

# End-to-End Training of Hybrid CNN-CRF Models for Stereo

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## Abstract

We propose a novel method for stereo estimation, combining advantages of convolutional neural networks (CNNs) and optimization-based approaches. The optimization, posed as a conditional random field (CRF), takes local matching costs and consistency-enforcing (smoothness) costs as inputs, both estimated by CNN blocks. To perform the inference in the CRF we use an approach based on linear programming relaxation with a fixed number of iterations. We address the challenging problem of training this hybrid model end-to-end. We show that in the discriminative formulation (structured support vector machine) the training is practically feasible. The trained hybrid model with shallow CNNs is comparable to state-of-the-art deep models in both time and performance. The optimization part efficiently replaces sophisticated and not jointly trainable (but commonly applied) post-processing steps by a trainable, well-understood model.

## 1. Introduction

Stereo matching is a fundamental low-level vision problem. It is an ill-posed inverse problem, asking to reconstruct the depth from a pair of images. This requires robustness to all kinds of visual nuisances as well as a good prior model of the 3D environment. Different methods for this problem have been proposed and improved throughout the whole history of computer vision. Prior to deep NN data-driven approaches, progress has been made using global optimization techniques [20, 24, 36, 40, 48] featuring robust surface models and occlusion mechanisms. Typically, these methods had to rely on engineered cost matching and involved a number of parameters to be chosen experimentally.

Recent deep CNN models for stereo [12, 28, 53] learn from data to be robust to illumination changes, occlusions, reflections, noise, *etc.* A deep and possibly multi-scale ar-

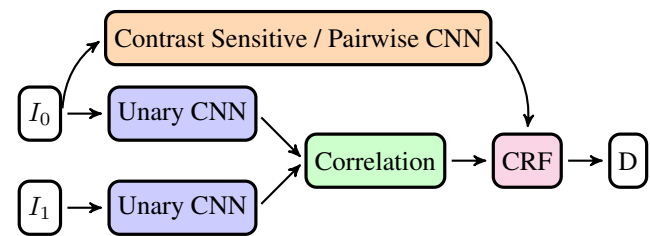


Figure 1: Architecture: A convolutional neural network, which we call *Unary-CNN* computes features of the two images for each pixel. The features are compared using a *Correlation* layer. The resulting matching cost volume becomes the unary cost of the CRF. The pairwise costs of the CRF are parametrized by edge weights, which can either follow a usual contrast sensitive model or estimated by the *Pairwise-CNN*.

chitecture is used to leverage the local matching to a global one. However, to produce a final accurate result they still rely a lot on post-processing that combines a set of filters and optimization-like heuristics.

In this work we combine CNNs with a discrete optimization model for stereo, allowing complex local matching costs and parametrized geometric priors to be put together in a global optimization approach and to be learned end-to-end from the data. Even though our model contains CNNs, it is still easily interpretable. This property allows us to shed more light on the learning our network performs. We start from a CRF formulation and replace all hand-crafted terms with learned ones.

## 2. Overview

We propose a hybrid CNN-CRF model illustrated in Fig. 1. Our *Unary-CNN* computes local features of both images which are then compared in a fixed correlation metric. Our *Pairwise-CNN* can additionally estimate contrast-sensitive pairwise costs in order to encourage or discourage label jumps. Using the learned unary and pairwise costs, the CRF tries to find a joint solution optimizing the to-

tal sum of all unary and pairwise costs in a 4-connected graph. This model is designed to generalize existing engineered approaches in stereo as well as augment existing fully learned ones. The *Unary-CNN* straightforwardly generalizes manually designed matching costs such as those based on differences of colors, sampling-insensitive variants [5], local binary patterns (e.g., Census transform [49]), etc. The *Pairwise-CNN* generalizes a contrast-sensitive regularizer [7], which is the best practice in MRF/CRF models for segmentation and stereo.

To perform inference in the CRF model we apply the fast method of [42], which improves over heuristic approaches combining multiple post-processing steps as used in [12, 28, 53]. We deliberately chose not to use any post-processing in order to show that the large part of its performance gain can be covered by a well-trained CRF model. While previously, methods based on LP-relaxation were considered prohibitively expensive for stereo, [42] reports a near real-time performance, which makes this choice definitely faster than a full deep architecture [53] and competitive in speed with inference heuristics such as SGM [16], MGM [14], etc.

To this end, we can train the complete model shown in Fig. 1 using the structured support vector machine (SSVM) formulation and propagating its subgradient through the networks. Training a non-linear CNN+CRF model of this scale is a challenging problem that has not been addressed before. We show this is practically feasible having a fast inference method and using an approximate subgradient scheme. Since at test time the inference is applied to complete images, we train it on complete images as well. This is in contrast to the works [28, 50, 53] which sample patches for training. The SSVM approach optimizes more directly the inference performance on complete images of the training set. While with the maximum likelihood it is important to sample hard negative examples (hard mining) [43], the SSVM determines labellings that are hard to separate as the most violated constraints.

We observed that the hybrid CNN+CRF network performs very well already with shallow CNN models, such as 3-7 layers. With the CRF layer the generalization gap is much smaller (less overfitting) than without. Therefore a hybrid model can achieve a competitive performance using much fewer parameters than the state of the art. This leads to a more compact model and a better utilization of the training data.

We report competitive performance on benchmarks using a shallow hybrid model. Qualitative results demonstrate that our model is often able to delineate object boundaries accurately as well as it is also often robust to occlusions, despite our CRF did not include explicit occlusion modeling. **Contribution** We propose a hybrid CNN+CRF model for stereo, which utilizes the expressiveness of CNNs to com-

pute good unary- as well as pairwise-costs and uses the CRF to easily integrate long-range interactions. We propose an efficient approach to train our CNN+CRF model. The trained hybrid model is shown to be fast and yields competitive results on challenging datasets. We do not use any kind of post-processing. The code will be made publicly available.

### 3. Related Work

**CNNs for Stereo** Most related to our work are CNN matching networks for stereo proposed by [12, 28] and the *fast* version of [53]. They use similar architectures with a siamese network [8] performing feature extraction from both images and matching them using a fixed correlation function (product layer). Parts of our model Fig. 1 denoted as *Unary-CNN* and *Correlation* closely follow these works. While [12, 28, 53] train by sampling matching and non-matching image patches, following the line of work on more general matching / image retrieval, we train from complete images. Only in this setting it is possible to extend to a full end-to-end training of a model that includes a CRF (or any other global post-processing) optimizing specifically for the best performance in the dense matching. The *accurate* model of [53] implements the comparison of features by a fully connected NN, which is more accurate than their *fast* model but significantly slower. Lastly, all these methods make an extensive use of post-processing steps that are not jointly-trainable with the CNN: [53] applies cost cross aggregation, semi-global matching, subpixel enhancement, median and bilateral filtering; [28] uses window-based cost aggregation, semi-global matching, left-right consistency check, subpixel refinement, median filtering, bilateral filtering and slanted plane fitting; [12] uses semi-global matching, left-right consistency check, disparity propagation and median-filtering. Experiments in [28] comparing bare networks without post-processing show that their fixed correlation network outperforms the *accurate* version of [53].

**CNN Matching** General purpose matching networks are also related to our work. [50] used a matching CNN for patch matching, [13] used it for optical flow and [29] used it for stereo, optical flow and scene flow. Variants of networks [13, 29] have been proposed that include a correlation layer explicitly, however it is then used as a stack of features and followed by up-convolutions regressing the dense matching. Overall, these networks have a significantly larger number of parameters and require a lot of additional synthetic training data.

