ITSG-Grace2014s: Combined Estimation of Earth's Static and Time Variable Gravity Field

Andreas Kvas, Torsten Mayer-Gürr, Norbert Zehentner, Beate Klinger

Institute of Theoretical Geodesy and Satellite Geodesy,
Graz University of Technology

Geodätische Woche 2014
Berlin, Germany

Introduction

- Goal of ITSG-Grace2014s:
  estimate Earth's static and time-variable
  gravity field in a single least squares adjustment

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  estimate Earth's static and time-variable gravity field in a single least squares adjustment

- Why?
  - account for unmodelled mass variations and model errors
  - consider correlations between each gravity field component
  - provide realistic formal errors

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$$\text{with} \quad a_{nm}(t) = \sum_{k=0}^{\infty} a_{nm}^{(k)} \cdot \Phi_k(t)$$

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$$a_{nm}(t) = a_{nm}^{(0)} + (t - t_0) a_{nm}^{(1)} + e^{i\omega_A (t - t_0)} a_{nm}^{(2)} + \sum_i a_{nm,i}^{(3)} \cdot B_{i,1}(t)$$

$B_{i,1}$ ... degree zero B-spline with support of one day

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straightforward solution not desirable

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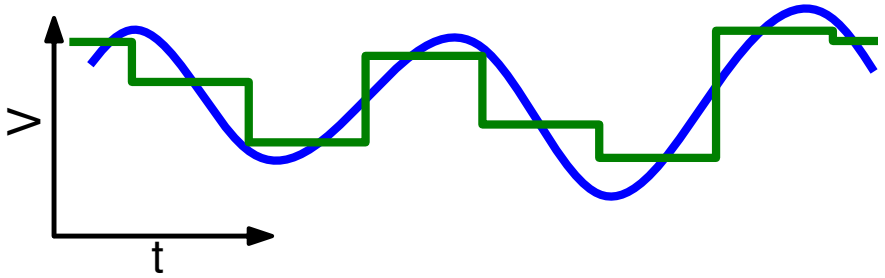

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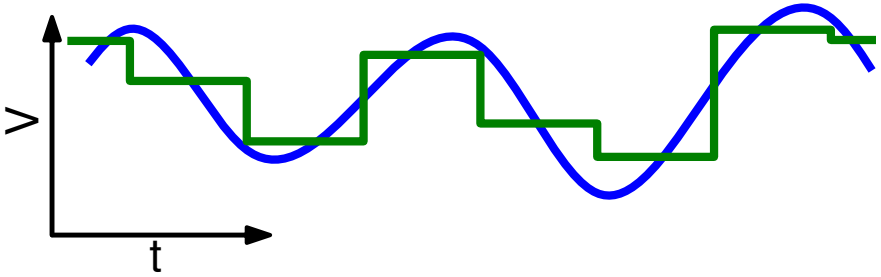
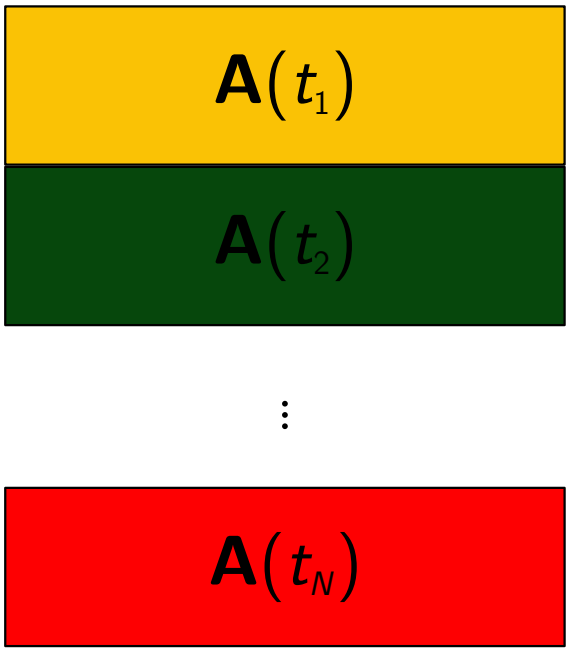
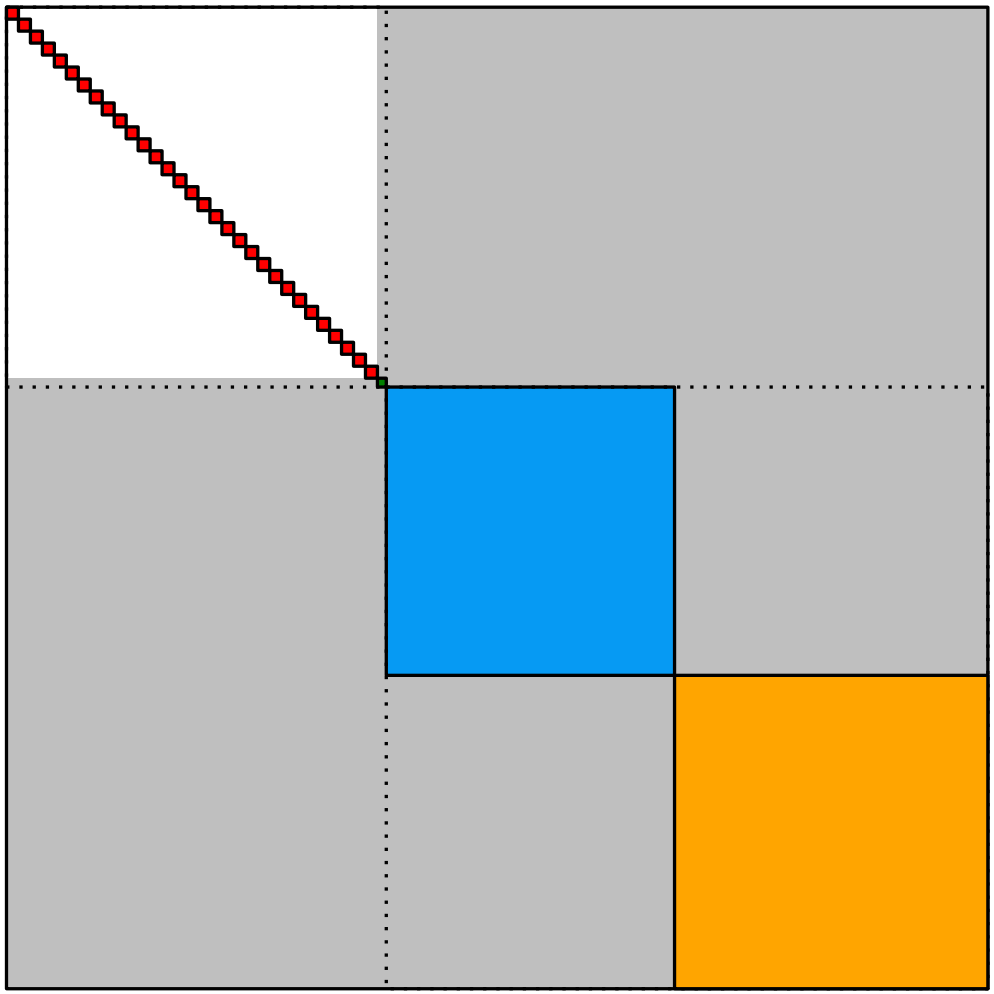




- observation equations are divided into corresponding intervals

$$\mathbf{l}(t_i) = \sum_{k=0}^{\infty} [\Phi_k(t_i)\mathbf{A}(t_i)]\mathbf{x}^{(k)} + \mathbf{e}(t_i)$$

Solution Strategy (2) - Normal Equations



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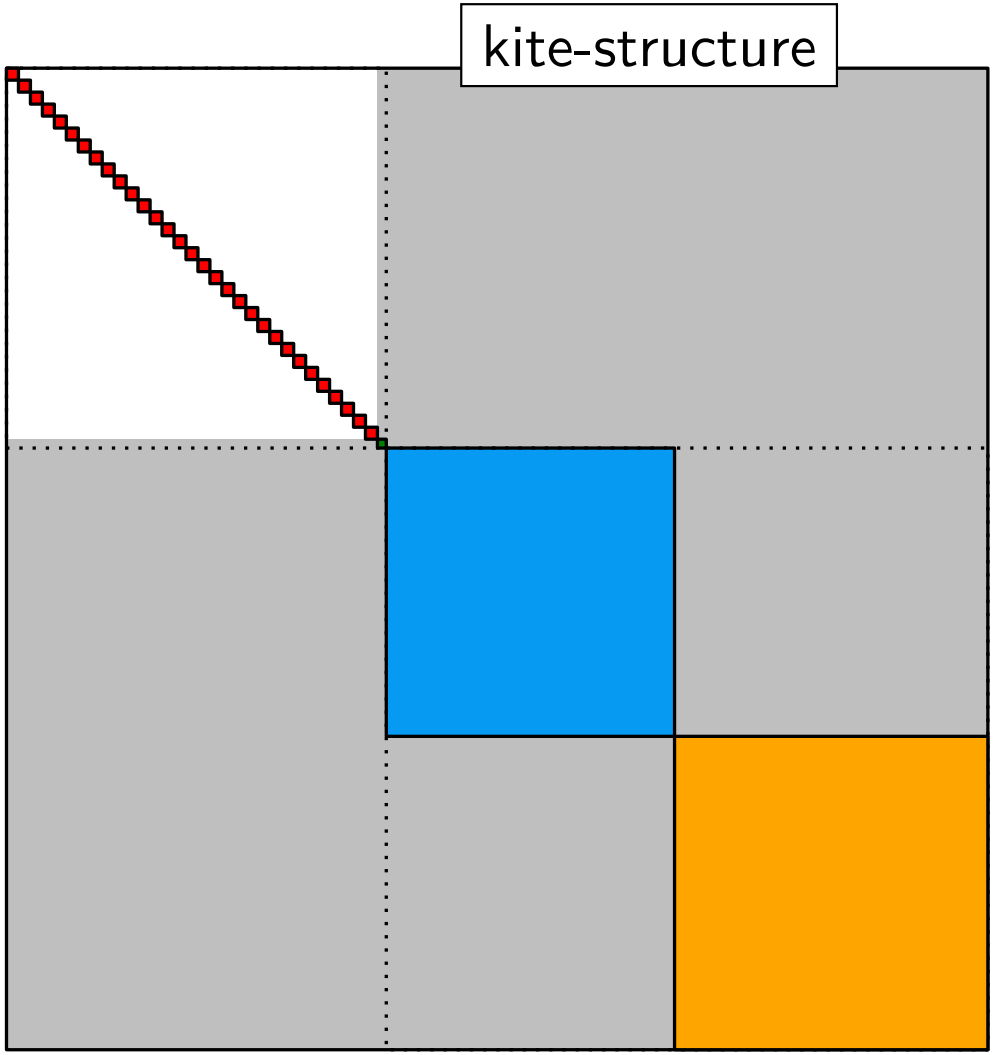

