Consistent combination of satellite and terrestrial gravity field observations in regional geoid modeling

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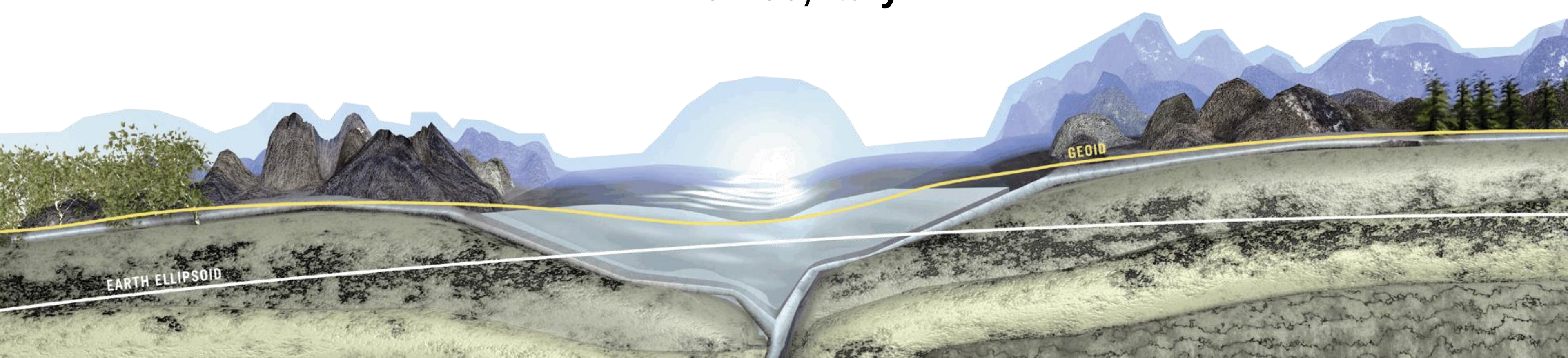
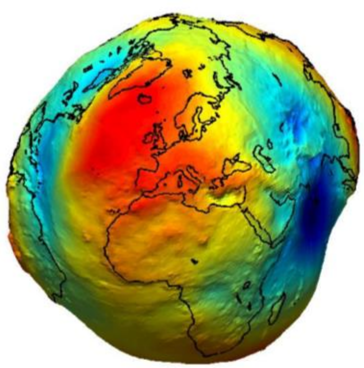


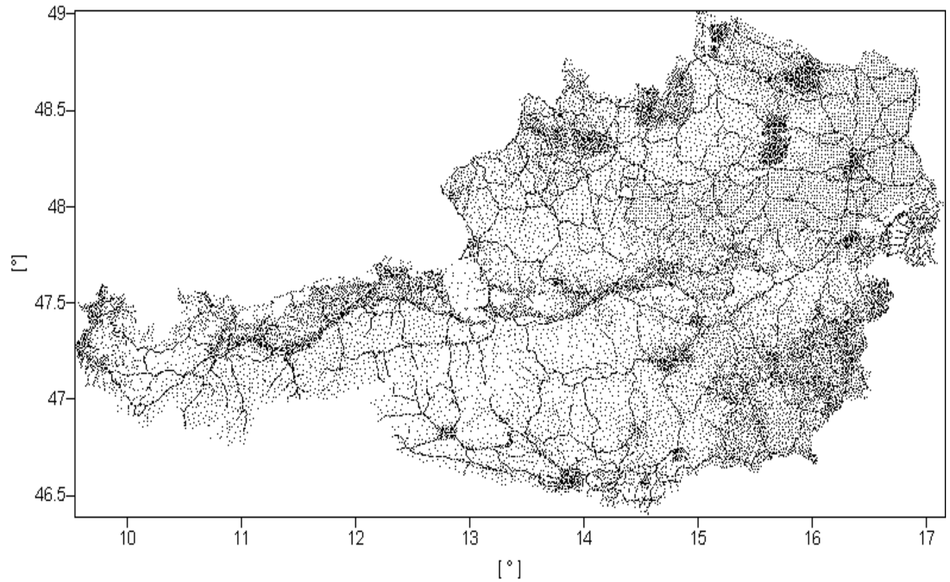
Introduction

- For regional geoid modeling the spatial resolution of satellite-only models is often insufficient and a higher resolution is required
- Local terrestrial data represent high-spectral components but lack from long- wavelength information
- Regional refinement of the global satellite models using Radial Basis Functions

Realization - Consistent Reduction

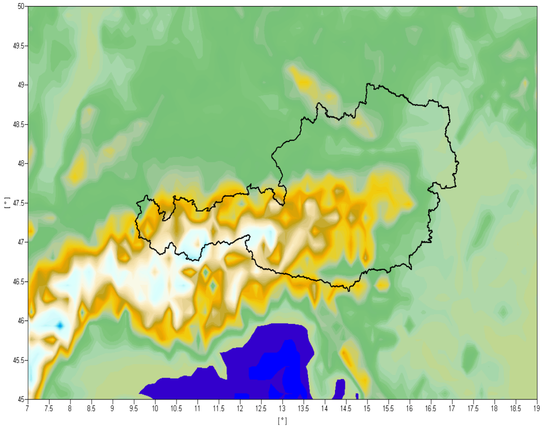
- Measured gravity has to be reduced
  - Satellite data (long-wavelength)
  - Topography (short-wavelength)
- Hence combining terrestrial gravity data with global satellite observations
  - In frame of Remove/Restore Technique

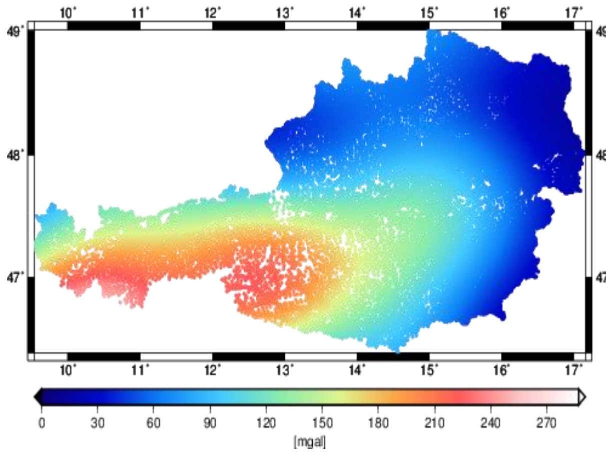




Realization - Consistent Reduction (1)

- Idea:
  - Gravitational influence of topography is affecting global satellite models
  - Avoid spectral overlap: $V(\lambda,\vartheta,r) = \sum_{n=0}^{250} Y_{nm}Global + \sum_{251}^{\infty} Y_{nm}DTM$
  - Therefore using potential coefficients derived from topography
  - Add gravity disturbances obtained from topography to Remove/Restore step



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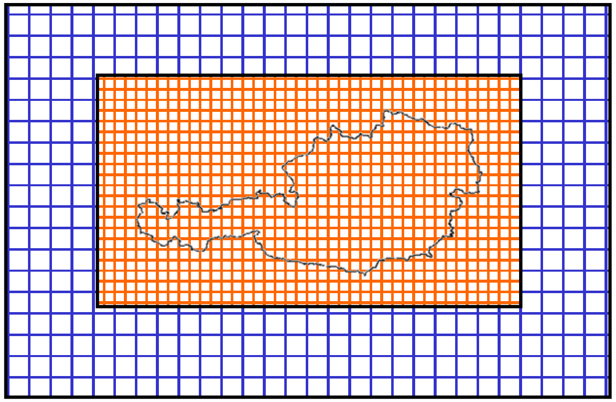