**Joint Training (CNN+CRF training)** End-to-end training of CNNs and CRFs is helpful in many applications. The fully connected CRF [23], performing well in semantic segmentation, was trained jointly in [10, 55] by unrolling iterations of the inference method (mean field) and backpropagating through them. Unfortunately, this model does not

seem to be suitable for stereo because typical solutions contain slanted surfaces and not piece-wise constant ones (the filtering in [23] propagates information in fronto-parallel planes). Instead simple heuristics based on dynamic programming such as SGM [16] / MGM [14] are typically used in engineered stereo methods or as post-processing. However they suffer from various artifacts as shown in [14]. A trained inference model, even a relatively simple one, such as dynamic programming on a tree [35], can become very competitive. Scharstein [38] and Pal et al. [34] have considered training CRF models for stereo, linear in parameters. To the best of our knowledge, training of inference techniques with CNNs has not yet been demonstrated for stereo. We believe the reason for that is the relatively slow inference for models over pixels with hundreds of labels. Employing the method proposed in [42], which is a variant of a LP-relaxation on the GPU, allows us to overcome this limitation. In order to train this method we need to look at a suitable learning formulation. Specifically, methods approximating marginals are typically trained with variants of approximate maximum likelihood [1, 18, 26, 31, 34, 38]. Inference techniques whose iteration can be differentiated can be unrolled and trained directly by gradient descent [27, 32, 33, 37, 41, 45, 55]. Inference methods based on LP relaxation can be trained discriminatively, using a structured SVM approach [11, 15, 21, 46], where parameters of the model are optimized jointly with dual variables of the relaxation (blended learning and inference). We discuss the difficulty of applying this technique in our setting (memory and time) and show that instead performing stochastic approximate subgradient descent is more feasible and practically efficient.

## 4. CNN-CRF Model

In this section we describe the individual blocks of our model (Fig. 1) and how they connect.

We consider the standard rectified stereo setup, in which epipolar lines correspond to image rows. Given the left and right images  $I^0$  and  $I^1$ , the left image is considered as the *reference* image and for each pixel we seek to find a matching pixel of  $I^1$  at a range of possible disparities. The disparity of a pixel  $i \in \Omega = \text{dom } I^0$  is represented by a discrete label  $x_i \in \mathcal{L} = \{0, \dots, L-1\}$ .

The *Unary-CNN* extracts dense image features for  $I^0$  and  $I^1$ , respectively, denoted as  $\phi^0 = \phi(I^0; \theta_1)$  and  $\phi^1 = \phi(I^1; \theta_1)$ . Both instances of the *Unary-CNN* in Fig. 1 share the parameters  $\theta_1$ . For each pixel, these extracted features are then correlated at all possible disparities to form a correlation-volume (a matching confidence volume)  $p: \Omega \times \mathcal{L} \rightarrow [0, 1]$ . The confidence  $p_i(x_i)$  is interpreted as how well a window around pixel  $i$  in the first image  $I^0$  matches to the window around pixel  $i + x_i$  in the second image  $I^1$ . Additionally, the reference image  $I^0$  is used to

estimate contrast-sensitive edge weights either using a pre-defined model based on gradients, or using a trainable pairwise CNN. The correlation volume together with the pairwise weights are then fused by the CRF inference, optimizing the total cost.

### 4.1. Unary CNN

We use 3 or 7 layers in the *Unary-CNN* and 100 filters in each layer. The filter size of the first layer is  $(3 \times 3)$  and the filter size of all other layers is  $(2 \times 2)$ . We use the tanh activation function after all convolutional layers. Using tanh i) makes training easier, *i.e.*, there is no need for intermediate (batch-)normalization layers and ii) keeps the output of the correlation-layer bounded. Related works [2, 9] also have found that tanh performs better than ReLU for patch matching with correlation.

### 4.2. Correlation

The cross-correlation of features  $\phi^0$  and  $\phi^1$  extracted from the left and right image, respectively, is computed as

$$p_i(k) = \frac{e^{\langle \phi_i^0, \phi_{i+k}^1 \rangle}}{\sum_{j \in \mathcal{L}} e^{\langle \phi_i^0, \phi_{i+j}^1 \rangle}} \quad \forall i \in \Omega, \forall k \in \mathcal{L}. \quad (1)$$

Hence, the correlation layer outputs the softmax normalized scalar products of corresponding feature vectors. In practice, the normalization fixes the scale of our unary-costs which helps training the joint network. Since the correlation function is homogeneous for all disparities, a model trained with some fixed number of disparities can be applied at test time with a different number of disparities. The *pixel-wise independent estimate* of the best matching disparity

$$x_i \in \arg \max_k p_i(k) \quad (2)$$

will be used for the purpose of comparison with the full model.

### 4.3. CRF

The CRF model optimizes the total cost of complete disparity labelings,

$$\min_{x \in \mathcal{X}} (f(x) := \sum_{i \in \mathcal{V}} f_i(x_i) + \sum_{ij \in \mathcal{E}} f_{ij}(x_i, x_j)). \quad (3)$$

where  $\mathcal{V}$  is the set of all nodes in the graph, *i.e.*, the pixels,  $\mathcal{E}$  is the set of all edges and  $\mathcal{X} = \mathcal{L}^{\mathcal{V}}$  is the space of labelings. Unary terms  $f_i: \mathcal{L} \rightarrow \mathbb{R}$  are set as  $f_i(k) = -p_i(k)$ , the matching costs. The pairwise terms  $f_{ij}: \mathcal{L} \times \mathcal{L} \rightarrow \mathbb{R}$  implement the following model:

$$f_{ij}(x_i, x_j) = w_{ij} \rho(|x_i - x_j|; P_1, P_2). \quad (4)$$

The weights  $w_{ij}$  may be set either as manually defined contrast-sensitive weights [6]:

$$w_{ij} = \exp(-\alpha |I_i - I_j|^\beta) \quad \forall ij \in \mathcal{E}, \quad (5)$$

allowing cheaper disparity jumps across strong image gradients, or using the learned model of the *Pairwise-CNN*. The function  $\rho$  is a robust penalty function defined as

$$\rho(|x_i - x_j|) = \begin{cases} 0 & \text{if } |x_i - x_j| = 0, \\ P_1 & \text{if } |x_i - x_j| = 1, \\ P_2 & \text{otherwise,} \end{cases} \quad (6)$$

popular in stereo [17]. Cost  $P_1$  penalizes small disparity deviation of one pixel representing smooth surfaces and  $P_2$  penalizes larger jumps representing depth discontinuities.

We use only pairwise-interactions on a 4-connected grid (as opposed to the popular fully connected CRF often used in semantic segmentation).

**Inference** Despite the direct solution of (3) is intractable [25], there are a number of methods to perform approximate inference [11, 19] as well as related heuristics designed specifically for stereo such as [14, 17]. We apply the dual minorize-maximize method (`Dual_MM`) [42], which is sound because it is based on LP-relaxation, similar to TRW-S [19], and massively parallel, allowing a fast GPU implementation<sup>1</sup>.