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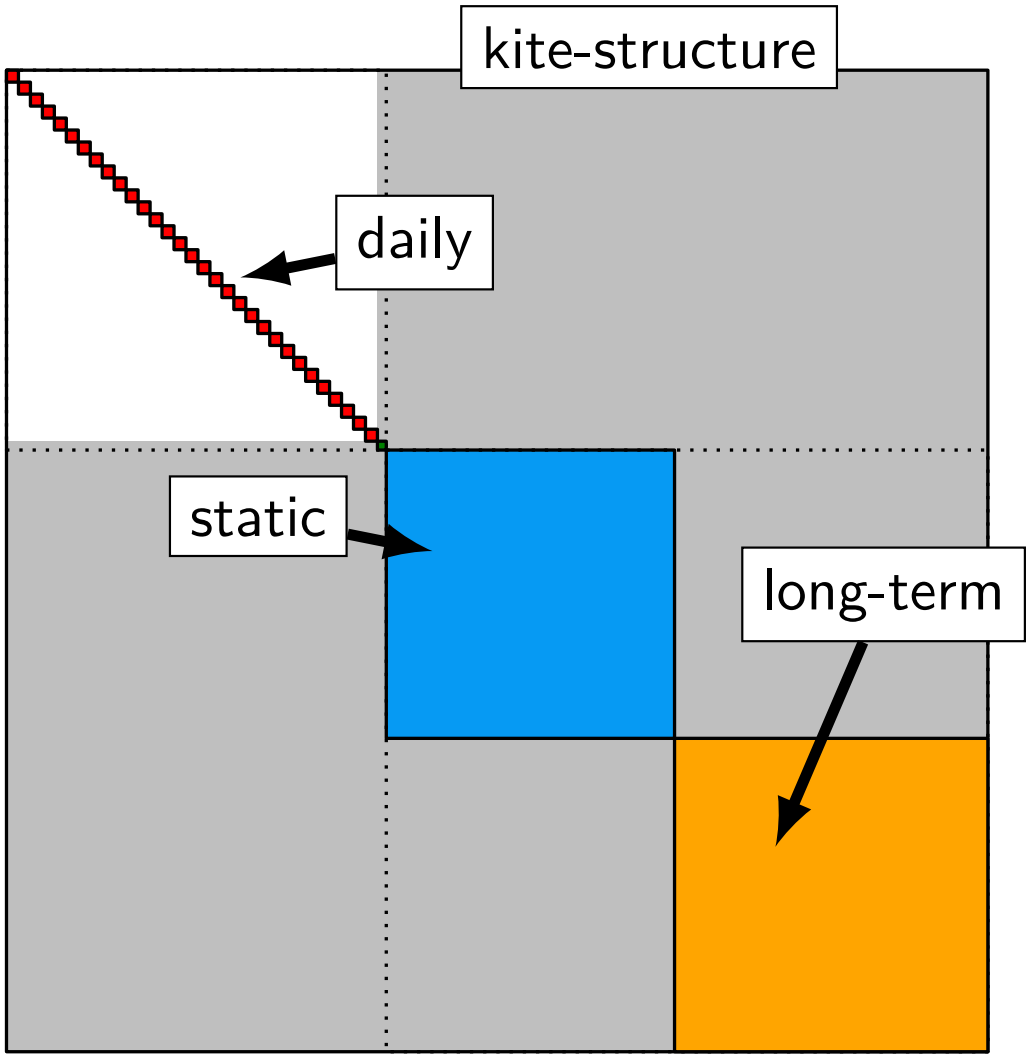

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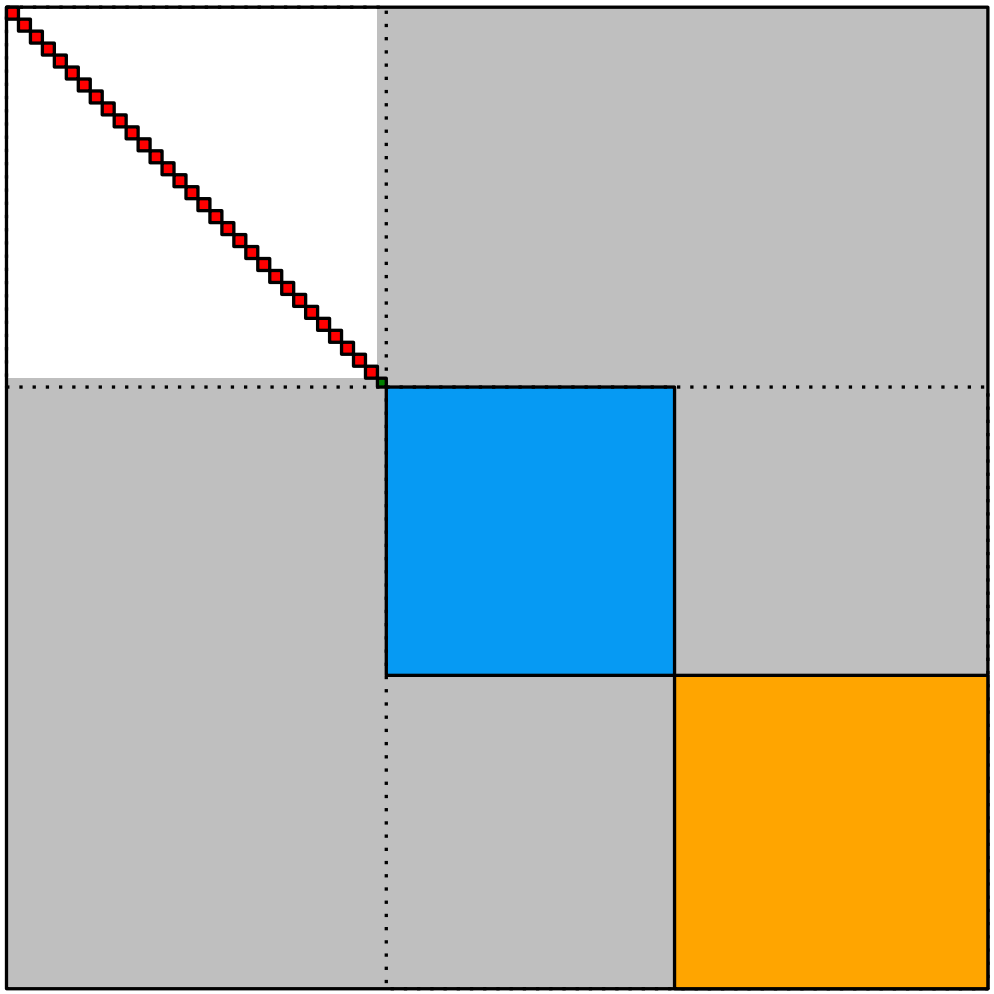

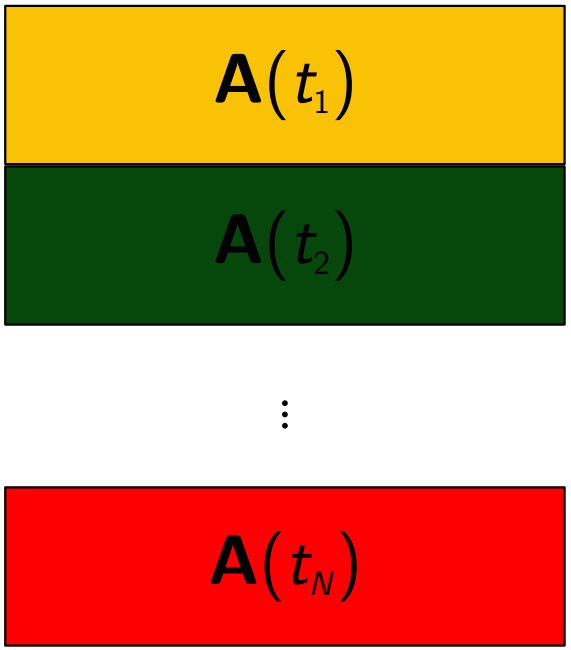

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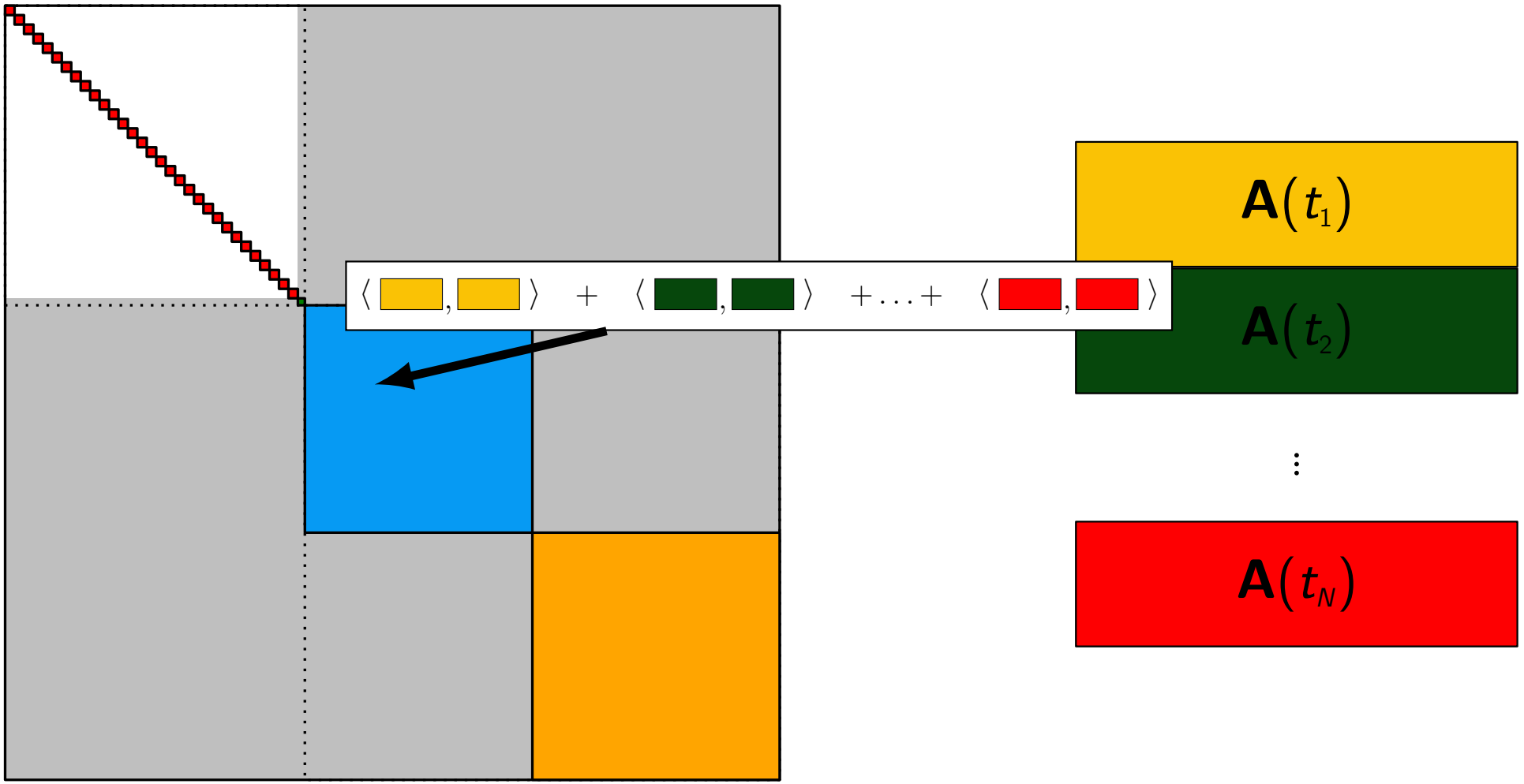