modelname                  GROOPS
product_type               gravity_field
earth_gravity_constant     3.9860044150e+14
radius                     6.3781363000e+06
max_degree                 250
norm                       fully_normalized
errors                     formal

key   L   M       C                  S
end_of_head ==========================================
gfc    0    0   8.657807121139e-08  0.000000000000e+00
gfc    1    0   3.660743917028e-08  0.000000000000e+00
gfc    1    1   3.308803634487e-08  7.276150209311e-09
gfc    2    0   1.186336263775e-08  0.000000000000e+00
gfc    2    1   3.250171035949e-08  7.151406659581e-09
gfc    2    2   1.396960929671e-08  6.450844188304e-09
gfc    3    0  -3.670191281414e-09  0.000000000000e+00
gfc    3    1   2.234023249920e-08  4.921918017090e-09
```

$$\begin{pmatrix} c_{nm} \\ s_{nm} \end{pmatrix} = \frac{1}{M(2n+1)} \iiint_{\Omega} \left(\frac{r'}{R}\right)^n \begin{pmatrix} C_{nm}(\theta',\lambda') \\ S_{nm}(\theta',\lambda') \end{pmatrix} \rho(r') d\Omega$$

Realization - Computation Parameter (2)

- Input data: ~30.000 gravity measurements
- Used global gravity field model: GOCO03S [Mayer-Gürr T., et al. (2012)]
  - D/O 250
- Topographic reduction is done using the well known prism formula
  - Standard crustal density of 2.670 kg/m^3
  - Without isostatic compensation
  - Coarse DTM: ~500 x 500m λ=5°- 22°, φ =43°- 53°

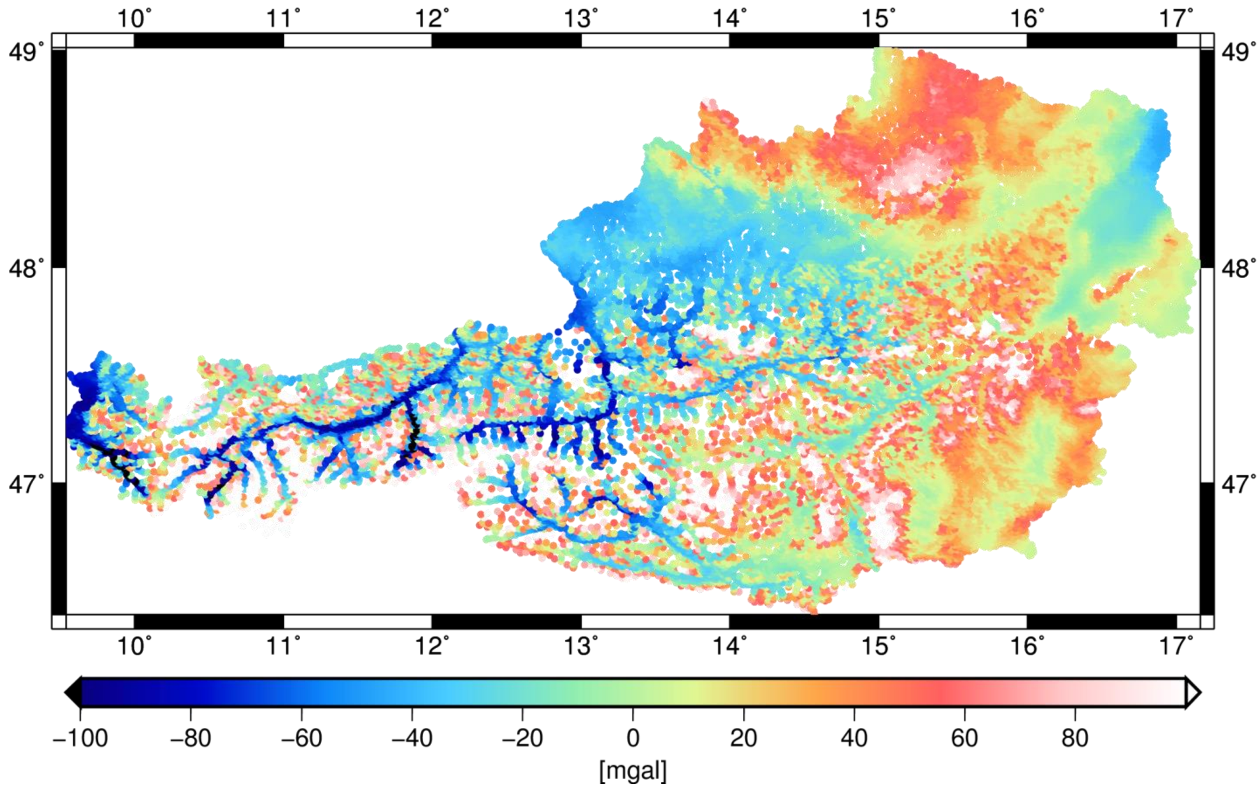


Dense DTM contains only height differences!