We give a brief description of `Dual_MM`, which will also be needed when considering training. Let  $f$  denote the concatenated *cost vector* of all unary and pairwise terms  $f_i, f_{ij}$ . The method starts from a decomposition of  $f$  into horizontal and vertical chains,  $f = f^1 + f^2$  (namely,  $f^1$  includes all horizontal edges and all unary terms and  $f^2$  all vertical edges and zero unary terms). The value of the minimum in (3) is lower bounded by

$$\max_{\lambda} (D(\lambda) := \min_{x^1} (f^1 + \lambda)(x^1) + \min_{x^2} (f^2 - \lambda)(x^2)), \quad (7)$$

where  $\lambda$  is the vector of Lagrange multipliers corresponding to the constraint  $x^1 = x^2$ . The bound  $D(\lambda) \leq (3)$  holds for any  $\lambda$ , however it is tightest for the optimal  $\lambda$  maximizing the sum in the brackets. The `Dual_MM` algorithms performs iterations towards this optimum by alternatively updating  $\lambda$  considering at a time either all vertical or horizontal chains, processed in parallel. Each update monotonously increases the lower bound (7). The final solution is obtained as

$$x_i \in \underset{k}{\operatorname{argmin}} (f_i^1 + \lambda_i)(k), \quad (8)$$

*i.e.* similar to (2), but for the reparametrized costs  $f^1 + \lambda$ . If the inference has converged and the minimizer  $x_i$  in (8) is unique for all  $i$ , then  $x$  is the optimal solution to the energy minimization (3) [22, 47].

#### 4.4. Pairwise CNN

In order to estimate edge weights with a pairwise CNN, we use a 3-layer network. We use 64 filters with size  $(3 \times 3)$

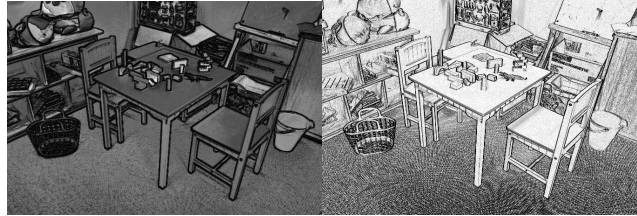


Figure 2: Learned vs fixed pairwise costs: Visualization of the pairwise costs between two neighboring pixels in  $x$ -direction using the learned pairwise-net (left) and a fixed edge-function (right). Dark pixels indicate a low cost for changing the label and bright pixels indicate a high cost for a label-switch. Note, how the dark pixels follow object outlines (where depth discontinuities are likely) and how texture-edges tend to be suppressed (*e.g.* at the floor) in the learned version.

and the tanh activation function in the first two layers to extract some suitable features. The third layer maps the features of pixel  $i$  to weights  $(w_{ij} \mid ij \in \mathcal{E})$  corresponding to the two edge orientations, where we use the absolute value function as activation. This ensures, that the pairwise costs are always larger than 0 and that our *Pairwise-CNN* has the ability to scale the output freely. In practice this is desirable because it allows to automatically learn the optimal trade-off between data-fidelity and regularization. The parameters of this network will be denoted as  $\theta_2$ . The weights  $w$  can be stored as a 2-channel image (one channel per orientation). These weights contribute to the CRF pairwise terms  $f_{ij}$  as follows:

$$f_{ij}(x_i, x_j) = w_{ij} \rho(|x_i - x_j|; P_1, P_2). \quad (9)$$

The values  $P_1, P_2$  remain as global parameters. This network generalizes the manually engineered model (6) with contrast-sensitive terms (5) and can learn to apply it adaptively based on the image content in a wider neighborhood. Fig. 2 shows an example output of the *Pairwise-CNN*.

## 5. Training

One major goal of this work is the training of the complete model in Fig. 1 end-to-end. For the purpose of comparison of different components we train 3 types of models, of increasing generality:

- Pixel-wise *Unary-CNN*: model in which CRF interactions are set to zero and *Pairwise-CNN* is switched off.
- Joint *Unary-CNN* +CRF model in which the *Pairwise-CNN* is fixed to replicate exactly the contrast-sensitive model (5). Trained parameters are: *Unary-CNN* and global parameters  $P_1, P_2$ .
- Joint model with trained *Unary-CNN* and *Pairwise-CNN* (=complete model). Trained Parameters are: *Unary-CNN*, *Pairwise-CNN* and global parameters  $P_1, P_2$ .

<sup>1</sup>GPU implementation of [42] provided by its authors.

## 5.1. Training Unary CNN in the Pixel-wise Model

For the purpose of comparison, we train a Unary CNN in a pixel-wise mode, similarly to [12, 28, 53]. For this purpose we let the CRF interactions to be fixed to zero (e.g., by letting  $P_1 = P_2 = 0$ ), in which case the resulting decision degenerates to the pixel-wise independent argmax decision rule (2). Training such models can be formulated in different ways, using gradient of the likelihood / cross-entropy [28, 51], reweighted regression [12] or hinge loss [52]. Following [28, 51] we train parameters of the *Unary-CNN*  $\theta_1$  using the cross-entropy loss,

$$\min_{\theta_1} \sum_{i \in \Omega} \sum_{k \in \mathcal{X}} p_i^{gt}(k) \log p_i(k; \theta_1), \quad (10)$$

where  $p_i^{gt}(k)$  is the one-hot encoding of the ground-truth disparity for the  $i$ -th pixel.

## 5.2. Training Joint Model

We apply the structured support vector machine formulation, also known as maximum margin Markov network [44, 46], in a non-linear setting. After giving a short overview of the SSVM approach we discuss the problem of learning when no exact inference is possible and the blended learning and inference approach of [11, 21], which as we argue is not feasible for models of our size. We then discuss the proposed training scheme approximating a subgradient of a fixed number of iterations of **Dual-MM**.

**SSVM** Assume that we have a training sample consisting of an input image pair  $I = (I^0, I^1)$  and the true disparity  $x^*$ . Let  $x$  be a disparity prediction that we make. We consider an additive loss function

$$l(x, x^*) = \sum_i l_i(x_i, x_i^*), \quad (11)$$

where the pixel loss  $l_i$  is taken to be  $l_i(x_i, x_i^*) = \min(|x_i - x_i^*|, \tau)$ , appropriate in stereo reconstruction. The empirical risk is the sum of losses (11) over a sample of several image pairs, however for our purpose it is sufficient to consider only a single image pair. When the inference is performed by the CRF *i.e.*, the disparity estimate  $x$  is the minimizer of (3), training the optimal parameters  $\theta = (\theta_1, \theta_2, P_1, P_2)$  can be formulated in the form of a *bilevel optimization*:

$$\min_{\theta} l(x, x^*) \quad (12a)$$

$$\text{s.t. } x \in \arg \min_{x \in \mathcal{X}} f(x; \theta). \quad (12b)$$

Observe that any  $x \in \arg \min f(x)$  in (12b) necessarily satisfies  $f(x) \leq f(x^*)$ . Therefore, for any  $\gamma > 0$ , the

scaled loss  $\gamma l(x, x^*)$  can be upper-bounded by

$$\max_{x: f(x) \leq f(x^*)} \gamma l(x, x^*) \quad (13a)$$

$$\leq \max_{x: f(x) \leq f(x^*)} [f(x^*) - f(x) + \gamma l(x, x^*)] \quad (13b)$$

$$\leq \max_x [f(x^*) - f(x) + \gamma l(x, x^*)]. \quad (13c)$$

A subgradient of (13c) w.r.t.  $(f_i | i \in \mathcal{V})$  can be chosen as

$$\delta(x^*) - \delta(\bar{x}), \quad (14)$$

where  $\delta(x)_i$  is a vector in  $\mathbb{R}^{\mathcal{L}}$  with components  $(\mathbb{1}_{x_i = k} | k \in \mathcal{L})$ , *i.e.* the 1-hot encoding of  $x_i$ , and  $\bar{x}$  is a (generally non-unique) solution to the *loss augmented inference* problem

$$\bar{x} \in \arg \min_x [\bar{f}(x) := f(x) - \gamma l(x, x^*)]. \quad (15)$$

In the case of an additive loss function, problem (15) is of the same type as (3) with adjusted unary terms.