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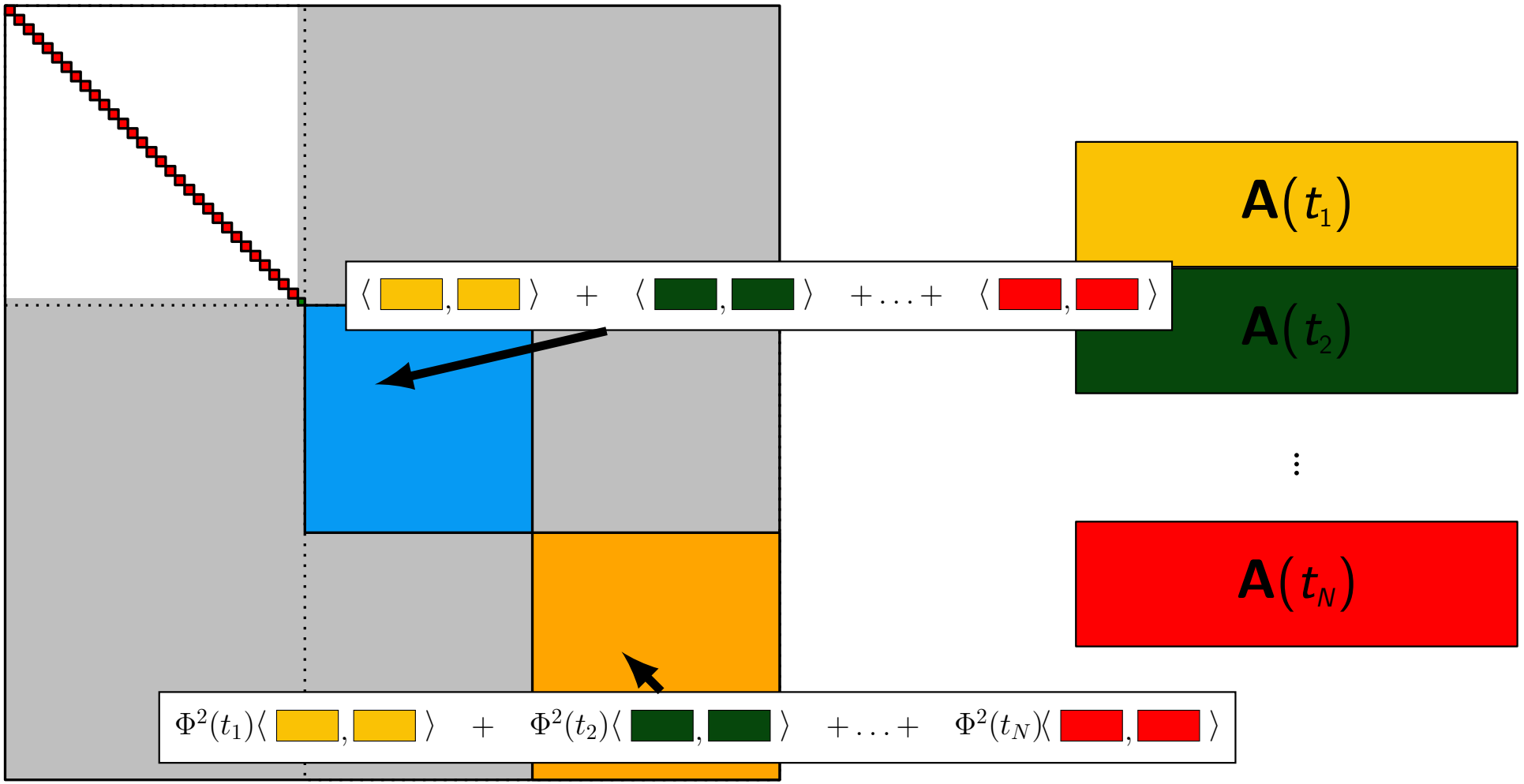

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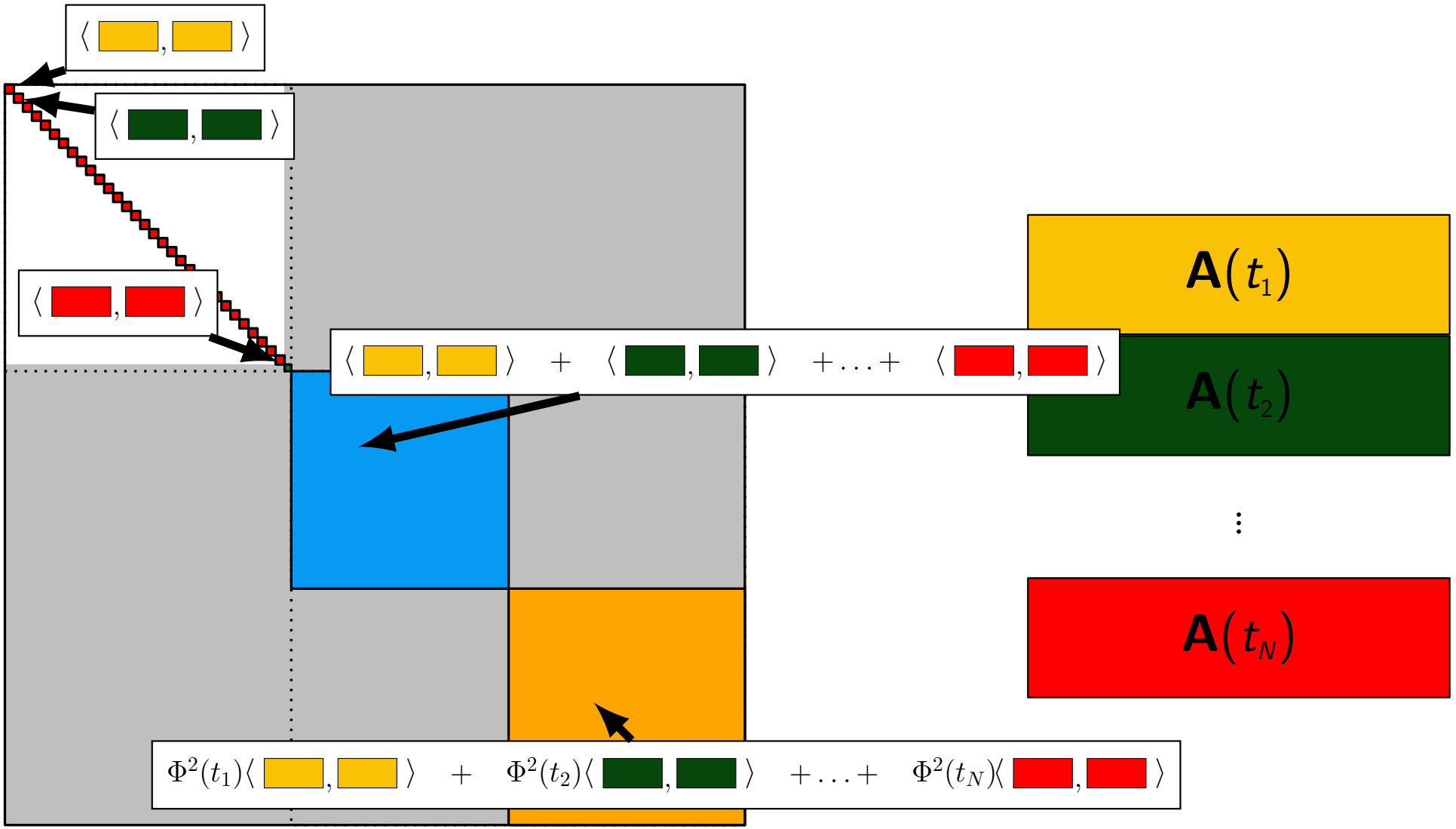

