Evaluation of a momentum based impact model and application in an effectivity study considering junction accidents

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Abstract - The advent of active safety systems calls for the development of appropriate testing methods. These methods aim to assess the effectivity of active safety systems based on criteria such as their capability to avoid accidents or lower impact speeds and thus mitigate the injury severity. For prospective effectivity studies, simulation becomes an important tool that needs valid models not only to simulate driving dynamics and safety systems, but also to resolve the collision mechanics.

This paper presents an impact model which is based on solving momentum conservation equations and uses it in an effectivity study of a generic collision mitigation system in reconstructed real accidents at junctions. The model assumes an infinitely short crash duration and computes output parameters such as post-crash velocities, delta-v, force directions, etc. and is applicable for all impact collision configurations such as oblique, excentric collisions. Requiring only very little computational effort, the model is especially useful for effectivity studies where large numbers of simulations are necessary. Validation of the model is done by comparison with results from the widely used reconstruction software PC-Crash.

Vehicles involved in the accidents are virtually equipped with a collision mitigation system for junctions using the software X-RATE, and the simulations (referred to as system simulations) are started sufficiently early before the collision occurred. In order to assess the effectivity, the real accident (referred to as baseline) is compared with the system simulations by computing the reduction of the impact speeds and delta-v.

NOMENCLATURE

ADAS	Advanced Driver Assistance System
CMS	Collision Mitigation System
TTC	Time to Collision
X-RATE	Extended Effectiveness Rating of Advanced Driver Assistance Systems
CEDATU	Central Database for In-Depth Accident Study
GIDAS	German In-Depth Accident Study
POI	Point of impact
CP	Contact plane
VFS	Virtual forward simulation
NCAP	New Car Assessment Protocol

INTRODUCTION

In recent years, ADAS (Advanced Driver Assistance Systems) have increasingly become standard equipment [1–3] for newly introduced cars, aiming to assist in different areas of active vehicle safety. The inclusion of test protocols for ADAS in Euro NCAP [4] or other NCAPs is not only a symptom of this development but also drives the development of such safety systems even further. For each assistance system, there are many questions that have to be addressed, such as: Which type of accident can the system avoid, which not? Which specifications are necessary at minimum to achieve a certain effect? What other positive or negative effects are there?

In order to answer those questions, there are several possibilities:

- Natural driving studies
- Field operational tests
- Physical experiments
- Driving simulator studies
- Retrospective statistical analysis
- Virtual forward simulation

Each method has its advantages and disadvantages and addresses different aspects of effectivity. Especially the simulative approach allows to test a huge variety of scenarios that are very hard if not at all testable. One of the key elements is the correct assessment of crash severity. For that purpose, valid impact models to resolve the resulting collisions are necessary. This can be achieved for example by finite element methods, which are time consuming and therefore applicable only to limited extent for very high volumes of simulations, or alternatively by less detailed but fast models such as the impact model presented in this paper.

OBJECTIVE

The objective of this paper is to present an impact model based on momentum conservation equations and restitution principles and to investigate its applicability for effectivity studies of ADAS. Furthermore, the model's sensitivity in respect to the point of impact and contact plane is studied.

METHOD

Impact model – basic principles

The impact model under investigation is based on the solution of momentum conservation equations balancing the total momentum of the vehicles before and after the crash. It can also be found Burg and Moser [5], Gilardi and Sharf [6] or Steffan and Moser [7], and is based on the works of Kudlich and Slibar [8,9].

For this impact model, the following assumptions are made:

- The model only considers the contact forces that are being exchanged between the two vehicles.
- Tire forces are being neglected.
- The crash duration is assumed to be infinitely small, which is why no position changes and acceleration pulses are resolved during the crash.
- No deformations are computed.
- The model only considers two spatial dimensions.