To facilitate the intuition of why SSVM chooses the most violated constraint, the hinge loss (13c) can be equivalently written in the form

$$\min\{\xi \in \mathbb{R} | (\forall x) \xi \geq f(x^*) - f(x) + \gamma l(x, x^*)\}, \quad (16)$$

which reveals the large margin separation property: the constraint in (16) tries to ensure that the training solution  $x^*$  is better than all other solutions by a margin  $\gamma l(x, x^*)$  and the most violated constraint sets the value of slack  $\xi$ . The parameter  $\gamma$  thus controls the margin: a large margin may be beneficial for better generalization with limited data. Finding the most violated constraint in (16) is exactly the loss-augmented problem (15).

**SSVM with Relaxed Inference** An obstacle in the above approach is that we cannot solve the loss-augmented inference (15) exactly. However, having a method solving its convex relaxation, we can integrate it as follows. Applying the decomposition approach to (15) yields a lower bound on the minimization: (15)  $\geq$

$$\bar{D}(\lambda) := \min_{x^1} (\bar{f}^1 + \lambda)(x^1) + \min_{x^2} (\bar{f}^2 - \lambda)(x^2) \quad (17)$$

for all  $\lambda$ . Lower bounding (15) like this results in an upper-bound of the loss  $\gamma l(x, x^*)$  and the hinge loss (13a):

$$\gamma l(x, x^*) \leq (13a) \leq f(x^*) - \bar{D}(\lambda). \quad (18)$$

The bound is valid for any  $\lambda$  and is tightened by maximizing  $\bar{D}(\lambda)$  in  $\lambda$ . The learning problem on the other hand minimizes the loss in  $\theta$ . Tightening the bound in  $\lambda$  and minimizing the loss in  $\theta$  can be written as a joint problem

$$\min_{\theta, \lambda} f(x^*; \theta) - \bar{D}(\lambda; \theta). \quad (19)$$

Using this formulation we do not need to find an optimal  $\lambda$  at once, it is sufficient to make a step towards minimizing it. This approach is known as blended learning and inference [11, 21]. Unfortunately, it is disadvantageous for our purpose for two reasons: i) at the test time we are going to use a fixed number of iterations instead of optimal  $\lambda$  ii) joint optimization in  $\theta$  and  $\lambda$  in this fashion will be slower and iii) it is not feasible to store intermediate  $\lambda$  for each image in the training set as  $\lambda$  has the size of a unary cost volume.

**Approximate Subgradient** We are interested in a subgradient of (18) after a fixed number of iterations of the inference method, *i.e.*, training the unrolled inference. A suboptimal  $\lambda$  (after a fixed number of iterations) will generally vary when the CNN parameters  $\theta$  and thus the CRF costs  $f$  are varied. While we do not fully backtrack a subgradient of  $\lambda$  (which would involve backtracking dynamic programming and recursive subdivision in `DualMM`) we can still inspect its structure and relate the subgradient of the approximate inference to that of the exact inference.

**Proposition 5.1.** Let  $\bar{x}^1$  and  $\bar{x}^2$  be minimizers of horizontal and vertical chain subproblems in (17) for a given  $\lambda$ . Let  $\Omega_{\neq}$  be a subset of nodes for which  $\bar{x}_i^1 \neq \bar{x}_i^2$ . Then a subgradient  $g$  of the loss upper bound (18) w.r.t.  $f_{\mathcal{V}} = (f_i \mid i \in \mathcal{V})$  has the following expression in components

$$g_i(k) = (\delta(x^*) - \delta(\bar{x}^1))_i(k) + \sum_{j \in \Omega_{\neq}} (J_{ij}(k, \bar{x}_i^2) - J_{ij}(k, \bar{x}_i^1)), \quad (20)$$

where  $J_{ij}(k, l)$  is a sub-Jacobian (matching  $\frac{d\lambda_j(l)}{df_i(k)}$  for a subset of directions  $df_i(k)$ ). See Suppl. A for more details.

We conjecture that when the set  $\Omega_{\neq}$  is small, for many nodes the contribution of the sum in (20) will be also small, while the first part in (20) matches the subgradient with exact inference (14).

**Proposition 5.2.** For training the approximate inference with dual decomposition such as `DualMM`, calculate the minimizer  $\bar{x}^1$  after a fixed number of iterations and approximate the subgradient as

$$\delta(x^*) - \delta(\bar{x}^1). \quad (21)$$

The assumption for the learning to succeed is to eventually have most of the pixels in agreement. The inference method works towards this by adjusting  $\lambda$  such that the constraints  $x_i^1 = x_i^2$  are satisfied. We may expect in practice that if the data is not too ambiguous this constraint will be met for a large number of pixels already after a fixed number of iterations. A good initialization of unary costs, such as those learned using the pixel-wise only method can help to improve the initial agreement and to stabilize the method.

### 5.3. Training Unary and Pairwise CNNs in Joint Model

In order to make the pairwise interactions trainable, we need to compute a subgradient w.r.t.  $w_{ij}$ ,  $P_1$ ,  $P_2$ . We will compute it similarly to how it was done for the unary terms assuming exact inference, and then just replace the exact minimizer  $\bar{x}$  with an approximate  $\bar{x}^1$ . A subgradient of (13c) is obtained by choosing a minimizer  $\bar{x}$  and evaluating the gradient of the minimized expression. Components of the later are given by

$$\frac{\partial}{\partial w_{ij}} = \rho(|x_i^* - x_j^*|; P_{1,2}) - \rho(|\bar{x}_i - \bar{x}_j|; P_{1,2}), \quad (22a)$$

$$\frac{\partial}{\partial P_1} = \sum_{ij} w_{ij} (\llbracket |x_i^* - x_j^*| = 1 \rrbracket - \llbracket |\bar{x}_i - \bar{x}_j| = 1 \rrbracket), \quad (22b)$$

$$\frac{\partial}{\partial P_2} = \sum_{ij} w_{ij} (\llbracket |x_i^* - x_j^*| > 1 \rrbracket - \llbracket |\bar{x}_i - \bar{x}_j| > 1 \rrbracket). \quad (22c)$$

We thus obtain an end-to-end trainable model without any hand-crafted parameters, except for the hyper-parameters controlling the training itself.

### 5.4. Implementation Details

We trained our models using Theano [4] with stochastic gradient descent and momentum. For training the model without pairwise costs we set the learn rate to  $1 \times 10^{-2}$ , for all other models we set the learn rate to  $1 \times 10^{-6}$ . Before feeding a sample into our model we normalize it such that it has zero-mean as well as unit-variance. Our full model is trained gradually. We start by training the models with lower complexity and continue then training more complex models, where we reuse previously trained parameters and initialize new parameters randomly (see Fig. 3). Since we use full RGB images for training, we have to take care of occlusions as well as invalid pixels, which we mask out during training. Additionally, we implemented the forward pass using C++/CUDA in order to make use of our trained models in a real-time environment in a streaming setting. We achieve 3-4 frames per second with our fully trained 3-layer model using an input-size of  $640 \times 480$  pixels<sup>2</sup>.

## 6. Experiments

In this section we will be testing different variants of our proposed method. In order not to confuse the reader, we will use the following naming convention: `CNN $x$`  is the argmax output of a network trained as described in § 5.1; `CNN $x$ +CRF` is the same network with `DualMM` as post-processing; `CNN $x$ +CRF+Joint` is the jointly trained network described in § 5.2; `CNN $x$ +CRF+Joint+PW` is the fully trained method described in § 5.3.  $x$  represents the respective number of layers in the CNN.

<sup>2</sup>A detailed breakdown of the timings can be found in the supplementary material.