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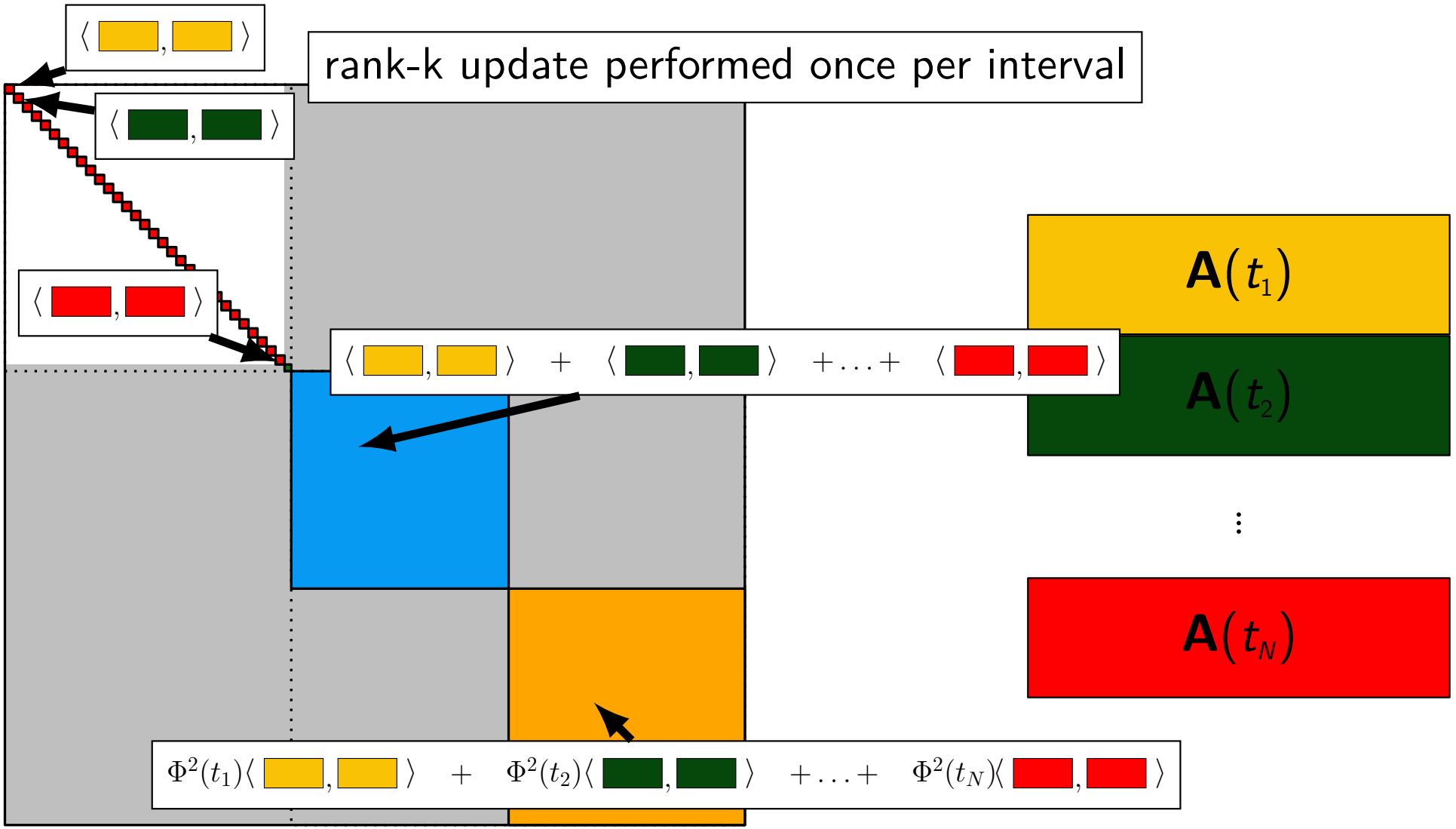

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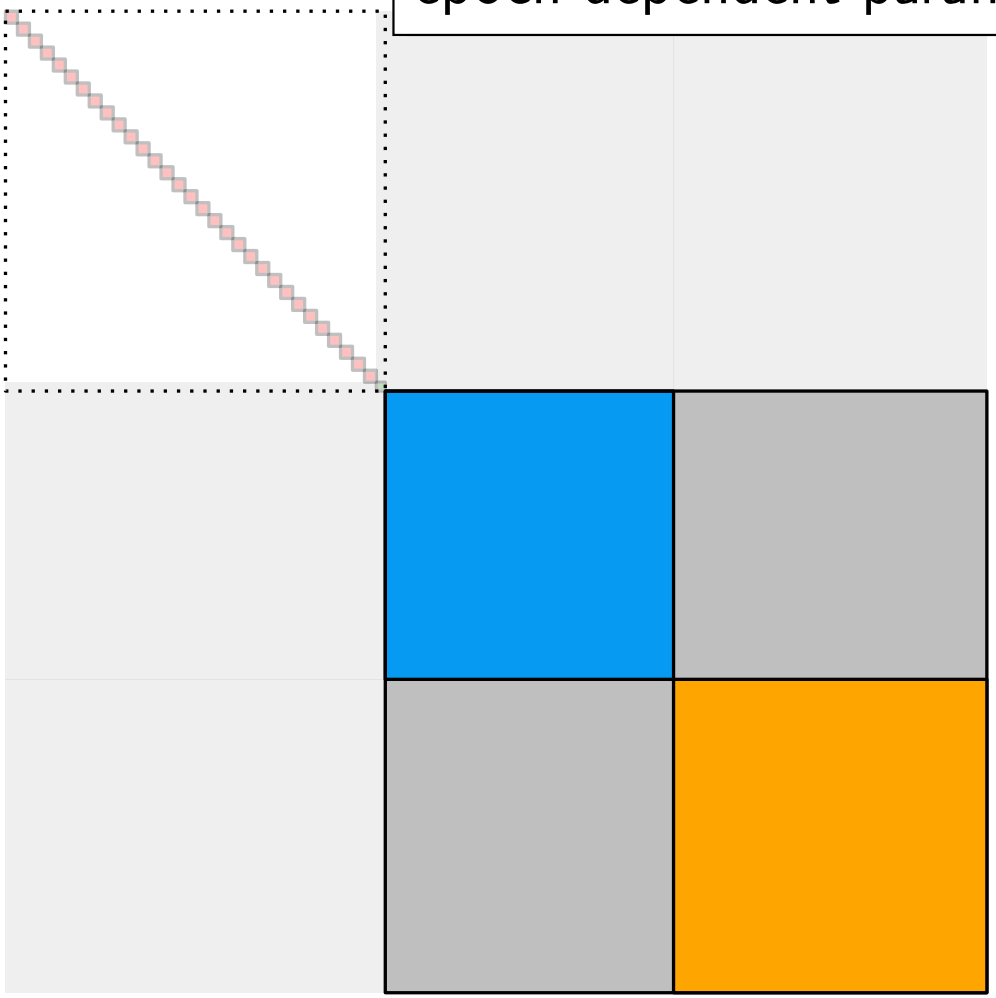
epoch-dependent parameters are eliminated



Performance Evaluation (1) - Data Set and functional Model

- comparison of gravity field parametrizations
  - static only
  - short-term variations only
  - short- and long-term variations
- GRACE data from 2004-01 to 2012-12
- short-arc approach (90 minute arcs)
  - accelerometer bias estimated per arc
  - accelerometer scale estimated monthly

Performance Evaluation (2) - Gravity Field Solutions

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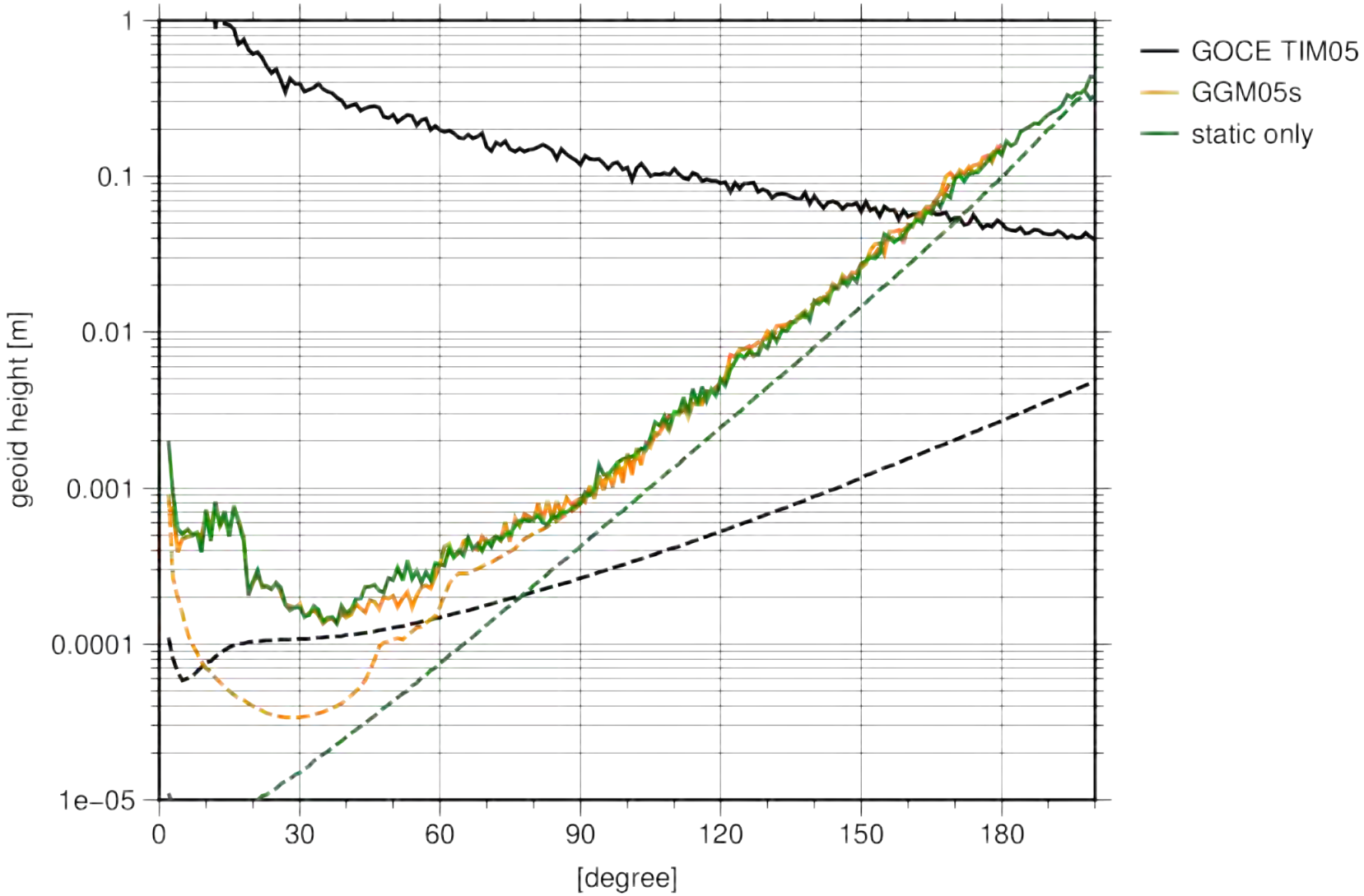