Conservation of momentum

Momentum conservation basically states that in a closed system, the total momentum remains constant if no external forces act on the system [10]. This principle is of fundamental importance for collision mechanics. The total pulse $\vec{p}$ of a system constituted of n interacting bodies, is the sum of the pulses of the individual bodies:

$$\vec{p} = \overrightarrow{p_1} + \overrightarrow{p_2} + \cdots + \overrightarrow{p_n}. \tag{1}$$

A conclusion of this is, that after a collision of two bodies in the system, the total momentum must still be the same. Analogously, this holds for the angular momentum as well:

$$\vec{L} = \overrightarrow{L_1} + \overrightarrow{L_2} + \cdots + \overrightarrow{L_n}. \tag{2}$$

Restitution and compression

In general, the collision phase can be separated into two phases: The compression and the restitution phase. During the compression phase, the kinetic energy of the colliding vehicles is transformed into deformation of the vehicles, until the relative velocity of the contact areas in normal direction is lowered to zero. Like with most other physical bodies, some of the relative kinetic energy is retained after a collision, which is called restitution. The restitution coefficient ϵ describes the ratio between the absolute of the exchanged momentum of both phases:

$$\epsilon = \frac{\vec{S}_{rest}}{\vec{S}_{komp}} \tag{3}$$

The total exchanged momentum amounts to:

$$\vec{S} = \vec{S}_{rest} + \vec{S}_{komp} = (1+\epsilon) * \vec{S}_{komp} \tag{4}$$

Momentum based impact model

For computation of the total exchanged momentum and the resulting postcrash parameters such as velocities and angular rates, the momentum conservation equations are used. In a first step, a new coordinate system is introduced, with its origin positioned in the point of impact. The point of impact represents the point at which the momentum is exchanged between the two colliding vehicles. Furthermore, a contact plane needs to be defined. For sliding impacts, the contact plane represents the plane along which the relative movement of vehicles occurs, while for non-sliding impacts, the orientation of the contact plane only affects intermediate results but not the end results such as delta-v or principal direction of force. The first axis of the new coordinate is tangential to the contact plane and the second axis is normal to it, see Figure 1 from Burg and Moser [5]. The lengths n_1, n_2, t_1 and t_2 represent the coordinates of the vehicles' centers of gravity in the new coordinate system.

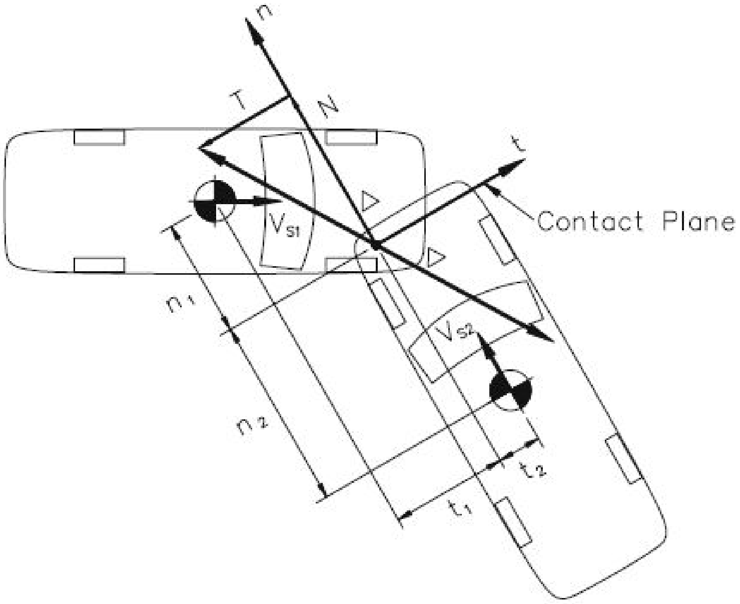

Figure 1. Coordinate system in the point of impact

The velocities of the centers of gravity v_{cog1} and v_{cog2} are transformed to the new coordinate system (here for vehicle 1, for vehicle 2 they are transformed analogously) by

$$v_{Tcog1} = v_{cog1} \cdot \begin{pmatrix} \cos(\phi_1) \\ \sin(\phi_1) \end{pmatrix} \cdot \vec{t}, \tag{5}$$

$$v_{Ncog1} = v_{cog1} \cdot \begin{pmatrix} -\sin(\phi_1) \\ \cos(\phi_1) \end{pmatrix} \cdot \vec{n}, \tag{6}$$

