On normals of ellipses and ellipsoids

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Distance computation is an important issue with applications to path planning, obstacle recognition and collision prevention. To compute distances between complex objects these objects are often decomposed into elementary and preferably convex components such as spheres and ellipsoids. Locally extremal distances between such objects occur on their common normals. In this presentation we discuss the task of finding all common normals between basic object pairs like point - ellipsoid, straight line - ellipsoid and ellipsoid - ellipsoid. For each of these pairs we present geometric proofs for the maximal number of common normals in case of generic relative position of the two objects. To that end we use tools from descriptive geometry, line geometry and algebraic geometry.
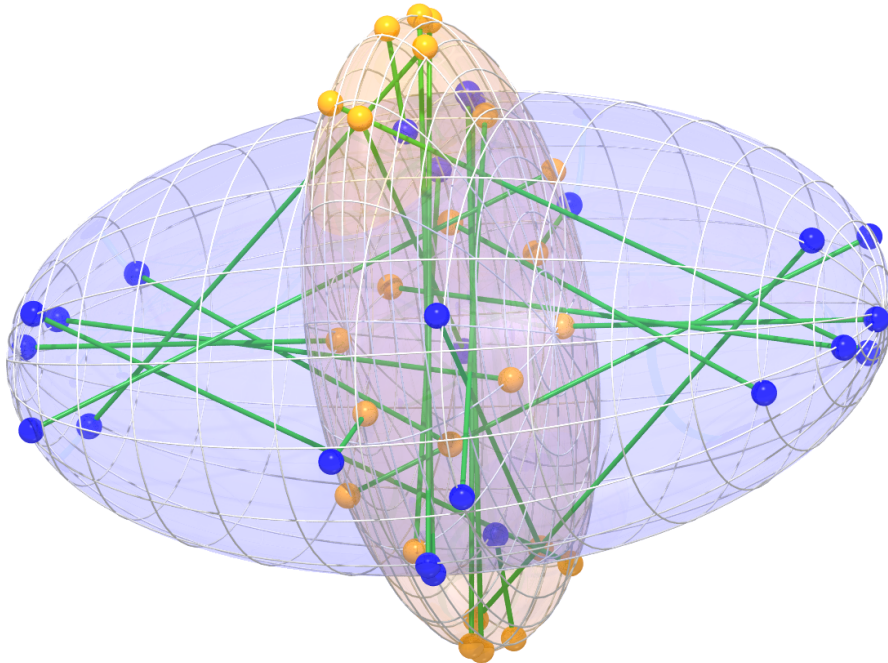
Figure 1: Two ellipsoids with 24 common normals

References

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