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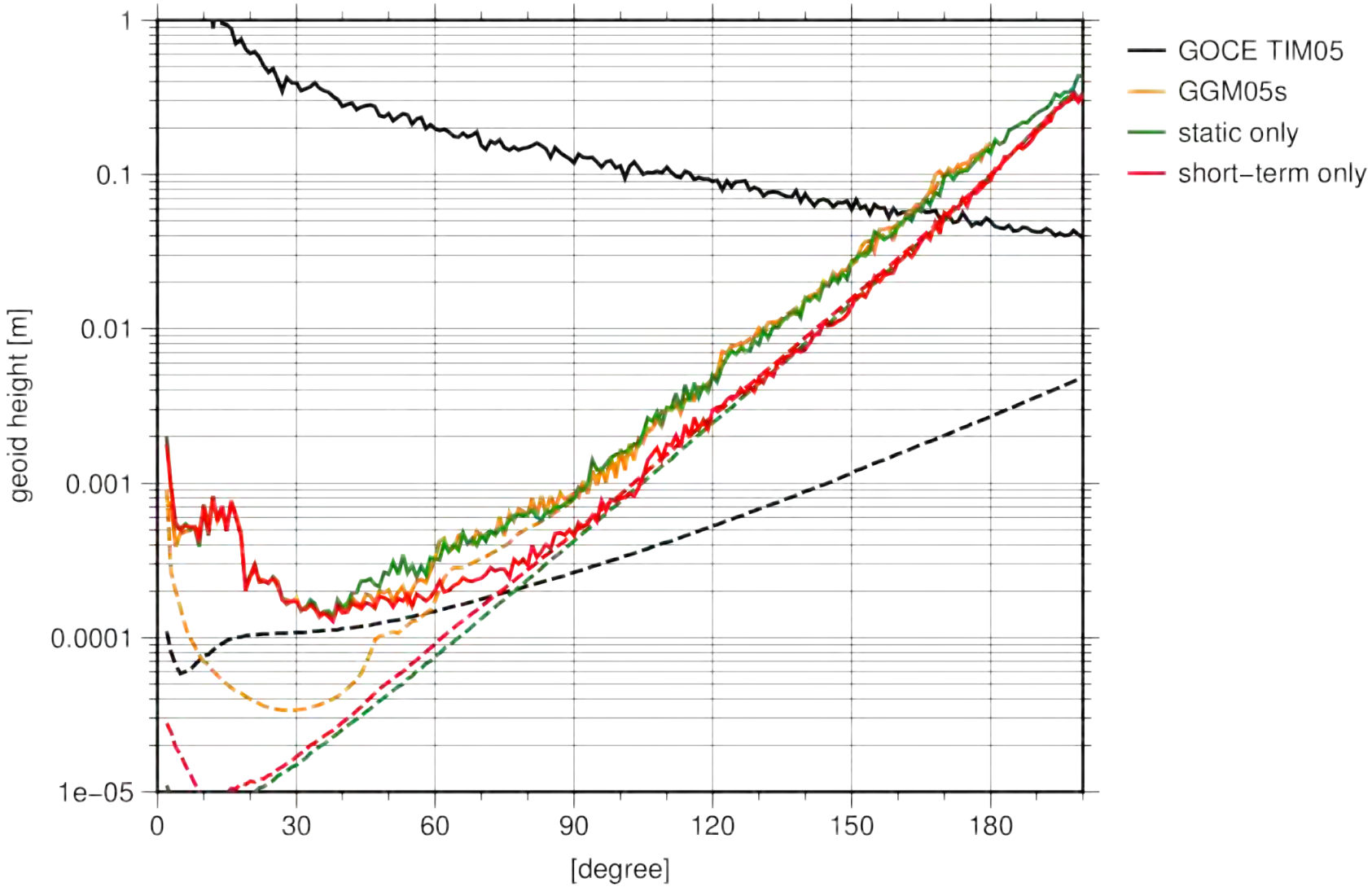

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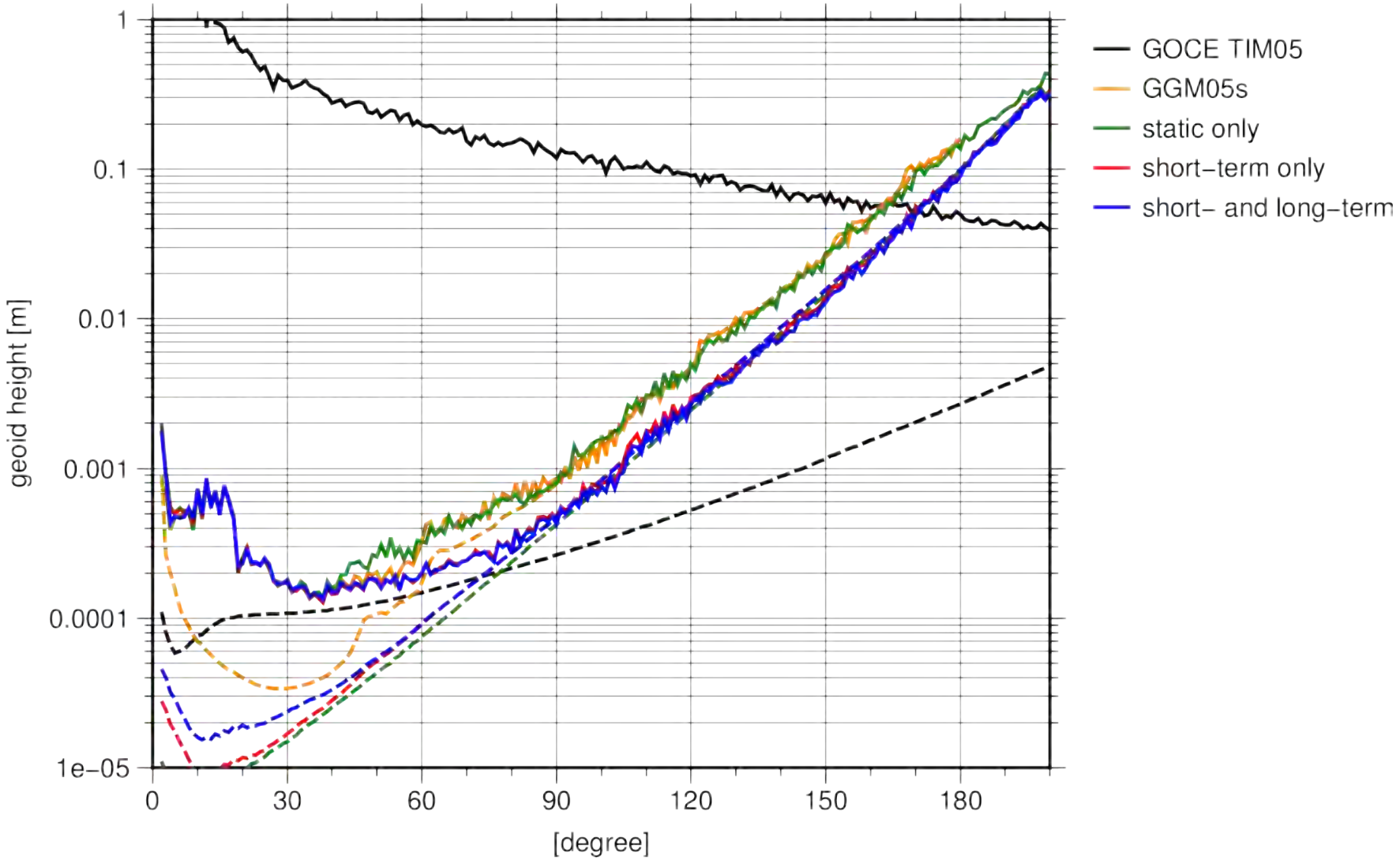

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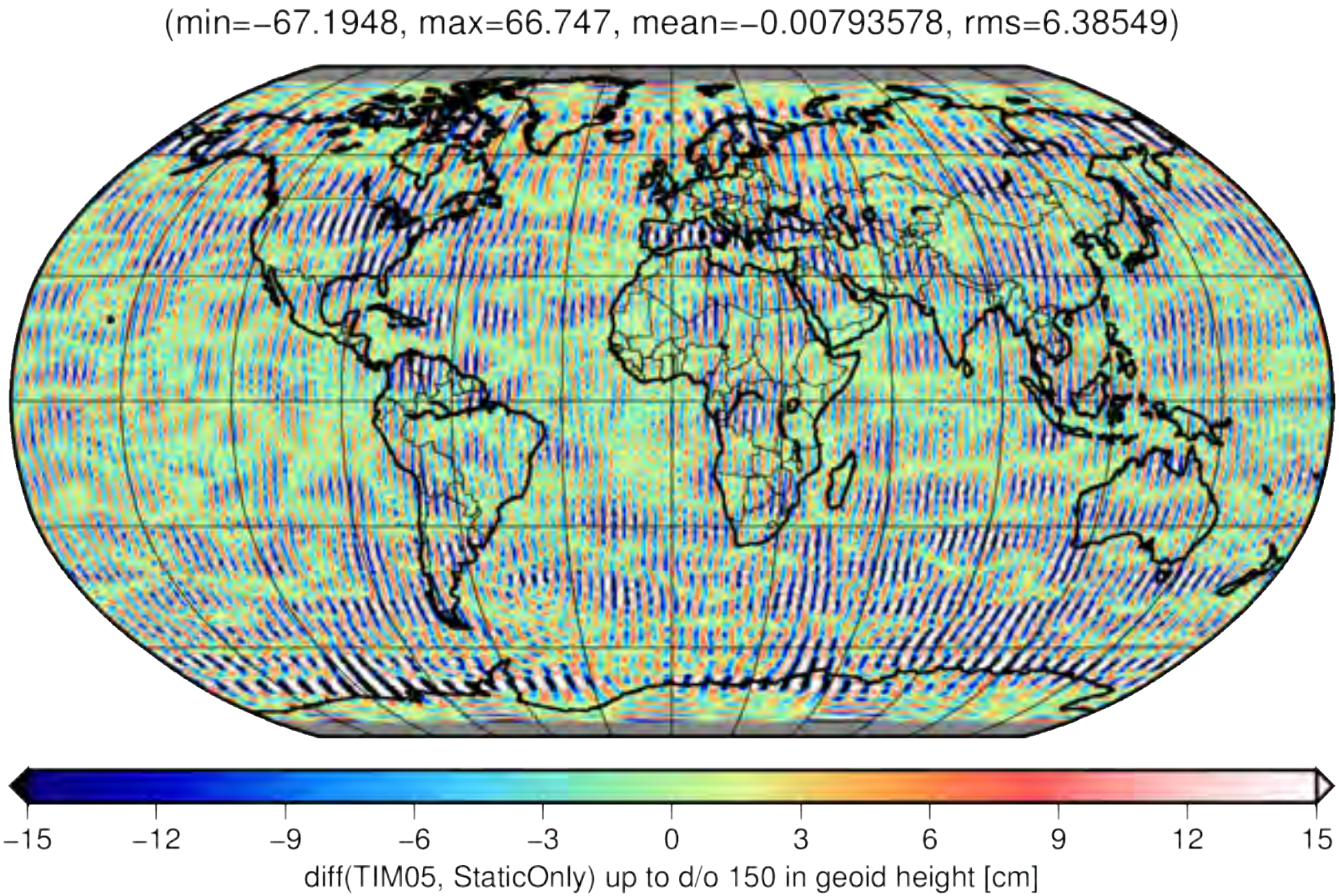