where ϕ_1 is the angle of the velocity direction. They are then projected to velocities in the point of impact (with ω_1 being the yaw-rate):

$$v_{T1} = v_{Tcog1} + \omega_1 n_1 \tag{7}$$

$$v_{N1} = v_{Ncog1} - \omega_1 t_1. \tag{8}$$

These velocities can be used to formulate the momentum conservation equations for the first vehicle

$$m_1\left(v'_{Tcog1} - v_{Tcog1}\right) = T, \tag{9}$$
$$m_1\left(v'_{Ncog1} - v_{Ncog1}\right) = N, \tag{10}$$

and the second vehicle,

$$m_2\left(v'_{Tcog2} - v_{Tcog2}\right) = -T, \tag{11}$$
$$m_2\left(v'_{Ncog2} - v_{Ncog2}\right) = -N, \tag{12}$$

where $v'_{Tcog1}$and v'_{Ncog1} denote the postcrash-velocities in tangential and normal direction for the first vehicle, v'_{Tcog2} and v'_{Ncog2} analogously for the second vehicle. Furthermore, assuming that the angular momenta I_1 and I_2 are known, the angular momentum balance equations read (with ω' being the postcrash yaw rate):

$$I_1(\omega'_1 - \omega_1) = T \cdot n_1 - N \cdot t_1, \tag{13}$$
$$I_2(\omega'_2 - \omega_2) = -T \cdot n_2 + N \cdot t_2. \tag{14}$$

By definition of the relative precrash velocities $v_T = v_{T1} - v_{T2}$ and $v_N = v_{N1} - v_{N2}$, the balance equations (9)-(14) can be rewritten into the following expression for the relative postcrash velocities:

$$v'_T = v_T + c_1 T - c_3 N, \tag{15}$$
$$v'_N = v_N - c_3 T + c_2 N, \tag{16}$$

with

$$c_1 = \frac{1}{m_1} + \frac{1}{m_2} + \frac{n_1^2}{I_1} + \frac{n_2^2}{I_2}, \tag{17}$$
$$c_2 = \frac{1}{m_1} + \frac{1}{m_2} + \frac{t_1^2}{I_1} + \frac{t_2^2}{I_2}, \tag{18}$$
$$c_3 = \frac{t_1 n_1}{I_1} + \frac{t_2 n_2}{I_2}. \tag{19}$$

Full impact

In case the collision is not a sliding collision, the relative velocities at the end of the compression phase (v'_T and v'_N) are zero. By rearranging the terms in (15)-(16), expressions for the crash pulse in the compression phase can be derived that only contain known values from the precrash phase:

$$T_C = \frac{v_N c_3 + v_T c_2}{c_3^2 - c_1 c_2}, \tag{20}$$
$$N_C = \frac{v_N c_1 + v_T c_3}{c_3^2 - c_1 c_2}. \tag{21}$$

The total exchanged momentum

$$T = T_C(1 + \epsilon), \tag{22}$$
$$N = N_C(1 + \epsilon), \tag{23}$$

can be used to compute the postcrash velocities of the center of gravity and the postcrash yaw velocities using the conservation equations (9)-(12).

Sliding collision

For sliding impacts, no common velocity at the contact zones is reached, i.e. the relative tangential velocity is not zero. In this case, the contact plane becomes relevant and an inter-vehicle friction coefficient μ is required to model the impact. The direction of momentum transfer is limited by this coefficient, such that:

$$T_C = \mu N_C, \tag{24}$$

which replaces equation (15). Rearranging equation (16) results to

$$N_C = \frac{v_N}{\mu c_3 + c_2}. \tag{25}$$

As long as $T_C \leq \mu N_C$ holds, there is no sliding involved and the pulse is given by equations (20)-(23). If equations (20) and (21) yield values that satisfy $T_C > \mu N_C$, instead equations (24) and (25) need to be used to define the crash pulse. The total momentum is still given by (22) and (23).