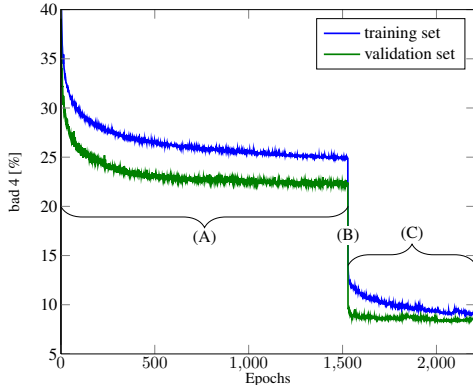


Figure 3: Performance w.r.t. the real objective (*bad4*, Middlebury) for key complexity steps of our model during training. (A) *Unary-CNN* (7 layers) is trained using ML § 5.1. (B) Adding the CRF with contrast-sensitive weights with an optimal choice of parameters ( $\alpha, \beta, P_1, P_2$ ). (C) Joint optimization of complete model §§ 5.2 and 5.3. Observe that the gap between training and validation errors is significantly smaller in (C).

### 6.1. Benchmark Data Sets

For our experiments we are using two stereo benchmark datasets: Kitti 2015 [30] and Middlebury [39]. Both benchmarks hold out the **test** set, where the ground truth is not accessible to authors. We call examples with ground truth available that can be used for training/validation the **design** set and split it randomly into 80% **training** set and 20% **validation** set. This way we obtain 160 + 40 examples for Kitti and 122+31 examples for Middlebury (including additionally provided images with different lightings, exposures and perfect/imperfect rectified stereo-pairs). The used error metric in all experiments is the percent of pixels with a disparity difference bigger  $x$  pixels (*bad<sub>x</sub>*).

### 6.2. Performance of Individual Components

In this experiment we measure the performance improvement when going from *CNN<sub>x</sub>* to the full jointly trained model. Since ground-truth of the test data is not available to us, this comparison is conducted on the complete design set. The results are shown in Table 1. This experiment demonstrates that an optimization or post-processing is necessary, since the direct output of all tested CNNs (after a simple point-wise minimum search in the cost volume) contains too many outliers to be used directly. A qualitative comparison on one of the training images of Middlebury is depicted in Fig. 4. One can observe that the quality of the CNN-only method largely depends on the number of layers whereas the CNN+CRF versions achieve good results even for a shallow CNN.

### 6.3. Benefits of Joint Training

In this experiment, we compare our method to two recently proposed stereo matching methods based on CNNs,

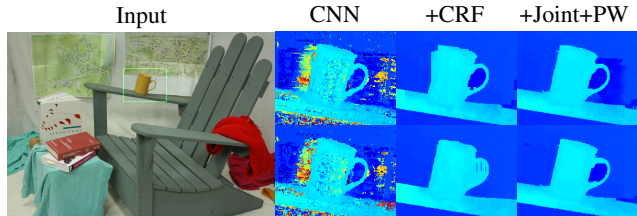


Figure 4: Qualitative comparison of *Unary-CNN*, CNN+CRF and CNN+CRF+Joint+PW on the Middlebury benchmark. Zoom-in of disparity with 3 layers (top) and 7 layers (bottom). Note how the jointly trained models inpaint occlusions correctly.

Benchmark	Method	CNN	+CRF	+Joint	+PW
Middlebury	CNN3	23.89	11.18	9.48	9.45
	CNN7	18.58	9.35	8.05	7.88
Kitti 2015	CNN3	28.38	6.33	6.11	4.75
	CNN7	13.08	4.79	4.60	4.04
	[28]	5.99	4.31	-	-
	[53]	13.56	4.45	-	-

Table 1: Influence of the individual components of our method (§ 6.2) and comparison with [28, 53] without post-processing (§ 6.3). Standard error metrics (*bad4* on official training data for Middlebury and *bad3* on the **design** set for Kitti) are reported.

the *MC-CNN* by Zbontar and LeCun [53] and the *Content-CNN* by Luo et al. [28]. In order to allow for a fair comparison among the methods, we disable all engineered post-processing steps of [28, 53]. We then unify the post-processing step by adding our CRF on top of the CNN outputs. We evaluate on the whole design set since we do not know the train/test split of the different methods. In favor of the compared methods, we individually tune the parameters  $P_1, P_2, \alpha, \beta$  of the CRF for each method using grid search. The results are shown in Table 1. While the raw output of our CNN is inferior to the compared methods, the post-processing with a CRF significantly decreases the difference in performance. Joint training of our CNN+CRF model further improves the performance, despite using a relatively shallow network, having fewer parameters. Specifically, our full joint model with 7 layers has 281k parameters, while the networks [28, 53] have about 700k and 830k parameters, respectively.

### 6.4. Benchmark Test Performance

Evaluation of our submission on test images is available in online suites of Middlebury [39] and Kitti 2015 [30]. The summary of this evaluation is presented in Fig. 5. We would like to stress that those results have been achieved without using any post-processing like occlusion detection and -inpainting. We fine-tuned our best performing model (Table 1, CNN7+PW) for *half* sized images and used it for the Middlebury evaluation. Some qualitative results are shown in Fig. 6. Since no ground-truth is handed out, we can only

## Middlebury

Method	Average performance	Middlebury														Kitti 2015				
		Australia	AustraliaP	Bicycle2	Classroom2	Classroom2E	Computer	Crusade	CrusadeP	Djembe	Djebel	Hoops	Livingroom	Newkuba	Plants	Staircase	Method	Non-occ	All Time	
[53] acc.	8.29	5.59	4.55	5.96	2.83	11.4	8.44	8.32	8.89	2.71	16.3	14.1	13.2	13.0	6.40	11.1	[53] acc.	3.33	3.89	67s
[3]	8.62	6.05	5.16	6.24	3.27	11.1	8.91	8.87	9.83	3.21	15.1	15.9	12.8	13.5	7.04	9.99	[28]	4.00	4.54	1s
[54]	13.4	5.90	4.88	10.8	12.9	10.6	13.6	12.2	9.01	5.39	27.4	23.5	17.7	21.0	15.4	20.9	Ours	4.84	5.50	1.3s
Ours	16.5	27.9	4.79	8.62	20.2	37.6	15.1	24.2	17.4	4.47	19.2	22.0	20.0	21.1	9.43	16.2				

Figure 5: Performance in benchmark **test** sets as of time of submission. For both benchmarks, we compare our results against work that is based on CNNs for matching costs and accepted for publication [3, 28, 53, 54]. We report the respective standard error metric *bad2* for the Middlebury benchmark and *bad3* for the Kitti benchmark.

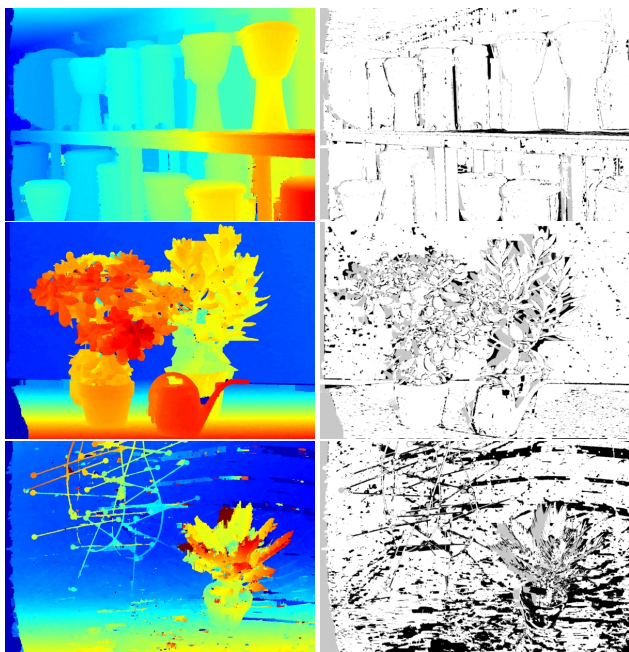


Figure 6: Qualitative comparison on selected *test* images (from top to bottom: *Djembe*, *Plants*, *Australia*) of the Middlebury Stereo Benchmark. The left column shows the generated disparity images in false color, the right column the *bad2* error image, where white = error smaller than 2 disparities, grey = occlusion and black = error greater than 2 disparities.