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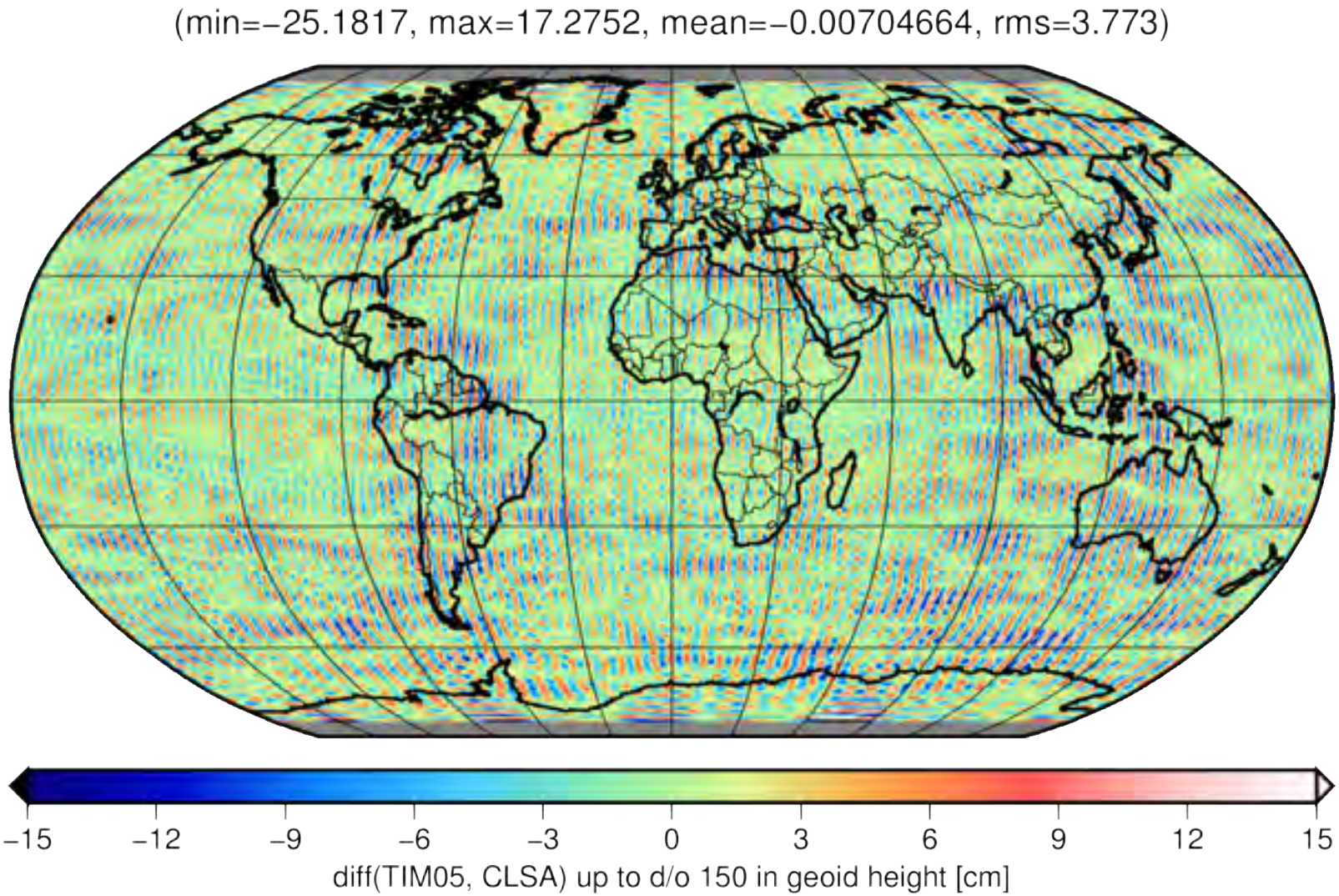

ITSG-Grace2014s - Outline

- Input
  - GRACE data from 2003-02 to 2013-12
  - ITSG orbit product (Zehentner et al. 2014)
  - improved satellite attitude (Klinger et al. 2014)
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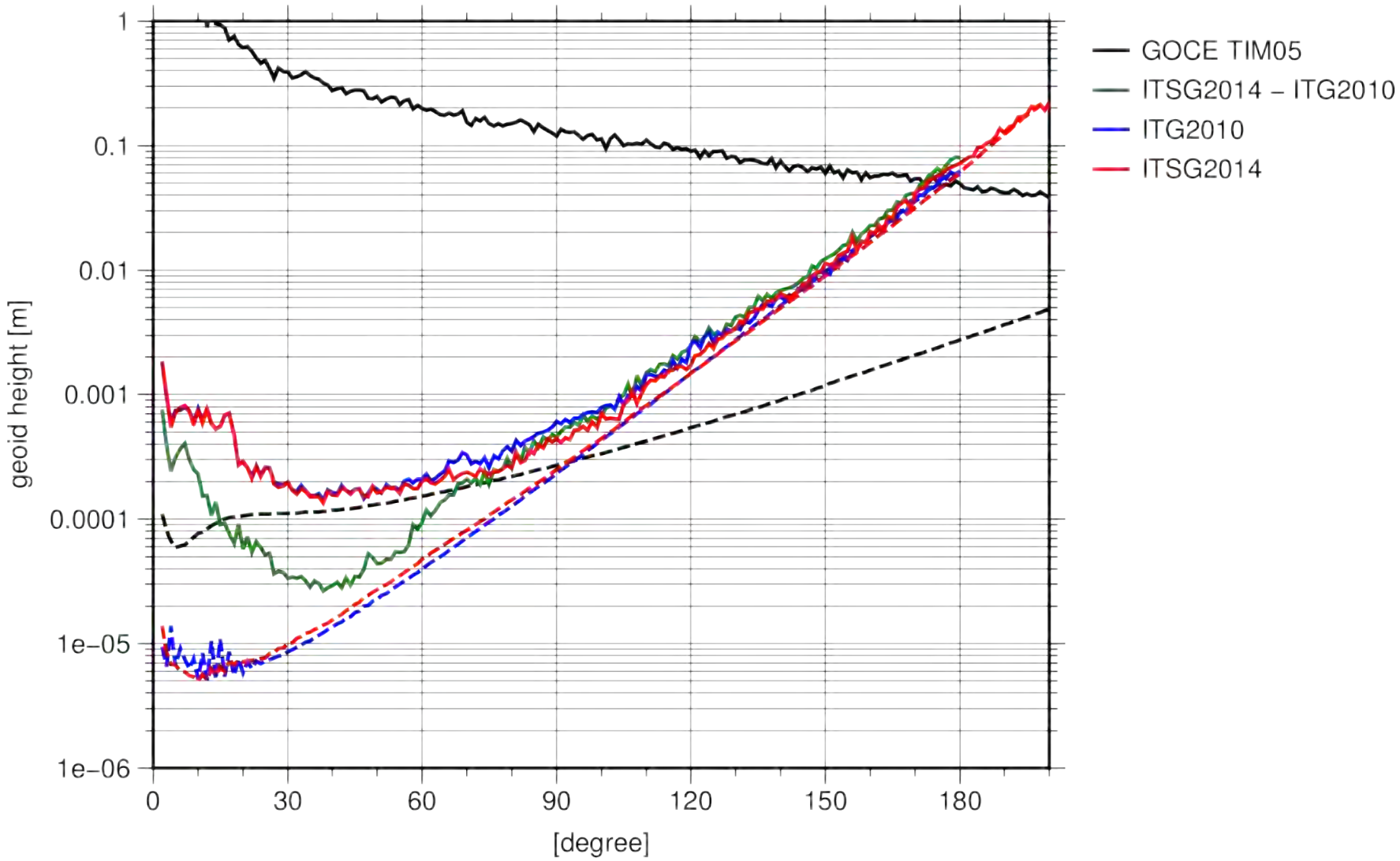
- Method
  - variational equations
  - 24h arc length, 1h covariance length
- non-gravity parameters
  - satellite state (once per day)
  - accelerometer bias/scale (daily degree-3 polynomial/daily)
  - K-band antenna center (monthly)

ITSG-Grace2014s - Static Field



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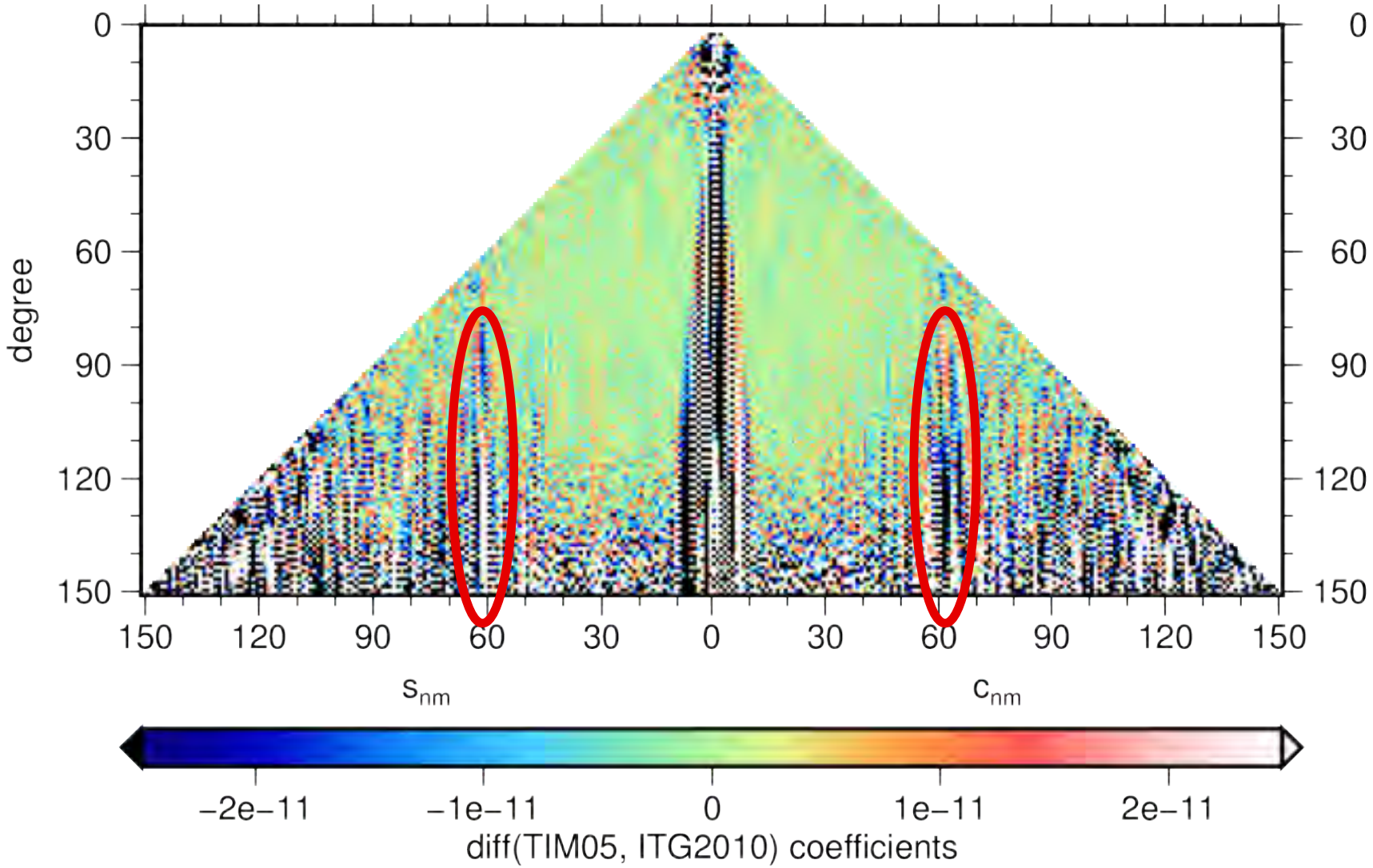