Definition of the point of impact by geometrical rules

1. One possibility to define the point of impact (POI) is to use a simple geometrical approach which uses the centroid of overlap of the vehicle outlines. For that purpose, the following steps are carried out, a graphical explanation is given by Figure 3. Each vehicle is represented by a rectangle, with their corners defined in the local vehicle coordinate system as shown in Figure 2. The corners 1 and 2 have a distance to the center of gravity of d in longitudinal direction and corners 3 and 4 have a distance of $Length - d$. Each corner has a lateral distance of $Width/2$ to the center of gravity. The coordinates of the corners are then transformed into the global coordinate system of the simulation.
2. *Calculation of the polygonal line overlap*. After initial contact, the simulation is continued without impact calculation for a certain amount of time (also called penetration depth). The result of this is that the rectangles representing the vehicles will overlap each other, see Figure 3. The overlap is calculated based on algorithm of Sutherland and Hodgman [11] which will then be given by a polygon. The corners of this remaining polygon are either corners of one of the colliding vehicles' exterior or the intersection of two edges of the vehicles' exterior.

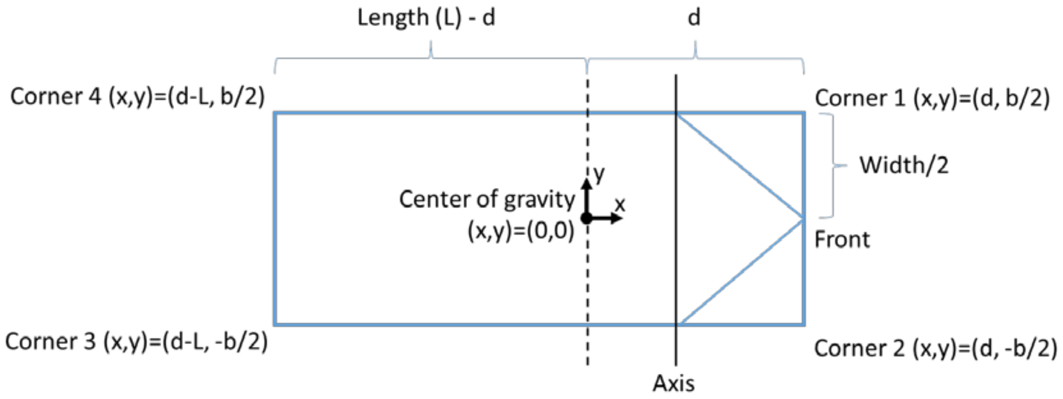

Figure 2. Definition of vehicle outline polygon.

3. *Calculation of the point of impact as the centroid of the overlap*. Here, the equations

$$C_x = \frac{1}{6A}\sum_{i=0}^{n-1}(x_i + x_{i+1})(x_i y_{i+1} - x_{i+1} y_i), \tag{26}$$

$$C_y = \frac{1}{6A}\sum_{i=0}^{n-1}(y_i + y_{i+1})(x_i y_{i+1} - x_{i+1} y_i), \tag{27}$$

$$A = \frac{1}{2}\sum_{i=0}^{n-1}(x_i y_{i+1} - x_{i+1} y_i) \tag{28}$$

are used which are based on [12]. C_x and C_y are the coordinates of the centroid and x_i and y_i the coordinates of the nodes of the polygon from step 2.