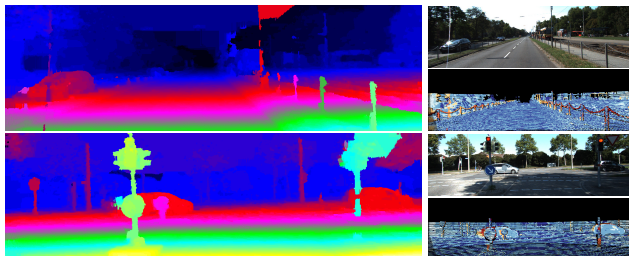


Figure 7: Qualitative comparison on the test set of Kitti 2015. More results can be viewed online and in the supplementary material.

print the error image that has been provided by the online system. Looking at the results in Figs. 5 and 6 we have identified the strengths and drawbacks of the proposed method. The first two rows of Fig. 6 show examples where our algorithm achieves a very low error as in majority of images. In the third row we can identify a failure case of our method. The *Australia* image is imperfectly rectified, *i.e.*, the epipolar lines do not correspond precisely to the rows anymore. This contradicts the assumption of our model that the corresponding pixel is always located in the same row and therefore implies some errors if the imperfectness is too large.

For Kitti we again used our best performing model (Table 1, CNN7+PW), including the  $x$ - and  $y$ -coordinates of the pixels as features. This is justified because *e.g.* the sky is always at the top of the image while the roads are always at the bottom. By looking at the error plots for Kitti in Fig. 7, one can see, that most of the incorrect predictions are in i) occluded areas or ii) along fine structures in the images which are *fattened* in the ground-truth. We include additional experiments as well as visual comparisons with other methods in the supplementary material.

## 7. Conclusion

We have proposed a fully trainable hybrid CNN+CRF model for stereo. This allows us to directly optimize for the final output. We utilize CNNs for computing both, unary costs and pairwise costs, to be used in a CRF. This architecture makes our model easily interpretable. We showed how this model can be trained using the SSVM formulation. Our model achieves competitive results on the Middlebury and Kitti stereo benchmarks. In contrast to other works, our model has much less parameters and more importantly, does not use any post-processing. Despite not modeling occlusions, they are often filled in correctly in practice. For future work we want to model occlusions explicitly and try to learn them in order to further improve the performance of our model. Also, it will be interesting to look into post-processing methods like sub-pixel enhancement that can be incorporated into the training process.



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# End-to-End Training of Hybrid CNN-CRF Models for Stereo

## Supplementary Material

### A. Technical Details

#### A.1. Approximate Subgradient

Here we proof Proposition 5.1, restated below for convenience.

**Proposition 5.1.** Let  $\bar{x}^1$  and  $\bar{x}^2$  be minimizers of horizontal and vertical chain subproblems in (17) for a given  $\lambda$ . Let  $\Omega_{\neq}$  be a subset of nodes for which  $\bar{x}_i^1 \neq \bar{x}_i^2$ . Then a subgradient  $g$  of the loss upper bound (18) w.r.t.  $f_{\mathcal{V}} = (f_i \mid i \in \mathcal{V})$  has the following expression in components

$$g_i(k) = (\delta(x^*) - \delta(\bar{x}^1))_i(k) + \sum_{j \in \Omega_{\neq}} (J_{ij}(k, \bar{x}_i^2) - J_{ij}(k, \bar{x}_i^1)), \quad (20)$$

*Proof.* The loss upper bound (18) involves the minimum over  $x^1, x^2$  as well as many minima inside the dynamic programming defining  $\lambda$ . A subgradient can be obtained by fixing particular minimizers in all these steps and evaluating the gradient of the resulting function. It follows that a subgradient of the point-wise minimum of  $(\bar{f}^1 + \lambda)(x^1) + (\bar{f}^2 - \lambda)(x^2)$  over  $x^1, x^2$  can be chosen as  $g =$

$$\nabla_{f_{\mathcal{V}}}(\bar{f}^1(\bar{x}^1) + \bar{f}^2(\bar{x}^2)) + \nabla_{\lambda}(\lambda(\bar{x}^1) - \lambda(\bar{x}^2))J, \quad (23)$$

where  $J_{i,j}(k, l)$  is a sub-Jacobian matching  $\frac{d\lambda_j(l)}{df_i(k)}$  for the directions  $df_{\mathcal{V}}$  such that  $\lambda(f + df_{\mathcal{V}})$  has the same minimizers inside dynamic programming as  $\lambda(f)$ .

In the first part of the expression (23), the pairwise components and the loss  $l(\bar{x}^1, x^*)$  do not depend on  $f_i$  and may be dropped, leaving only  $(\nabla_{f_{\mathcal{V}}} \sum_{j \in \mathcal{V}} f_j(\bar{x}_j^1))_i = \delta(\bar{x}^1)_i$ .

Let  $h$  denote the second expression in (23). Its component  $h_i(k)$  expands as

$$h_i(k) = \sum_{j \in \mathcal{V}} \sum_{l \in \mathcal{L}} \frac{\partial}{\partial \lambda_j(l)} (\lambda_j(\bar{x}_j^1) - \lambda_j(\bar{x}_j^2)) J_{ij}(k, l) \quad (24a)$$

$$= \sum_{j \in \Omega_{\neq}} \sum_{l \in \mathcal{L}} (\llbracket \bar{x}_j^1 = l \rrbracket - \llbracket \bar{x}_j^2 = l \rrbracket) J_{ij}(k, l) \quad (24b)$$

$$= \sum_{j \in \Omega_{\neq}} (J_{ij}(k, x_j^1) - J_{ij}(k, x_j^2)). \quad (24c)$$

□

Our intuition to neglect the sum (24c) is as follows. We expect that variation of  $f_i$  for a pixel  $i$  far enough from  $j \in \Omega_{\neq}$  will not have a significant effect on  $\lambda_j$  and thus  $J_{ij}$  will be small over  $\Omega_{\neq}$ .

Component	# Disp.	Kitti 2015	Middlebury	Real-Time
		0.4 MP	1.3 MP	0.3 MP
Input processing		7.58	6.40	6.02
Pairwise CNN		21.12	59.46	13.75
Unary CNN		262.48	664.19	62.54
Correlation	128	154.86	437.02	46.70
Correlation	256	286.87	802.86	—
CRF	128	309.48	883.57	155.85
CRF	256	605.35	1739.34	—
<b>Total</b>	128	755.52	2050.64	284.86
<b>Total</b>	256	1183.40	3272.25	—

Table B.1: Timing experiments for 7 layer CNN and 5 CRF iterations (3 layer and 4 iterations for **Real-Time**). Runtimes in ms.