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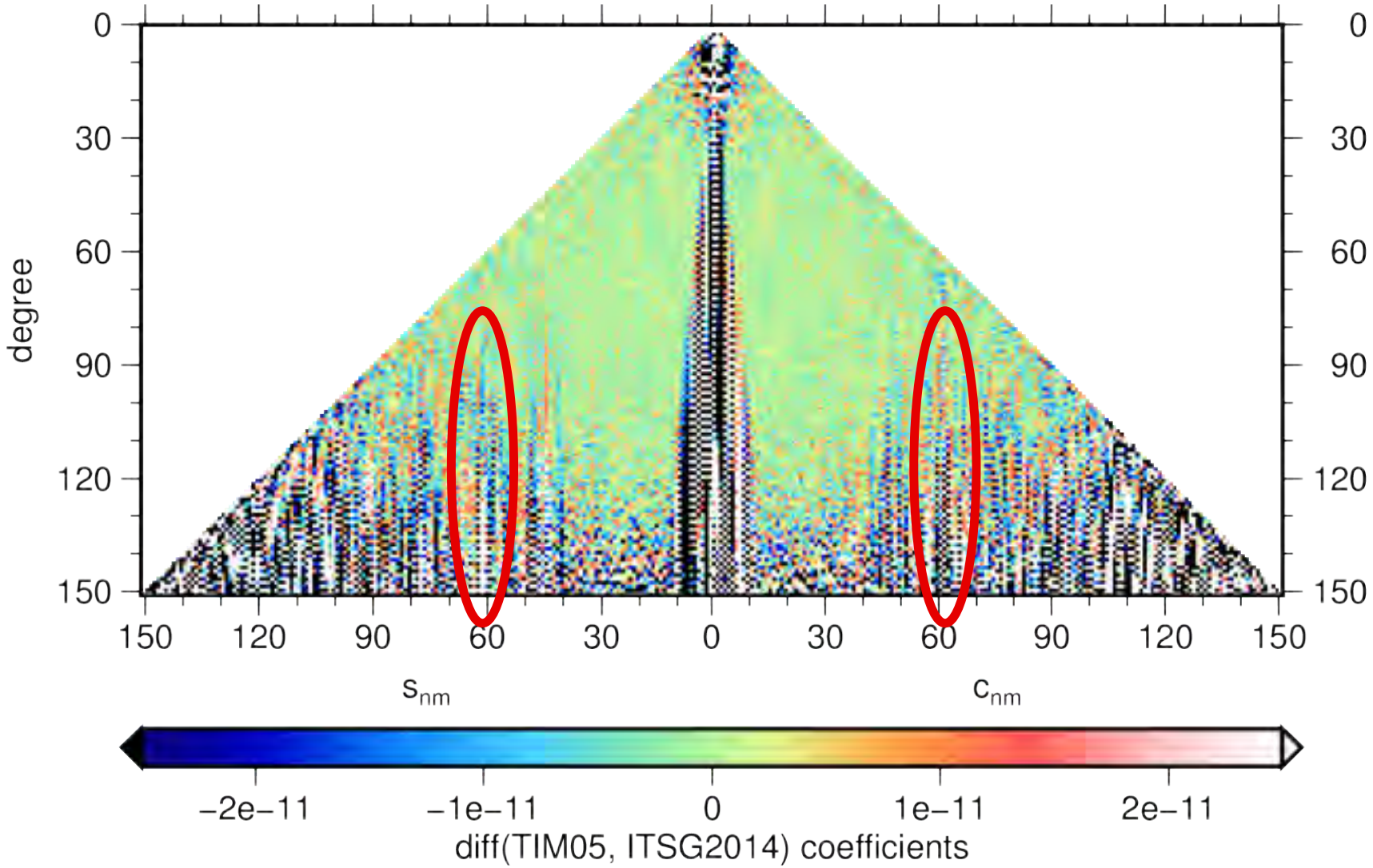
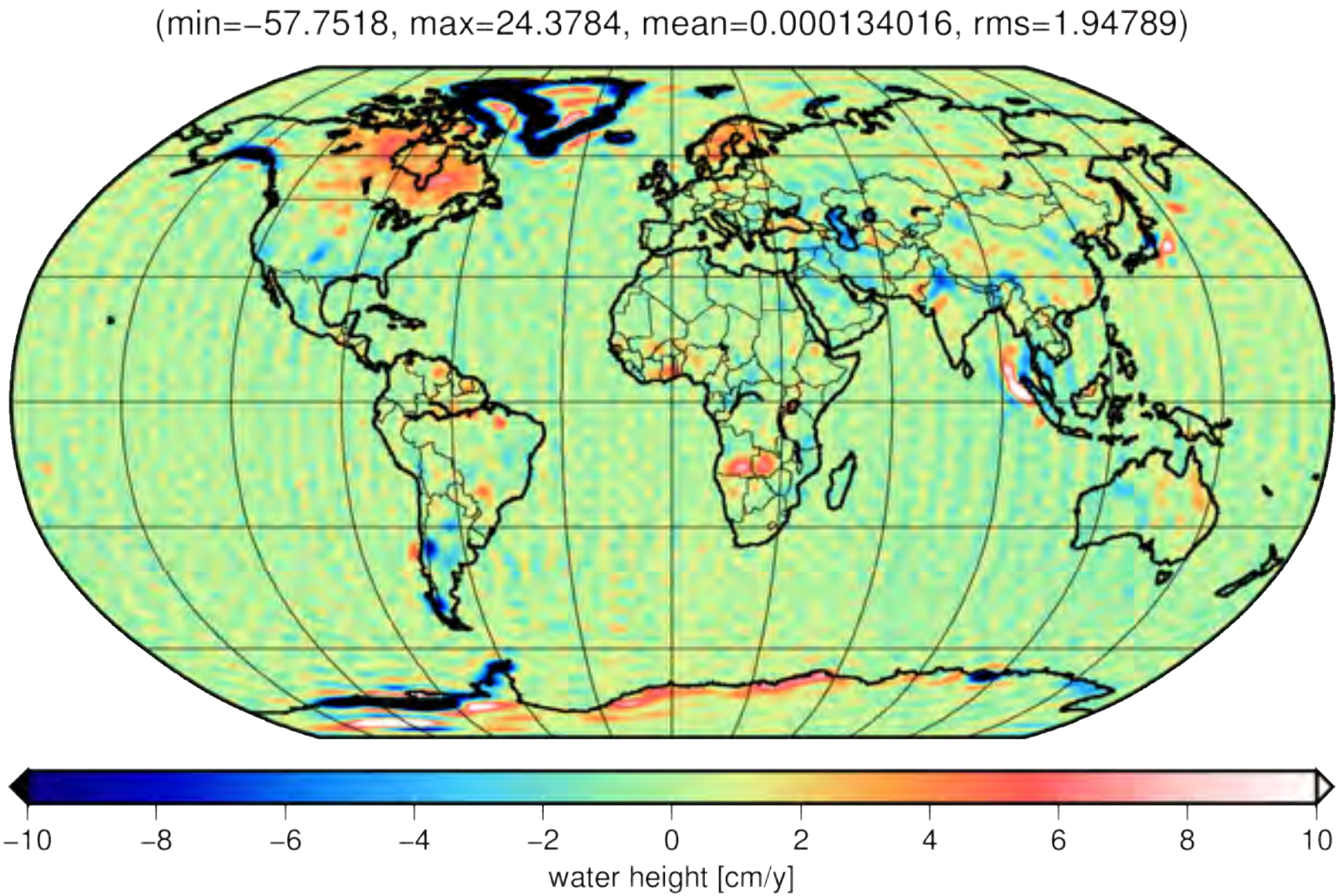


ITSG-Grace2014s - Long-term Variations



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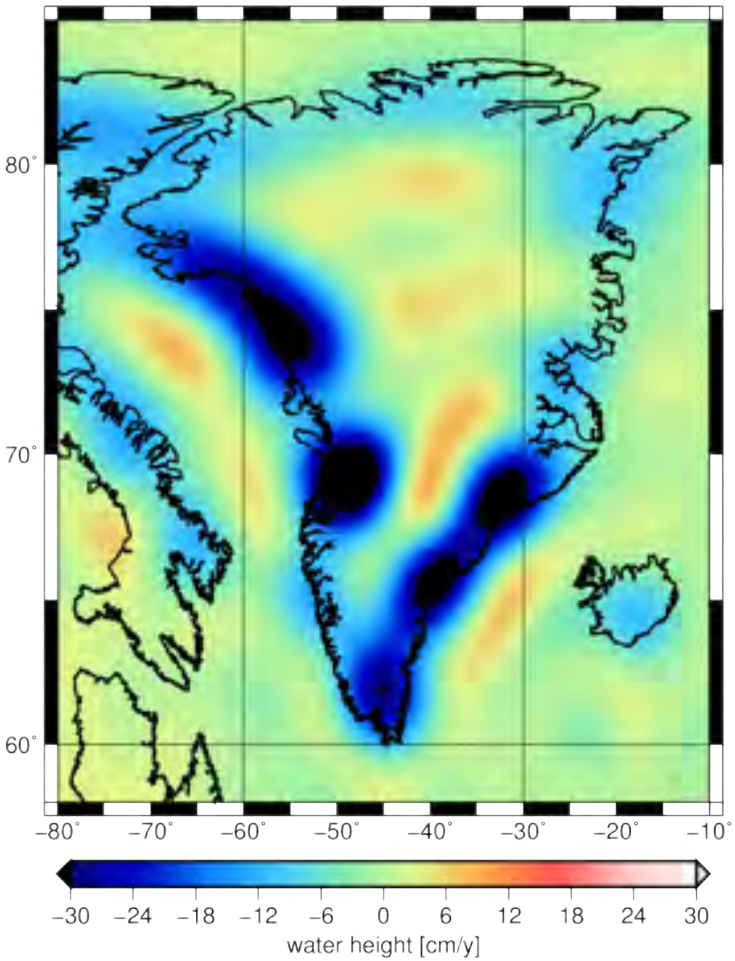