Realization - Consistent Reduction (3)

- Remove step:
  - $\triangle g_{red} = g_{abs} - \gamma - \triangle g_{global} - (\delta g_{Coarse} - \delta g_{CoarseDTM250}) - (\delta g_{Dense} - \delta g_{DenseDTM250})$

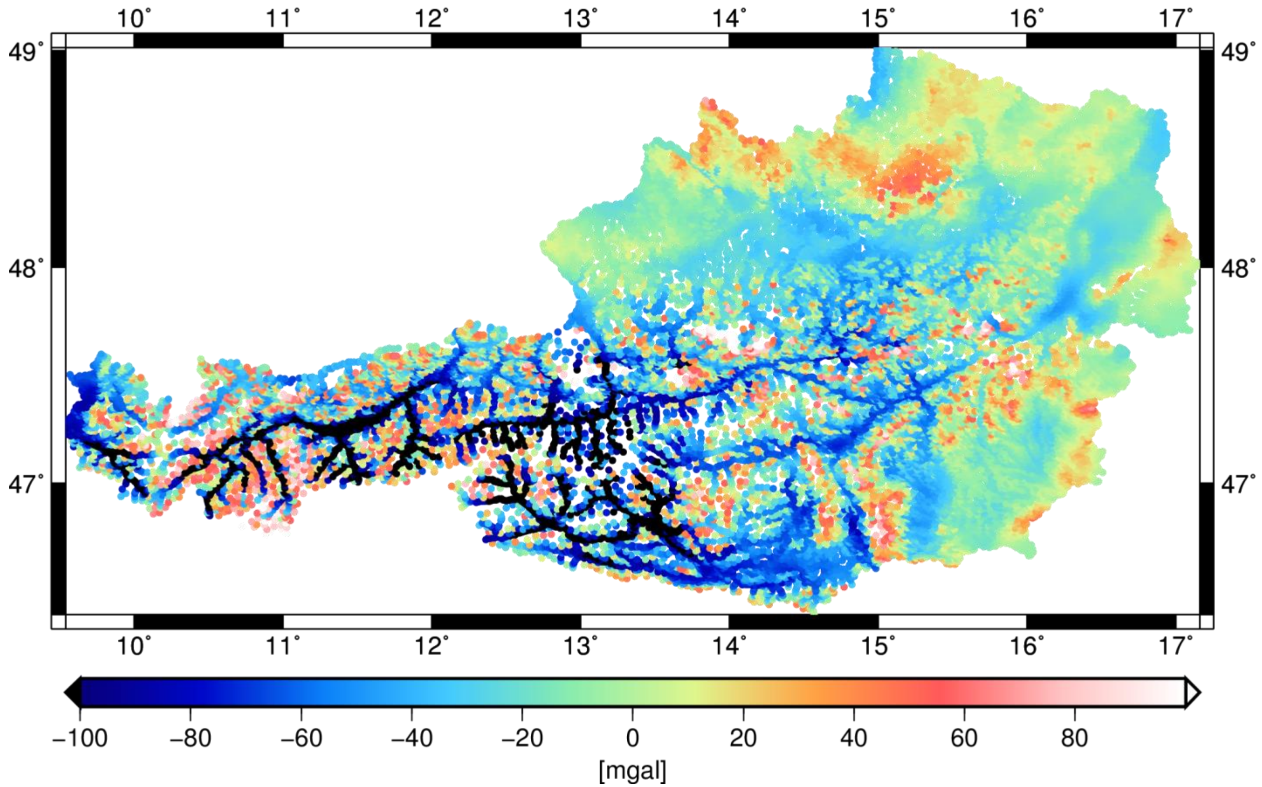
rms= 41.56 mgal



Realization - Consistent Reduction (4)

- Remove step:
  - $\triangle g_{red} = g_{abs} - \gamma - \triangle g_{global} - (\delta g_{Coarse} - \delta g_{CoarseDTM250}) - (\delta g_{Dense} - \delta g_{DenseDTM250})$

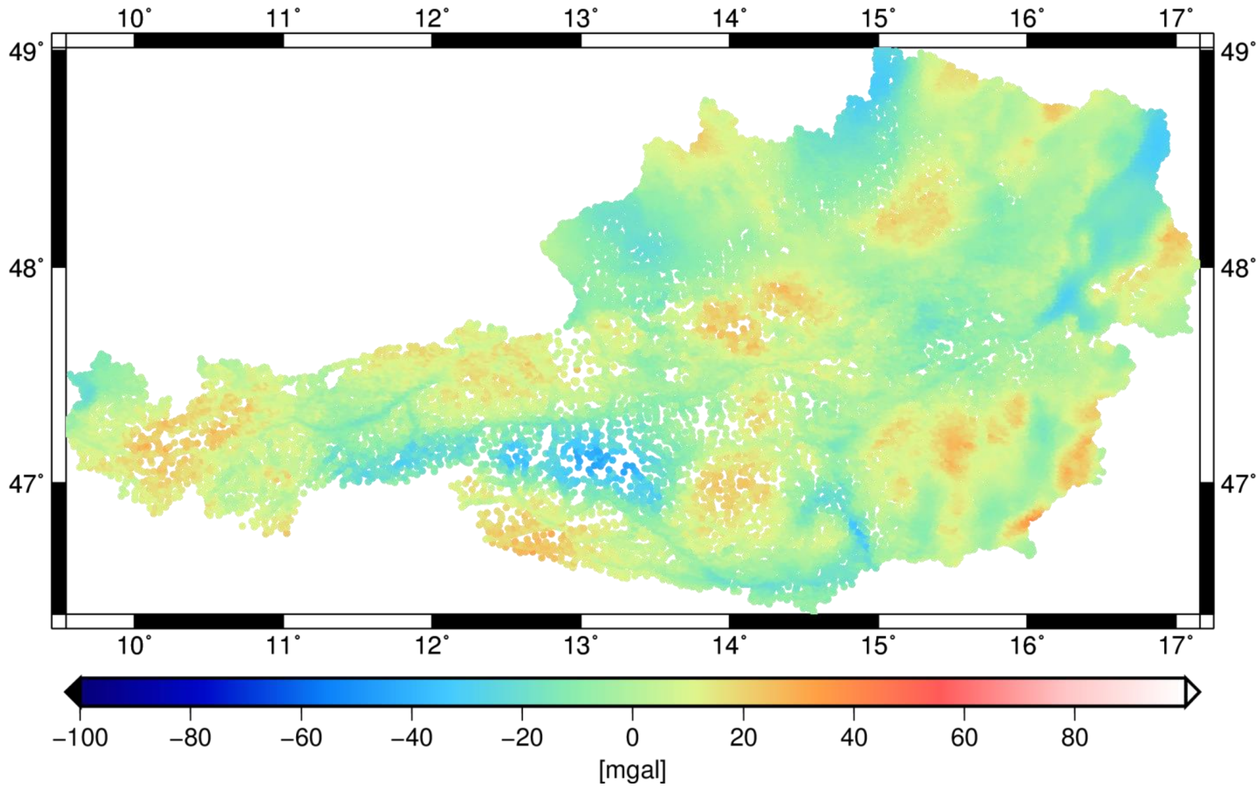
rms= 47.26 mgal



Realization - Consistent Reduction (5)

- Remove step:
  - $\triangle g_{red} = g_{abs} - \gamma - \triangle g_{global} - (\delta g_{Coarse} - \delta g_{CoarseDTM250}) - (\delta g_{Dense} - \delta g_{DenseDTM250})$

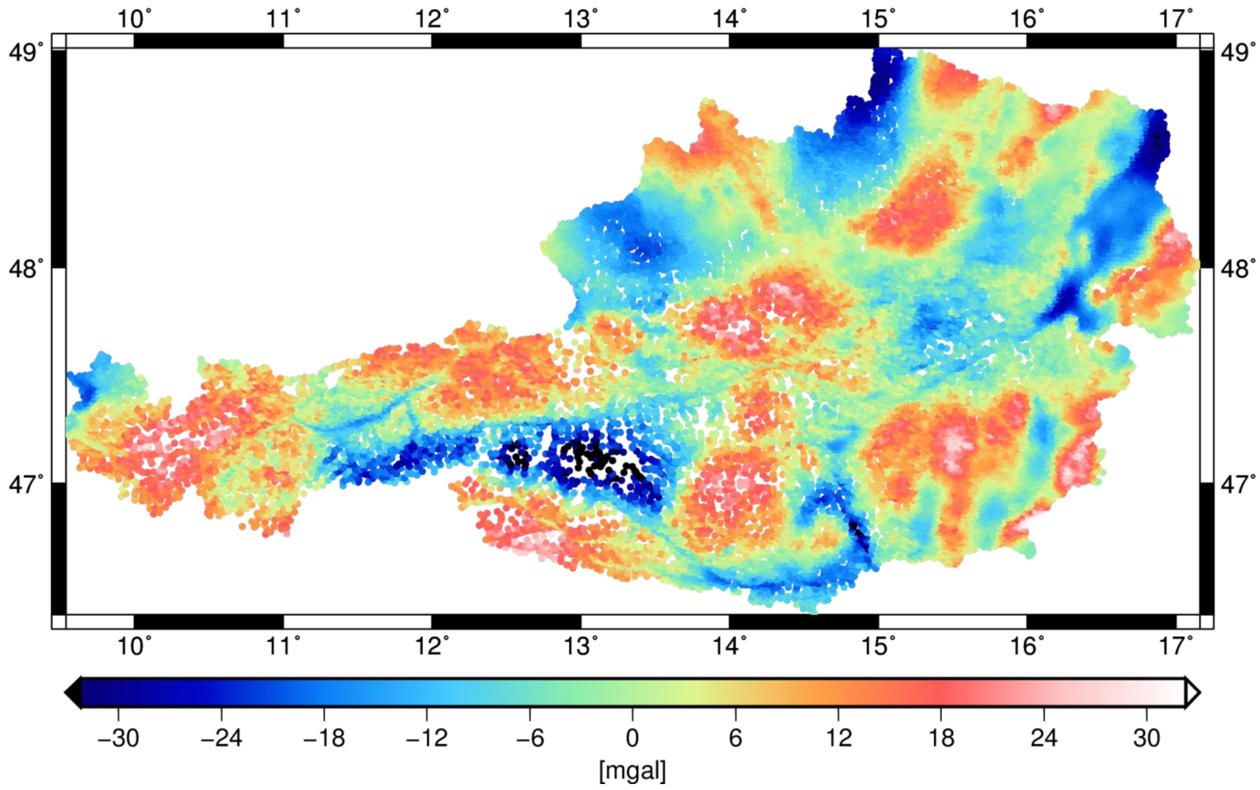
rms= 11.25 mgal



Realization - Consistent Reduction (6)

- Remove step:
  - $\triangle g_{red} = g_{abs} - \gamma - \triangle g_{global} - (\delta g_{Coarse} - \delta g_{CoarseDTM250}) - (\delta g_{Dense} - \delta g_{DenseDTM250})$

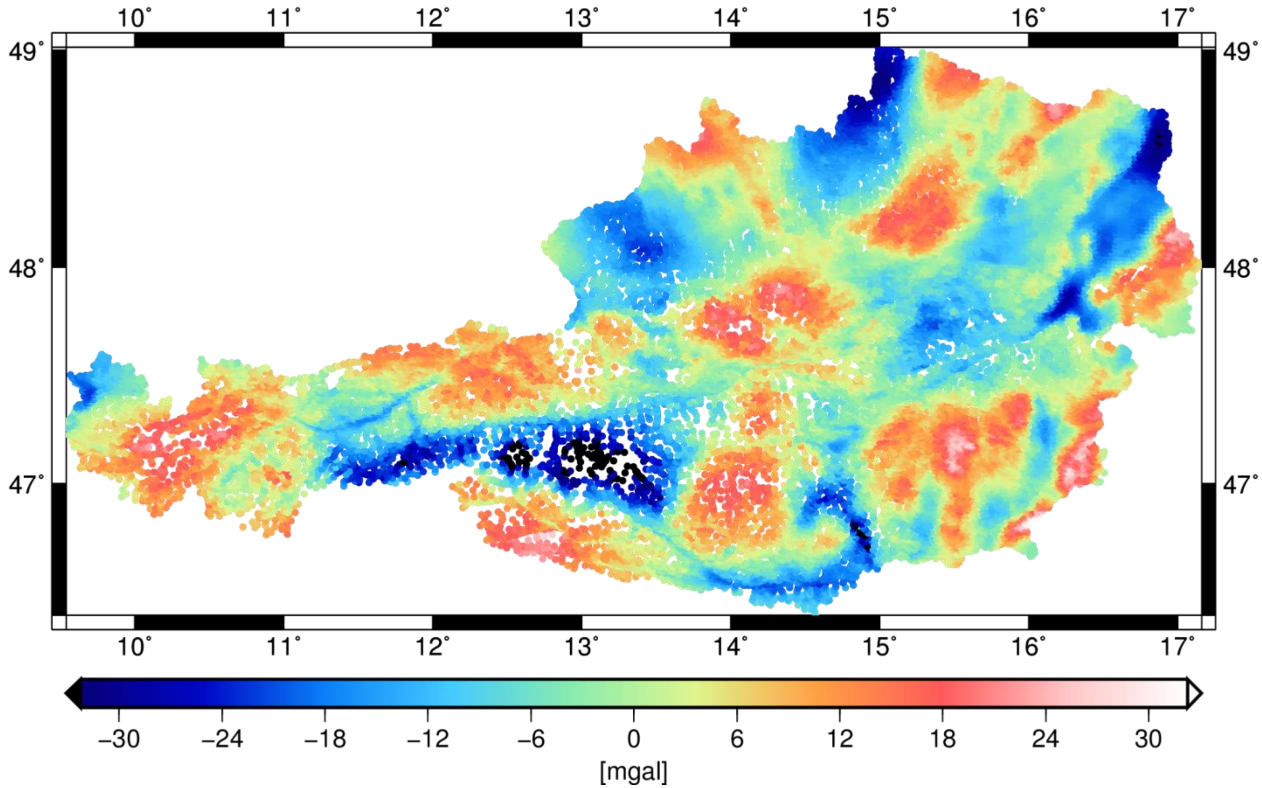
rms= 11.25 mgal



Realization - Consistent Reduction (7)

- Remove step:
  - $\triangle g_{red} = g_{abs} - \gamma - \triangle g_{global} - (\delta g_{Coarse} - \delta g_{CoarseDTM250}) - (\delta g_{Dense} - \delta g_{DenseDTM250})$

rms= 11.06 mgal



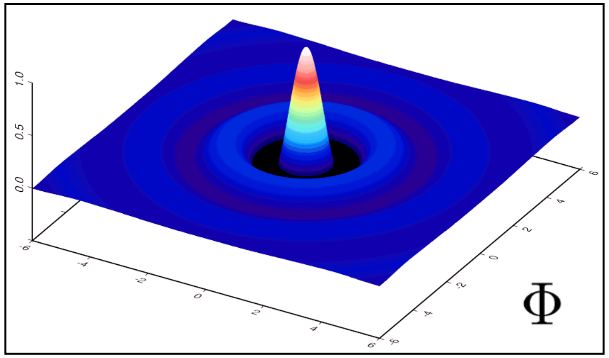
Radial Basis Functions (RBF)

- Used approach is based on [Eicker A. (2008)]

Signal:

$$s(x) = \sum_{i=1}^{N} a_i \Phi(x, x_i)$$

Unknown parameter a_i



RBF:

$$\Phi(\mathbf{x}, \mathbf{x}_i) = \sum_{n=2}^{\infty} \sum_{m=-n}^{n} k_n Y_{nm}(\mathbf{x}) Y_{nm}(\mathbf{x}_i)$$

Shape coefficients k_n
Spherical harmonics Y_{nm}
Grid points i

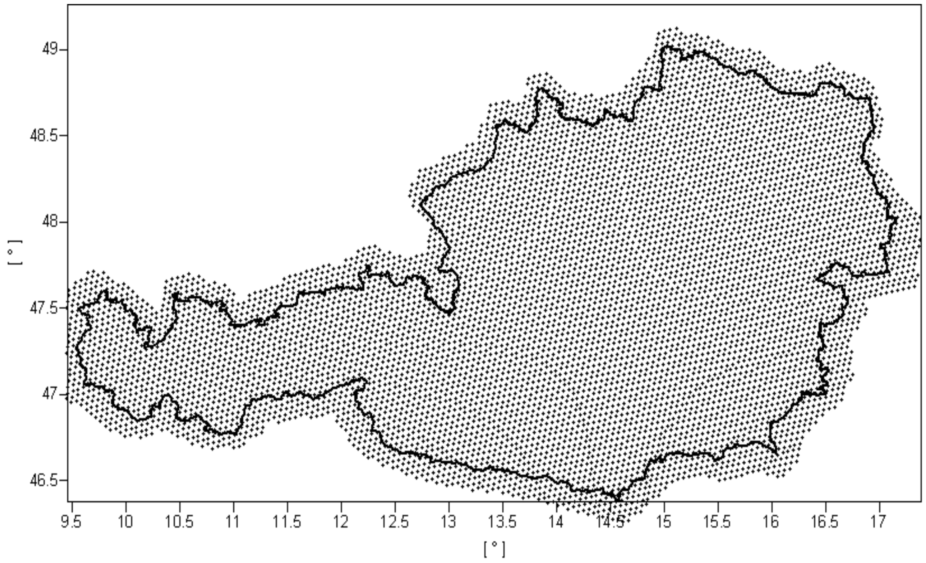
Shape coefficients:

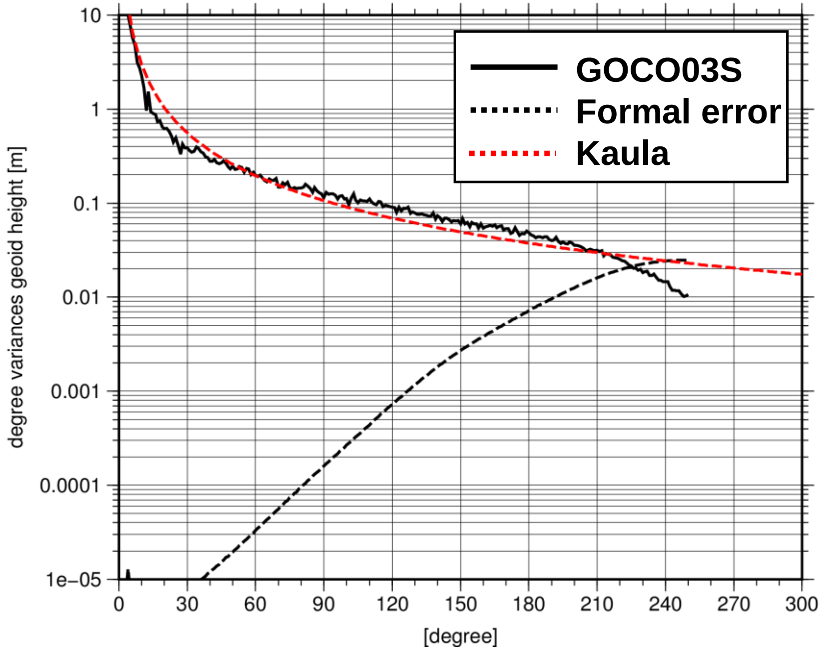