### B. Additional Experiments

#### B.1. Timing

In Table B.1 we report the runtime of individual components of our method for different image sizes and number of labels (=disparities). All experiments are carried out on a Linux PC with a Intel Core i7-5820K CPU with 3.30GHz and a NVidia GTX TitanX using CUDA 8.0. For Kitti 2015, the image size is  $1242 \times 375$ . For Middlebury V3 we selected the *Jadeplant* data set with *half* resolution, leading to an image size of  $1318 \times 994$ . We observe that with a constant number of layers in the Unary CNN and disparity range, the runtime depends linearly on the number of pixels in the input images. Correlation and CRF layer also depend on the number of estimated disparities, where we report numbers using 128 and 256 disparities.

#### B.2. Sublabel Enhancement

A drawback of our CRF method based on dynamic programming is the discrete nature of the solution. For some benchmarks like Middlebury the discretization artifacts negatively influence the quantitative performance. Therefore, most related stereo methods perform some kind of sub-label refinement (e.g. [28, 53]). For the submission to online benchmarks we deliberately chose to *discard* any form of non-trainable post-processing. However, we performed additional experiments with fitting a quadratic function to the output cost volume of the CRF method around the discrete solution. The refined disparity is then given by

$$d_{se} = d + \frac{C(d-h) - C(d+h)}{2(C(d+h) - 2C(d) + C(d-h))} \quad (25)$$

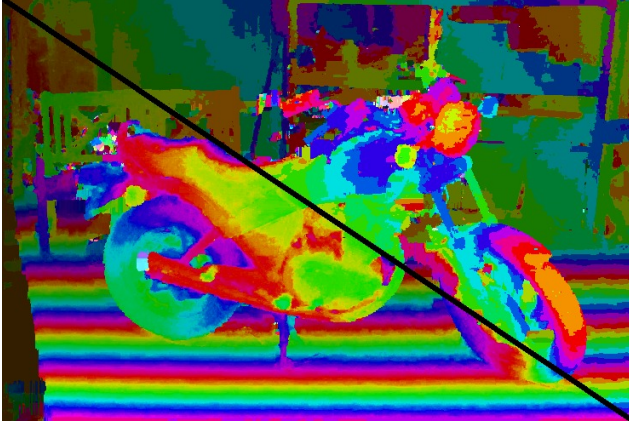


Figure B.1: Qualitative comparison on *Motorcycle* of discrete (upper-right) and sublabel enhanced (bottom-left) solution. Note how smooth the transitions are in the sublabel enhanced region (e.g. at the floor or the rear wheel).

where  $C(d)$  is the cost of disparity  $d$ . A qualitative experiment on the *Motorcycle* image of Middlebury stereo can be seen in Fig. B.1. Quantitative experiments have been conducted on both Kitti 2015 and Middlebury and will be reported in the follow sections (columns **w. ref.** in Tables B.2 and B.3). Again, in the main paper and in the submitted images we always report the performance of the *discrete* solution in order to keep the method pure.

### B.3. Middlebury Stereo v3

In this section we report a complete overview of all tested variants of our proposed hybrid CNN-CRF model on the stereo benchmark of Middlebury Stereo v3. We report the mean error (error metric *percent of non-occluded pixels with an error bigger 4 pixels*). All results are calculated on quarter resolution and upsampled to the original image size. We present the results in Fig. B.2 and Table B.2. Note, how the quality increases when we add more parameters and therefore allow a more general model (visualized from left to right in Fig. B.2). The last row shows the *Vintage* image, where our model produces a rather high error. The reason for that lies in the (almost) completely untextured region in the top-left corner. Our full model is able to recover some disparities in this region, but not all. A very interesting byproduct visible in Fig. B.2 concerns our small 3-layer model. Visually, one can hardly see any difference to the deeper 7-layer model, when our models are full jointly trained. Hence, this small model is suited very well for a real-time application.

Additionally, we compared to the performance of the model learned on Kitti, denoted Kitti-CNN in Table B.2. The performance is inferior, which means that the model trained on Kitti does not generalize well to Middlebury. Generalizing from Middlebury to Kitti, on the other hand

is much better, as discussed in the next section.

Method	w/o. ref.	w. ref.
CNN3	23.89	-
CNN3+CRF	11.18	10.50
CNN3 Joint	9.48	8.75
CNN3 PW+Joint	9.45	8.70
CNN7	18.58	-
CNN7+CRF	9.35	8.68
CNN7 Joint	8.05	7.32
CNN7 PW+Joint	7.88	7.09
Kitti-CNN	15.22	14.43

Table B.2: Comparison of differently trained models and their performance on the official training images of the Middlebury V3 stereo benchmark. The results are given in % of pixels farther away than 4 disparities from the ground-truth on all pixels.

### B.4. Kitti 2015

In this section we report a complete overview of all tested variants of our proposed hybrid CNN-CRF model on the stereo benchmark of KITTI 2015. We report the mean error (official error metric *percent of pixel with an error bigger 3 pixels*) on the complete design set. Table B.3 shows a performance overview of our models. In the last row of Table B.3 we apply our best performing model on Middlebury to the Kitti design set. Interestingly, the performance decreases only by  $\approx 1.5\%$  on all pixels. This experiment indicates, that our models generalize well to the scenes of the Kitti benchmark.

Method	w/o. ref.		w. ref.	
	all	non occ.	all	non occ.
CNN3	29.58	28.38	-	-
CNN3+CRF	7.88	6.33	7.77	6.22
CNN3 Joint	7.66	6.11	7.57	6.02
CNN3 PW+Joint	6.25	4.75	6.14	4.65
CNN7	14.55	13.08	-	-
CNN7+CRF	5.85	4.79	5.76	4.70
CNN7 Joint	5.98	4.60	5.89	4.50
CNN7 PW+Joint	5.25	4.04	5.18	3.96
[53]+CRF	6.10	4.45	5.74	4.08
[28]+CRF	5.89	4.31	5.81	4.21
[53]	15.02	13.56	-	-
[28]	7.54	5.99	-	-
MB-CNN	6.82	5.35	6.69	5.21

Table B.3: Comparison of differently trained models and their performance on the design set images of the KITTI 2015 stereo benchmark. The results are given in % of pixels farther away than 3 disparities from the ground-truth on all pixels.