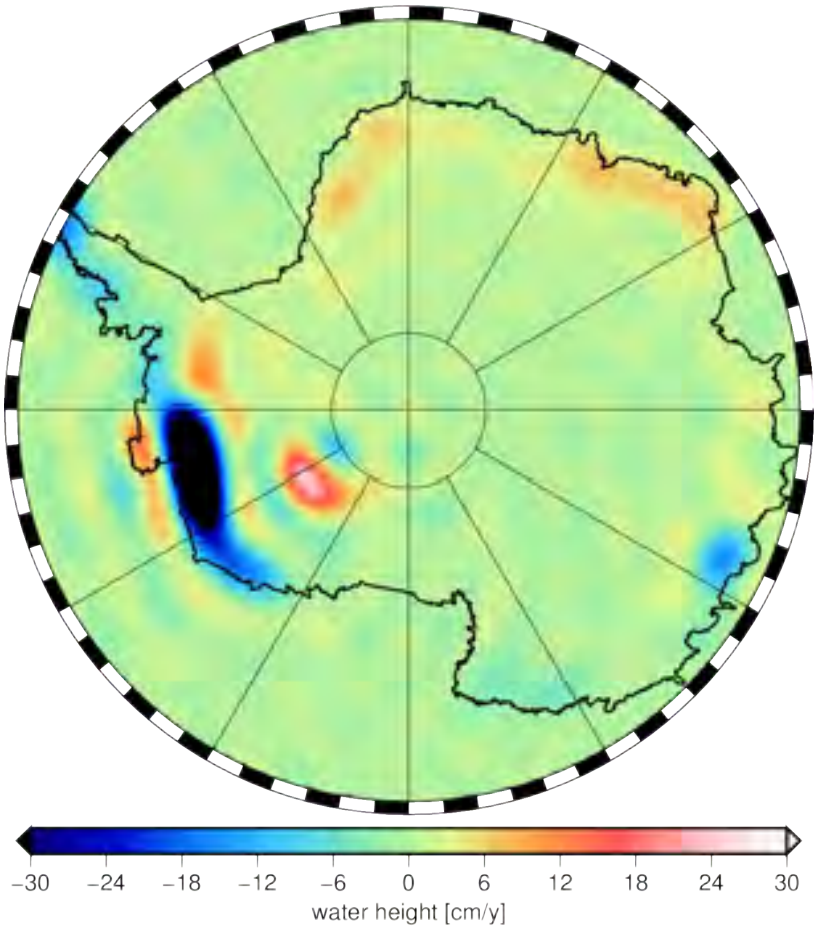

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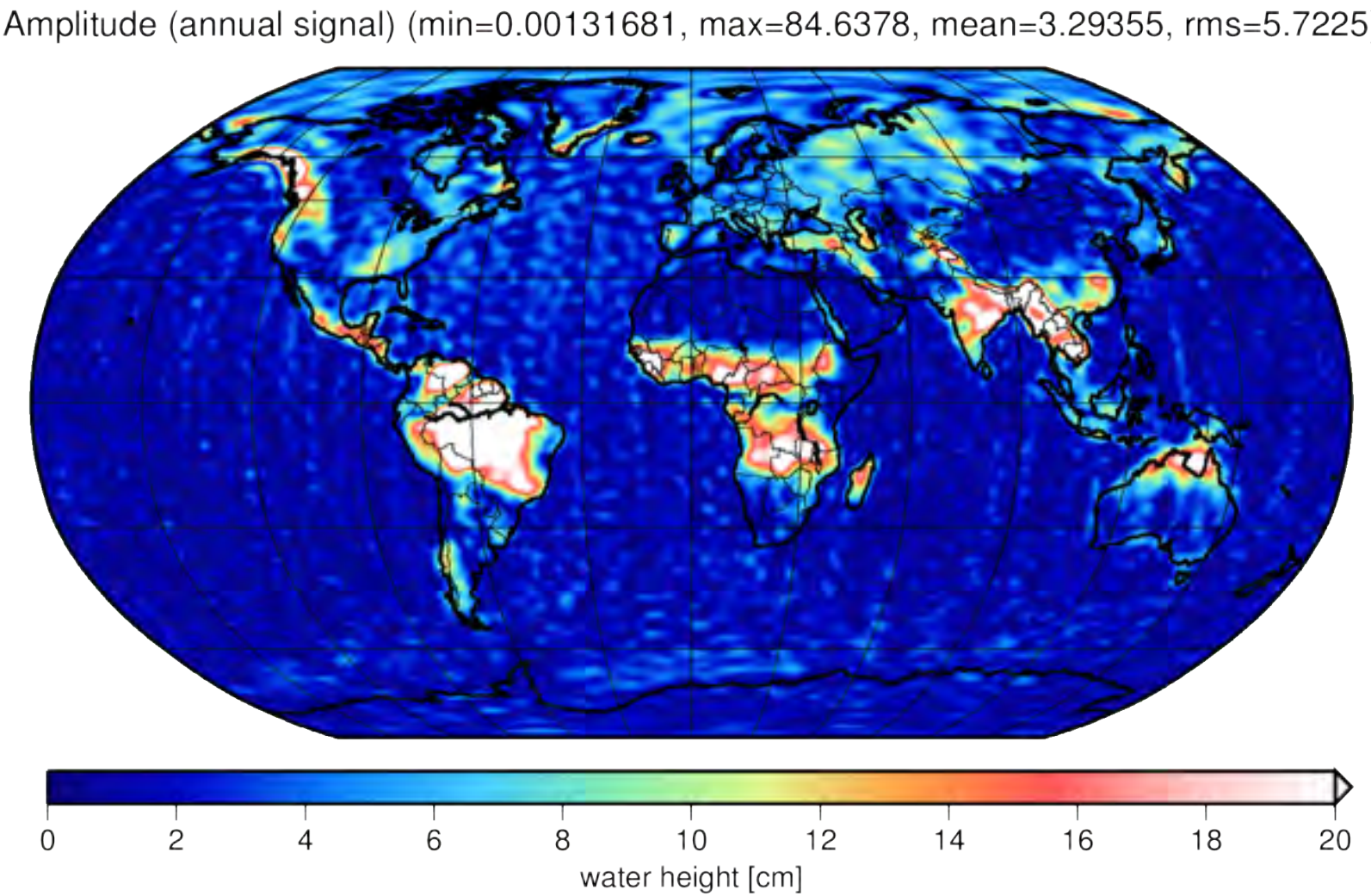

Conclusions

- Incorporation of short-term variations improves static solution significantly
- Combined estimation means high-resolution, long-term variations are directly accessible
- Method can easily be adapted to other spatial and temporal representations

- ITSG-Grace2014 is available at: `itsg.tugraz.at`

Thank you!

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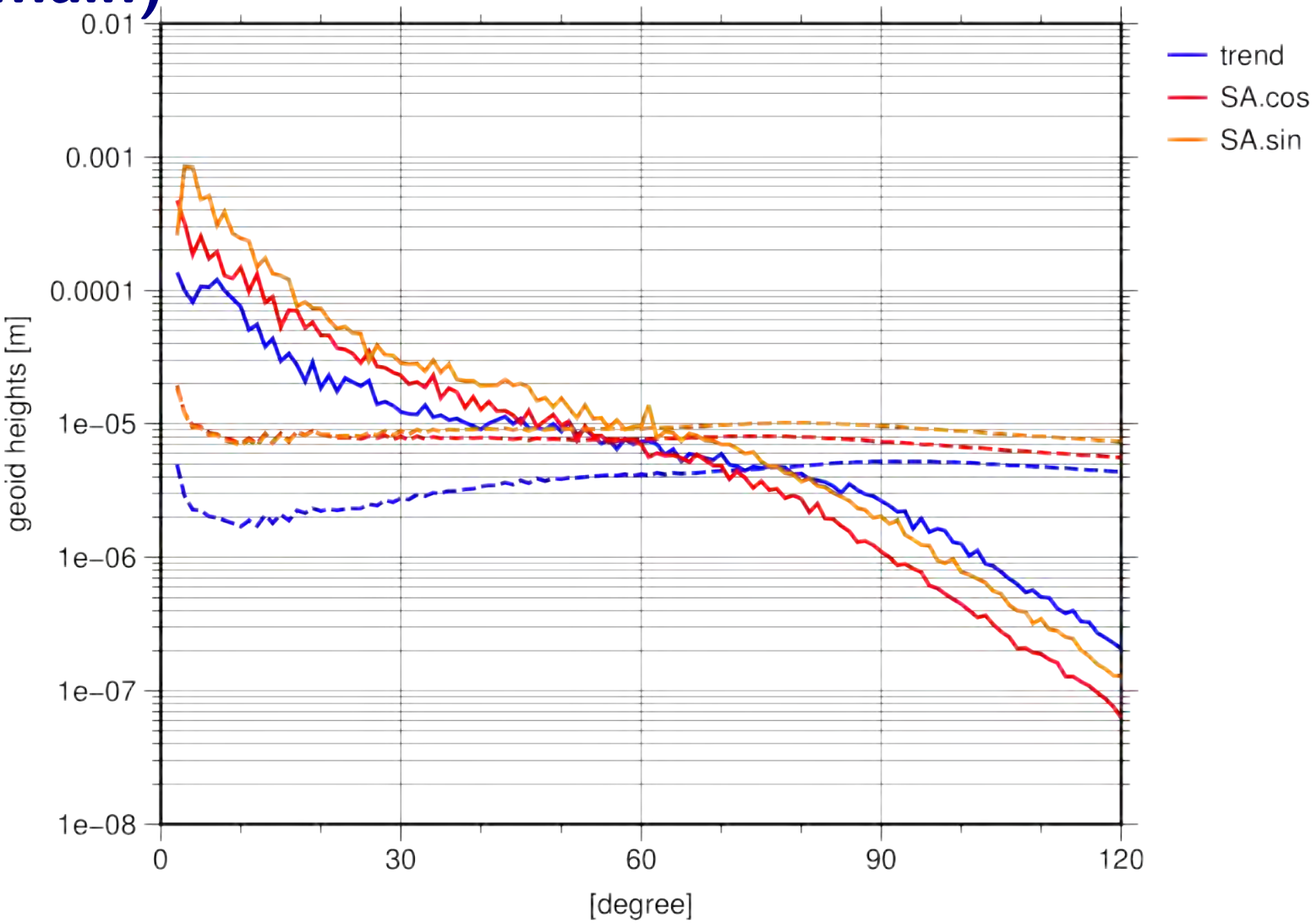
ITSG-Grace2014 - Long-term Variations (spectral domain)



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