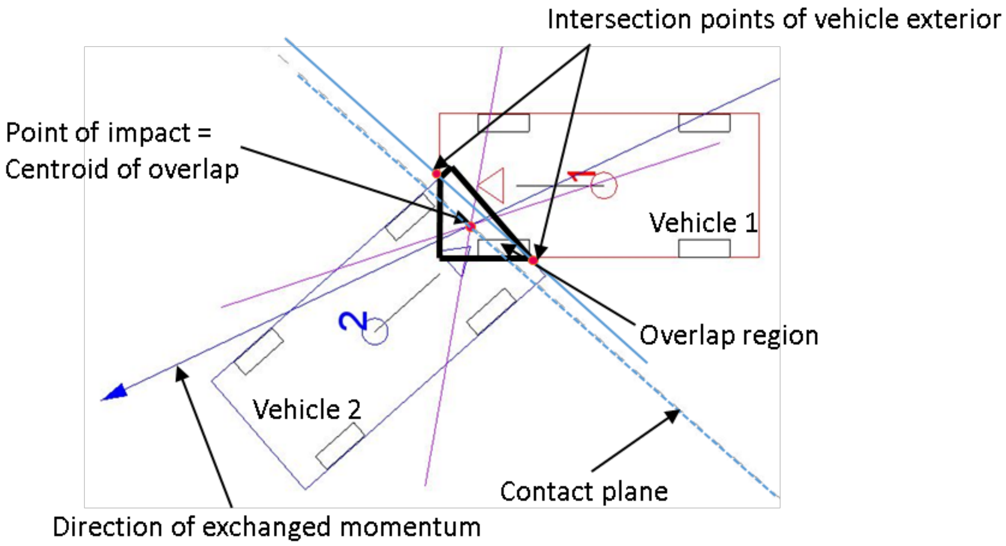

Figure 3: Determination of the point of impact and contact plane (dashed line). Source: own picture.

Definition of the contact plane by geometrical rules

For the definition of the contact plane (CP), a line can be interpolated through the intersection points between the polygonal lines that represent the vehicles' exterior and is translated in parallel such that it goes through the point of impact, i.e. the dashed blue line in Figure 3.

In the further content, these methods to define the POI and CP are simply referred to as the "geometrical rule".

Simulation toolbox: X-RATE

X-RATE (Extended Effectiveness Rating of Advanced Driver Assistance Systems) is a tool to investigate for example the effectivity of Advanced Driver Assistance Systems. For that purpose, it can be seen as an extended control platform that is able to automatically set up and run simulations in a driving dynamics simulation software (currently supported: PC-Crash). X-RATE interacts with a simulation software on a time-step basis, thus providing the possibility to simulate sensor detection including sight obstruction and different active safety strategies. Various parameters can be changed automatically according to user defined values or following a random distribution, such as initial velocities of vehicles, start positions, driver assistance system response times, sensor range and many more. Crash related parameters such as collision speeds or delta-V are gathered for further evaluation. X-RATE is being developed by TU Graz, Vehicle Safety Institute on the basis of Matlab and has been used for the evaluation of L6e or L7e vehicles by Kolk et al. [13] and Tomasch et al. [14] and pedestrian collision avoidance systems by Tomasch et al. [15].

Validation of the impact model using PC-Crash

The impact model in this study is validated to results computed by PC-Crash for the same accidents and simulation setups. PC-Crash is chosen as a reference due to its huge acceptance as an accident reconstruction software. The accuracy of PC-Crash in reproducing real accidents in simulations was examined by Cliff and Moser [16].

In order to validate the presented input model, the following steps are taken:

1. Reconstruction of a selection of junction accidents (the accidents considered in this paper were already reconstructed and available in the CEDATU).
2. Computation of crash related parameters such as delta-V, postcrash-velocity, etc. with PC-Crash for each accident.
3. Computation of the same parameters using the model presented in this study for each accident.
4. Evaluation of calculation results.

Two different methods for the POI and CP definition were used:

- Manual definition of POI and CP during the process of reconstruction. Simulations where this rule was used are called “reference simulations”.
- Automatic definition of POI and CP through the geometrical rule. These simulations are called “test simulations”.

Differences between the two different methods were examined to assess the model’s sensitivity on the parameters POI and CP.

Selection of reconstructed real accidents

The accidents used for the study were selected randomly from the CEDATU. Only junction accidents with two involved vehicles were used, and the probability for each junction accident to be chosen was equal.

Effectivity study

Virtual forward simulation

For the effectivity study, real accidents (the same accidents that were used in the validation study) are simulated starting in the precrash-phase while simulating a generic virtual ADAS, thus employing virtual forward simulation (VSF) as method to analyze the effectivity. VSF is one of several possibilities to examine the effectivity of ADAS. Other possibilities refer to natural driving studies, field operational tests or physical experiments. VFS has been used before for similar research questions, where other ADAS [17,18] or C2X systems [19] were investigated and was also used in the research project SafeEV (Safe Small Electric Vehicles through Advanced Simulation Methodologies [20]). Some of the advantages over the other effectivity assessment methods are that different system configurations and many different traffic situations can be analyzed in a relatively short time.