$$k_n = \frac{\sigma_n}{\sqrt{2n+1}} \sim \frac{1}{n^2}$$

Degree variances σ_n
Kaula's rule beyond max. D/O

Radial Basis Functions (RBF)

- Shape coefficients are obtained from GOCO03S satellite-only model
  - D/O 250: above padded by Kaula's rule
- RBF spatial distribution
  - Global spherical grid limited by Austrian borders: n=6000 ~ 3 x 3 km
- Estimation of the unknown parameter using a Gauß-Markov Model

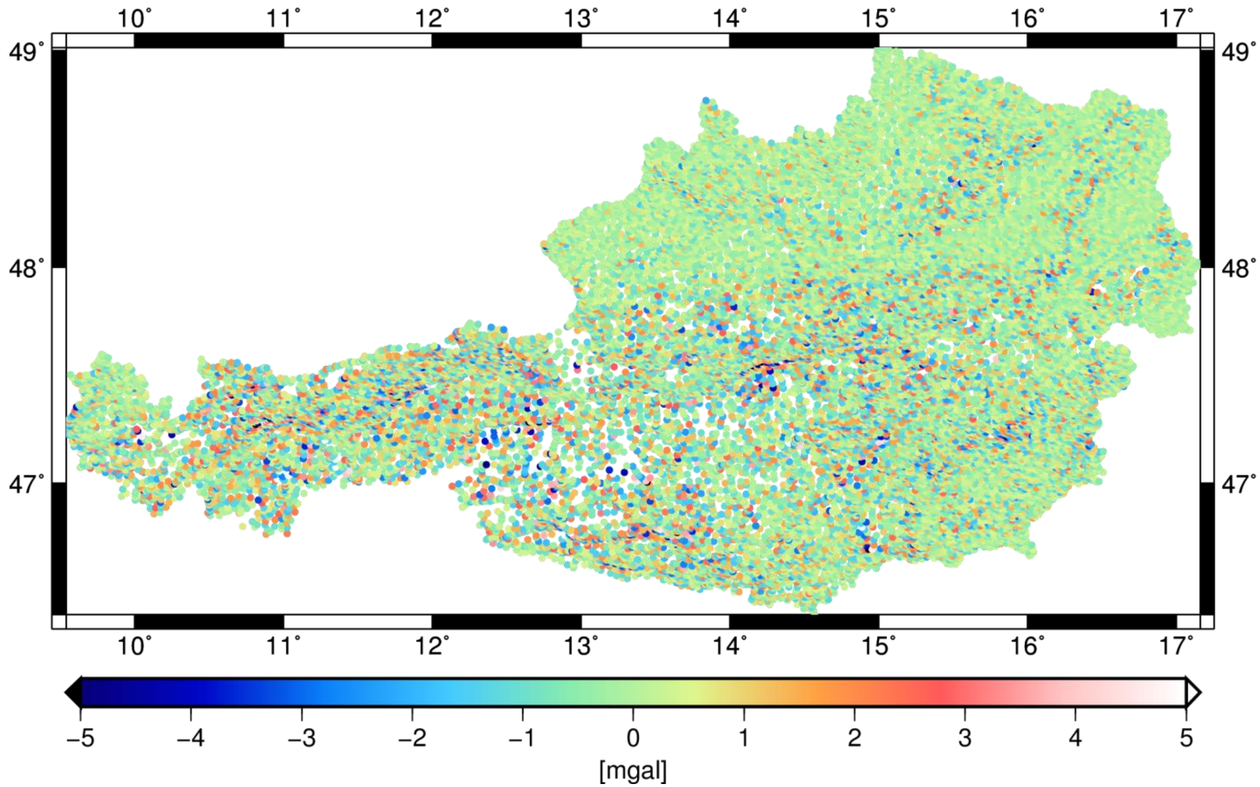




Radial Basis Functions - First results

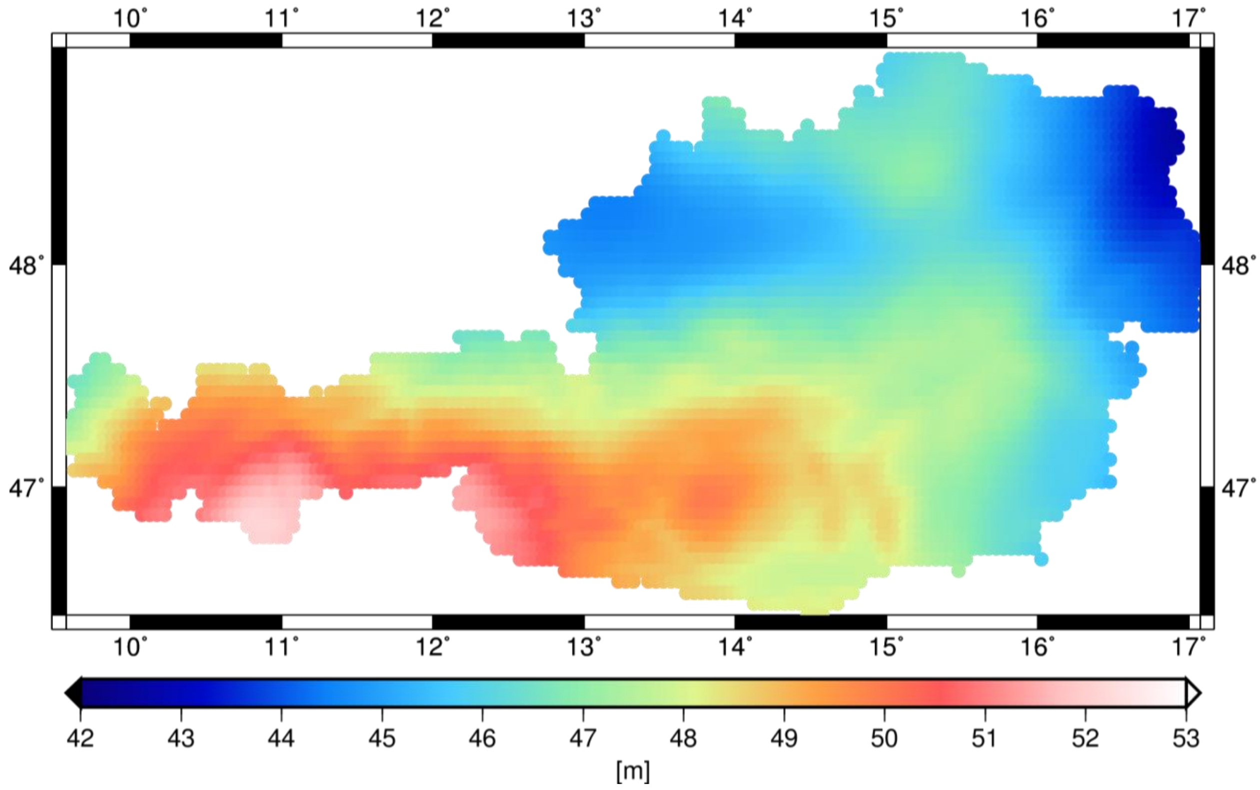
- Estimated gravity anomalies based on RBF parametrization
