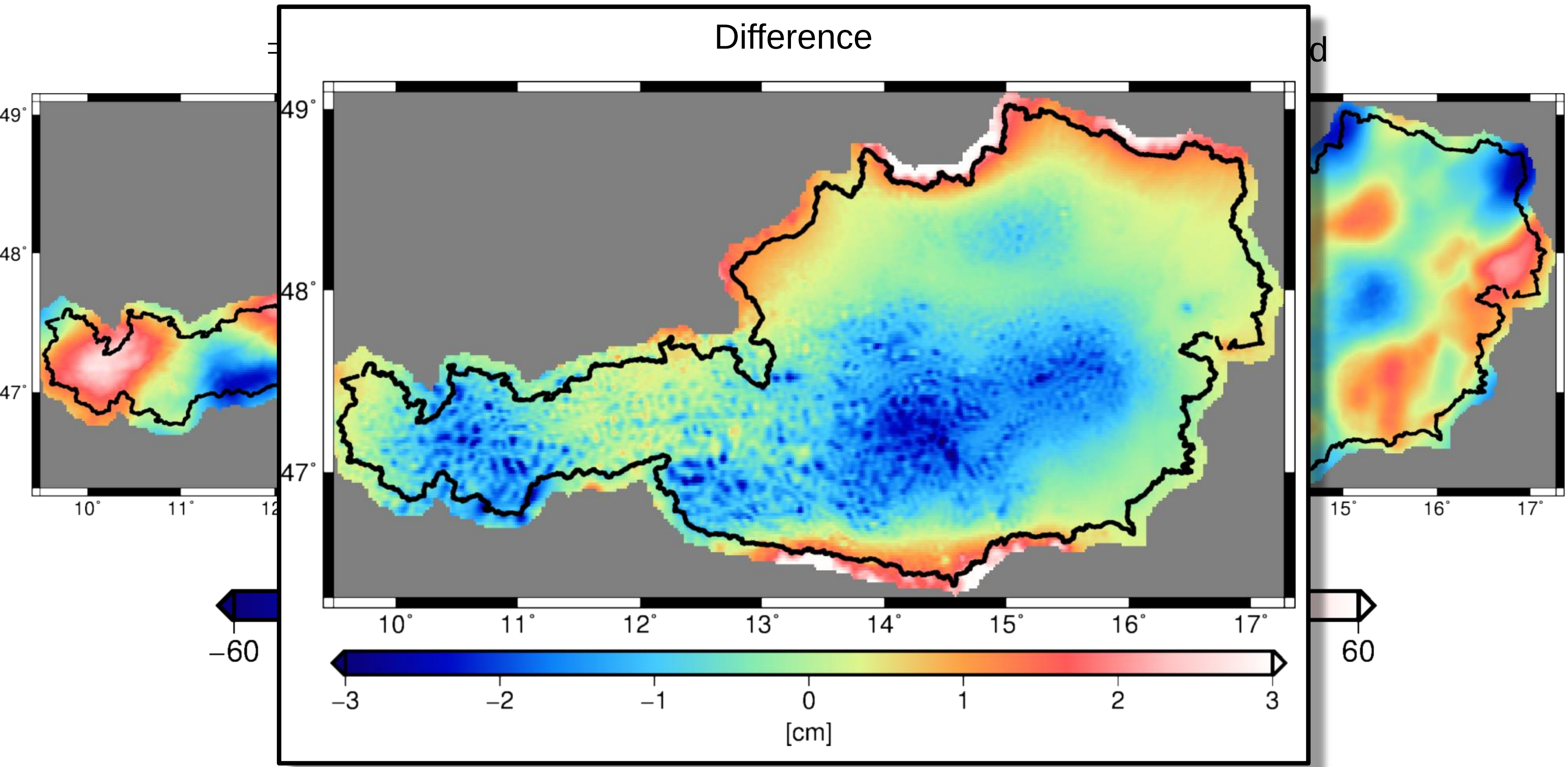

Summary

Classical definition:
Gravity anomalies = Observed absolute gravity
- Normal gravity

$$\Delta g = g - \gamma$$

$\Rightarrow$ Not accurate enough for geoid computation

Generalized definition:
Gravity anomalies = Observed absolute gravity
– Computed gravity from an approximate model

$$\Delta g = g - \left\| \mathbf{g}_{sat} \right\|$$