The simulations are started before the actual collision, such that the precrash-phase is included. The timeframe before the collision is chosen individually long enough such that the earliest possible activation point of the ADAS is included. For junction accidents, it suffices to start the simulation a couple seconds before the collision.

In the forward simulation, one of the accident vehicles was virtually equipped with a CMS (collision mitigation system) coupling the inhouse-tool X-RATE (simulation of the CMS) with PC-Crash (driving dynamics). In a repeated simulation, the other vehicle was equipped with a CMS, which yields in total two new situations for each real accident. Simulation of the original accident is referred to as baseline simulation, while the simulations including the CMS are referred to as system simulations.

The resulting collisions are then analyzed regarding impact parameters such as delta-v. The insights from the validation study are used to put the results of the effectivity study into perspective. In order to assess the effectivity of the CMS, the baseline simulations are compared to the system simulations.

Sensor representation

The simulation of the sensor is based on a simple geometrical consideration. Originating from the sensor, so-called “vision rays” were considered, which could represent e.g. the laser rays of a LiDAR-sensor. They are emitted horizontally in equal angular distances. The angular distances are chosen in such a way, that the whole range of the horizontal opening angle is covered for a defined number of rays. When a ray intersects an object, the object is only labeled by the algorithm as detected if the distance of the intersection point to the sensor is less than a defined sensor range. Only the closest intersecting object is detected by a ray, thus modelling sight occlusion, see Figure 4 [13].

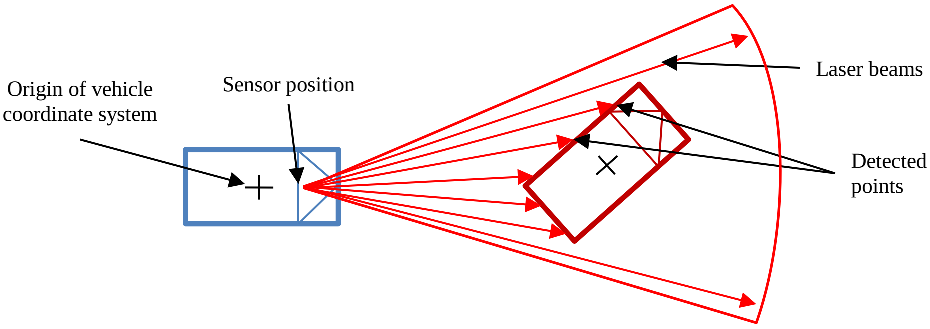

Figure 4. Top view of the geometrical sensor vision algorithm.

Due to the similarity of the geometrical sensor model to the LiDAR operation principle, the specifications for range (150m) and horizontal opening angle (145°) from the LiDAR sensor “ScaLa” by Ibeo were used as an example sensor [21].

ADAS simulation

For the effectivity study, a virtual CMS is considered that is able to estimate whether another object (vehicle) is on collision course with the ego vehicle under the assumption of constant velocity and velocity direction of both vehicles. A braking maneuver is triggered as soon as the time to collision is lower than 1.8s. The brake deceleration was defined as $8 m/s^2$ and a system latency time was taken into account as well as a lag time (time to full activation of brakes). This represents one of the strategies presented in Zauner et al. [22] and there is no reason why this specific strategy was chosen – for this paper, it serves just as an example strategy while the focus lies on the impact model. The values do not represent an existing system.

MATERIAL

The in-depth database CEDATU (Central Database for In-Depth Accident Study) was the source for the real accidents which were then analyzed [23,24]. Each individual traffic accident was reconstructed using the traffic accident reconstruction program PC-Crash and saved on CEDATU with all accident related data. Information on initial speed, collision speed, overlap, etc. were calculated on the basis of accident reports which consist of reports such as police reports and medical reports, attached photos and photogrammetric analyses of the accident site.