  - Residuals between reduced and estimated gravity anomalies $v = l - \hat{l}$

rms= 1.12 mgal



Radial Basis Functions - First results (1)

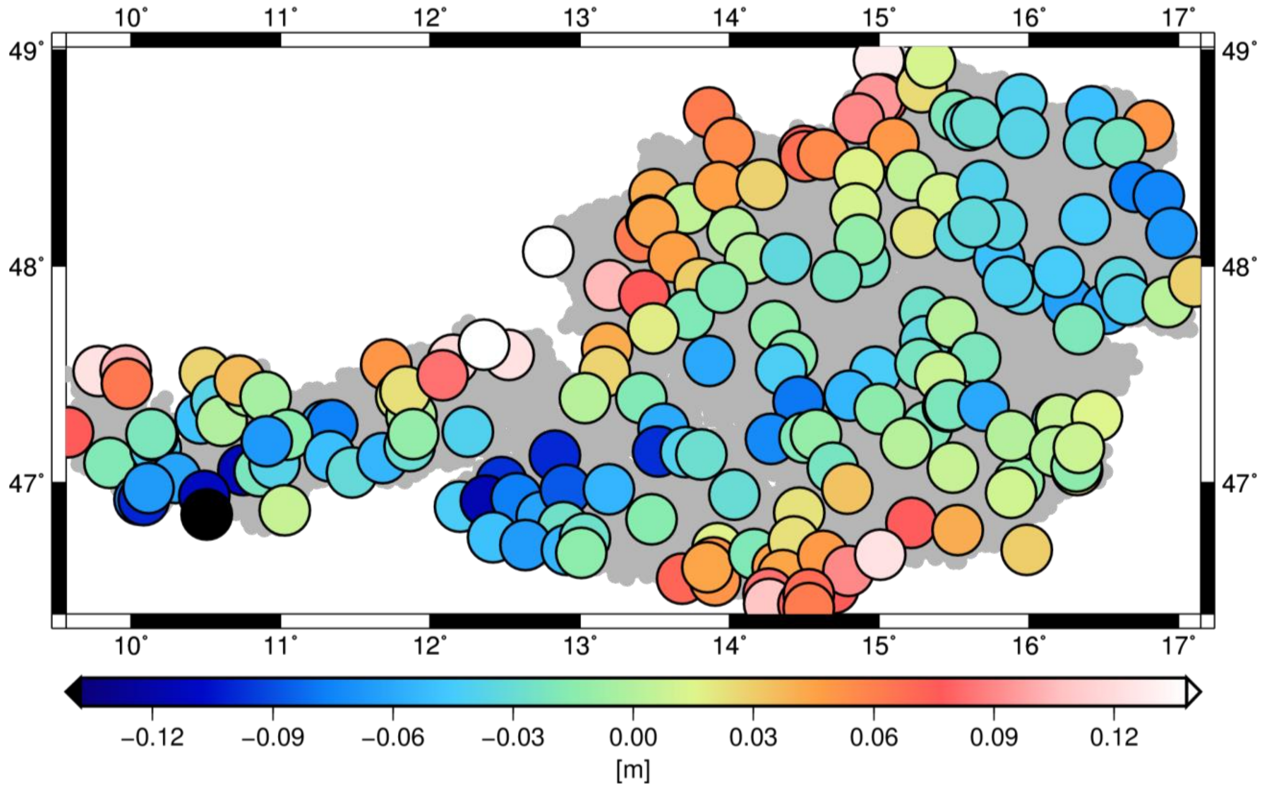
- Estimated gravimetric geoid based on RBF - full Restore step
  - Gravimetric geoid based on 4 x 4 km grid
  - Quality of solution?



Radial Basis Functions - First results (2)

- Estimated geoid heights based on RBF parametrization - full Restore step
  - 192 GPS/Leveling points are currently available to check the solution

rms= 5.8 cm



Summary & Outlook

- First results make us confident for further computation
- Full GOCO information is used in our approach
- Succeeding GOCO models will improve the solution
- Combined calculation with other gravity field quantities
    - Deflections of the vertical ξ, η
    - GPS/Leveling observations
- Global DTM model (DTM 2006) can contribute to an improvement

References

- Mayer-Gürr T., et al. (2012): The new combined satellite only model GOCO03S. Presentation at GGHS 2012, Venice, October 2012
- Pail R., et al. (2010): Combined satellite gravity field model GOCO01S derived from GOCE and GRACE, Geophys. Res. Lett., 37, doi:10.1029/2010GL044906
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