The data field basis of CEDATU is the STAIRS protocol (Standardization of Accident and Injury Registration System [25]) which was developed over the course of an EU project with the same name. Building on the STAIRS protocol, data fields were developed using information from the EU projects PENDANT (Pan-European Coordinated Accident and Injury Databases [26]), RISER (Roadside Infrastructure for Safer European Roads [27]) and ROLLOVER (Improvement of rollover safety for passenger vehicles [28]). The data fields from national statistics are considered to enable a direct connection to the latter [29]. Furthermore, the data fields of CEDATU correspond to the IGLAD database [30].

LIMITATIONS

The following limitations are valid for this study:

- The model only considers the contact forces that are being exchanged between the two vehicles.
- Tire forces are being neglected.
- The crash duration is assumed to be infinitely small, which is why no position changes and acceleration pulses are resolved during the crash.
- The crash force is not defined and is infinitely large due to the infinitely small crash duration.
- No deformations are computed.
- The model only considers two spatial dimensions.
- No occupant loads, rear cargo or roof cargo were considered.

Particular limitations for the effectivity study:

- Penetration depth is fixed to 30 ms.
- For the inter-vehicle friction coefficient, the value from the original reconstructed accident was used.
- For the point of impact and contact plane, the methods from the sections “Definition of the point of impact” and “Definition of the contact plane” were used. These methods still need to be investigated on how suitable they are for effectivity studies.

RESULTS

In total, 36 reconstructed real accidents at junctions were considered. They included full impacts (non sliding) as well as sliding impacts. For some of the accidents, originally the geometrical rule was used for the reconstruction, for some the POI and contact plane were defined manually. Furthermore, in some cases the POI was defined manually outside of the vehicle overlap in collision position. Detailed numbers of investigated accidents can be found in Table 1.

Number of accidents	***Full impact***	***Sliding impact***	***Sum***
Geometric rule	**9**	**7**	**15**
Manual POI	**11**	**5**	**17**
Sum	**20**	**12**	**32**

Table 1: Number of examined accidents for each category.

Impact model validation

For all of the above mentioned 32 cases, it was possible to show that the calculation results from PC-Crash and the impact model presented in this paper were the same with respect to a few decimal places.

Model sensitivity to the point of impact and contact plane

In order to assess how strong the influence of the location of the point of impact and contact plane is, all the accidents with a manually defined POI and CP were chosen. In a first step, the impact parameter delta-V was calculated for the manually defined POI and CP, and in a second step for a POI and CP defined by the geometrical rule for the same accidents. Next, the distances between the POIs defined with both methods are computed, see Figure 5.

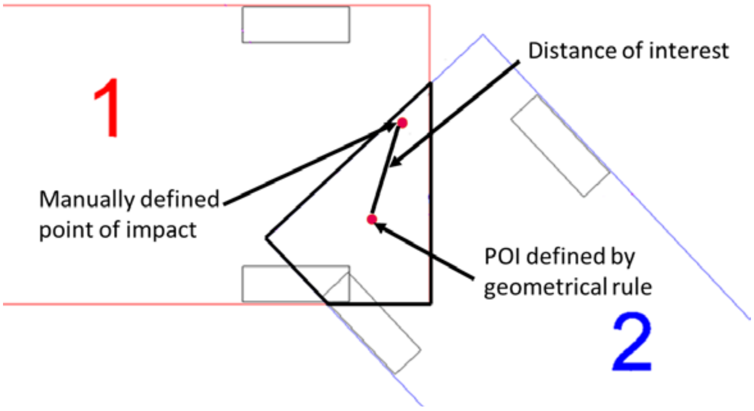

Figure 5: Calculation of distance between manually defined POI and POI defined by geometrical rule

Figure 6 shows the absolute difference in delta-V (difference of delta-V for each POI definition method) compared with the distance between points of impact for each definition method for each of the both vehicles involved in each accident.

A tendency can be seen, that for larger distances between the POIs, the deviation in delta-V will also be larger. While for full impacts the highest difference is slightly more than 10m/s, such a deviation can be seen also for small differences in POI for sliding impacts. The largest deviation in delta-V for sliding impacts was more than 25m/s.

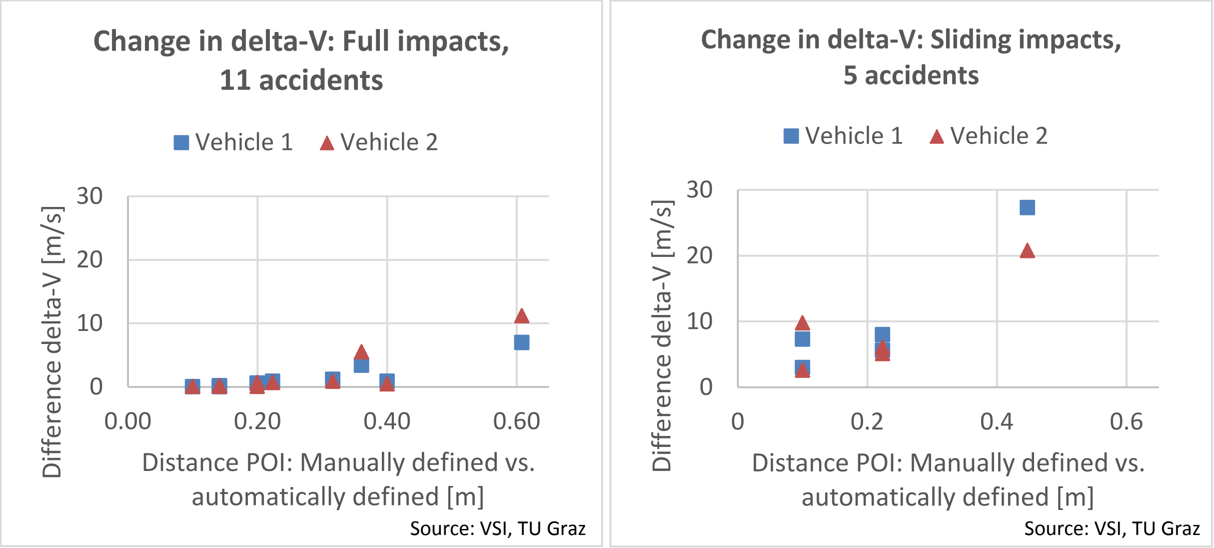

Figure 6: Change of delta-V in respect to changes in the location of POI. A tendency for larger deviations in delta-V can be seen when the difference between POI is larger. This effect is stronger for sliding impacts (right figure). Source: own picture.

For full impacts, the average distance between POI was 26 cm and 21 cm for sliding impacts. The average deviation in delta-V was 2.13 km/h, and 9.56 km/h for sliding impacts. This average deviation can be interpreted as an average error when using the geometrical rule.

Effectivity study

In an example effectivity study for a collision mitigation system, the accidents were simulated starting 5 seconds before the crash. The CMS was able to avoid 20 of 32 accidents, and reduce the average delta-V from 31.8 km/h in the baseline to 8.8 km/h in the system simulation (including avoided accidents in the calculation with a delta-V of 0) which is a reduction by around 72%. Using only the remaining accidents for the calculation of the average yields a delta-V of 22.9 km/h. Only 4 of the remaining 10 accidents resulted in a sliding collision.

CONCLUSION AND OUTLOOK

1. Impact model validation: The presented model produces the same results as PC-Crash under the mentioned limitations.
2. Model sensitivity: The results of the model are highly dependent to the input parameters point of impact and contact plane. For full impacts, the largest observed deviation in delta-V was around 11.2 km/h for a change of 61 cm in the POI, while it was much larger, 27 km/h, for sliding impacts for a change in POI of only 45 cm.
3. Effectivity study: Since the introduction of the CMS changes the original accident scenario dramatically, the manually defined POI and CP from the reconstruction could not be used, so the geometrical rule was employed. The CMS led to an impressive reduction of average delta-V from 31.8 to 8.8 km/h. However, since for full impacts the average error of the geometrical rule was 2.1 km/h and especially 9.6 km/h for sliding impacts, such hypothetical reduction rates of delta-V have to be taken with care.

In future research, the following things could be investigated:

- Extension to a 3D model.
- Further investigation of the model’s sensitivity to input parameters.
- Exploration for better rules to define POI and CP, especially a better rule for sliding impacts.

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