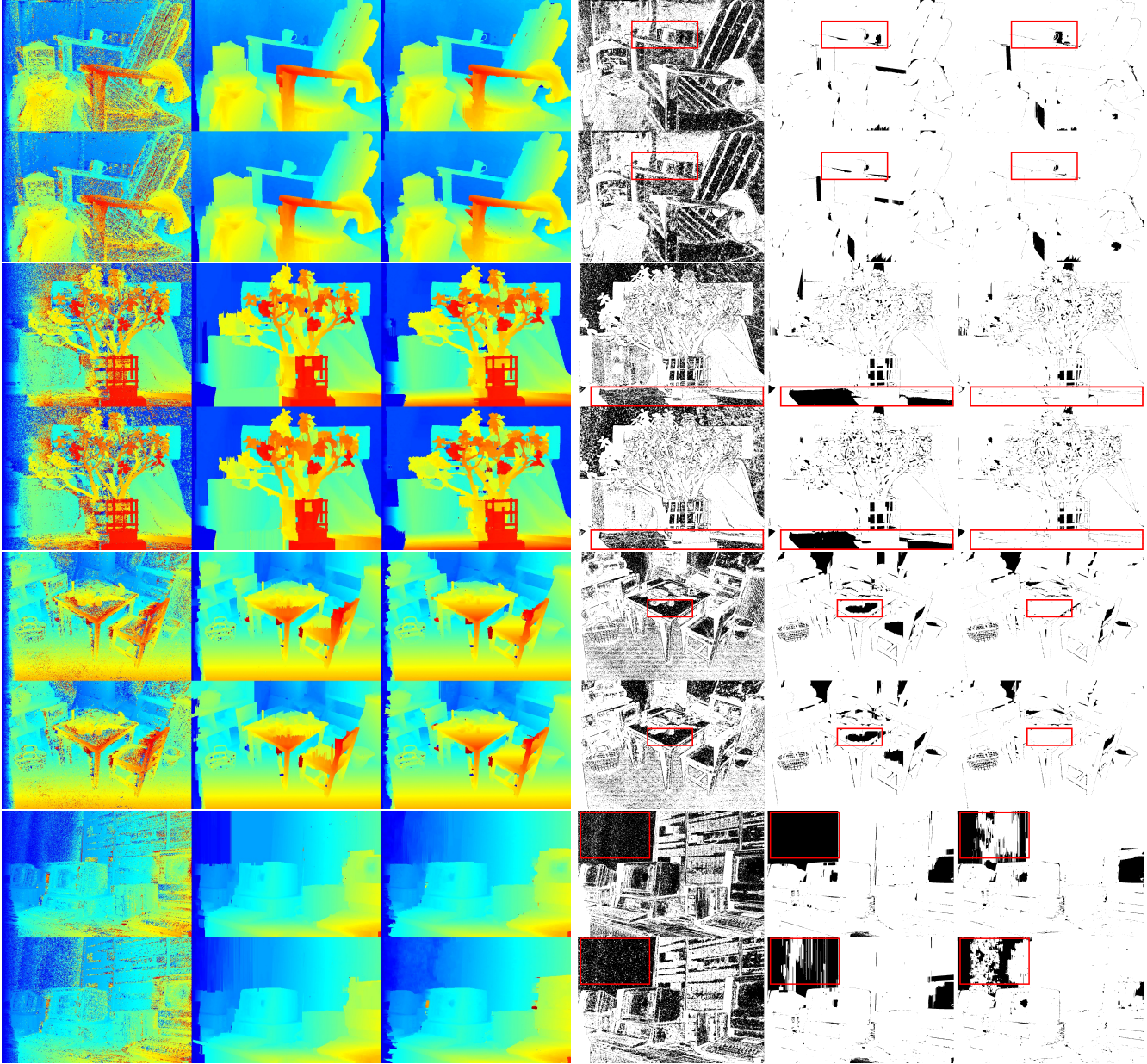


Figure B.2: Qualitative comparison of our models on Middlebury. For each image, the first row shows our 3-layer model and the second row shows the result of our 7-layer model. The first column shows out *Unary-CNN* with argmax decision rule, the second column *CNNx+CRF* and the third column shows the result of *CNNx+CRF+Joint+PW*. The remaining columns show the respective error-plots for the different models, where white indicates correct and black indicates wrong disparities. Disparity maps are color-coded from blue (small disparities) to red (large disparities).

Due to lack of space in the main paper, we could only show a few qualitative results of the submitted method. In Fig. B.4 we show additional results, more can be viewed online.

Looking at Kitti results in more detail, we observe that most of the errors happen in either occluded regions or due to a fattened ground-truth. Since we train edge-weights to courage label-jumps at strong object boundaries, our model

yields very sharp results. It is these sharp edges in our solution which introduce some errors on the benchmark, even when our prediction is correct. Fig. B.3 shows some examples on the *test* set (provided by the online submission system).

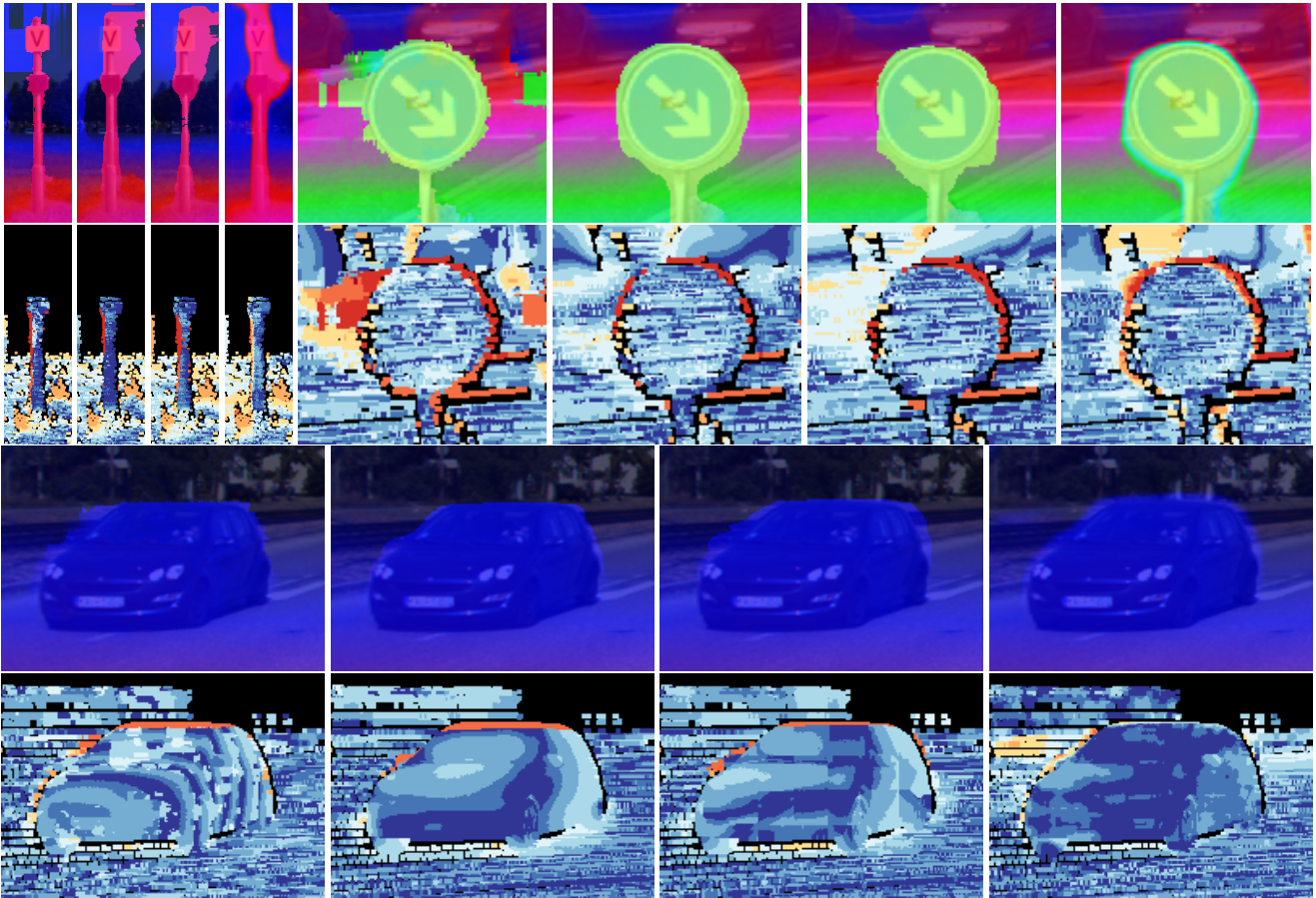


Figure B.3: Error comparison on magnified parts of Kitti 2015 test images: The first and third row show the color-coded disparity map of *Ours*, *MC-CNN* [53], *ContentCNN* [28] and *DispNetC* [29]. The second and last row show the corresponding error-plots, where shades of blue mean correct and shades of orange mean wrong. Note, how our model accurately follows object boundaries, whereas all other approaches fatten the object. Nevertheless, in terms of correct or wrong we make more wrong predictions, because the ground-truth seems to be fattened as well.

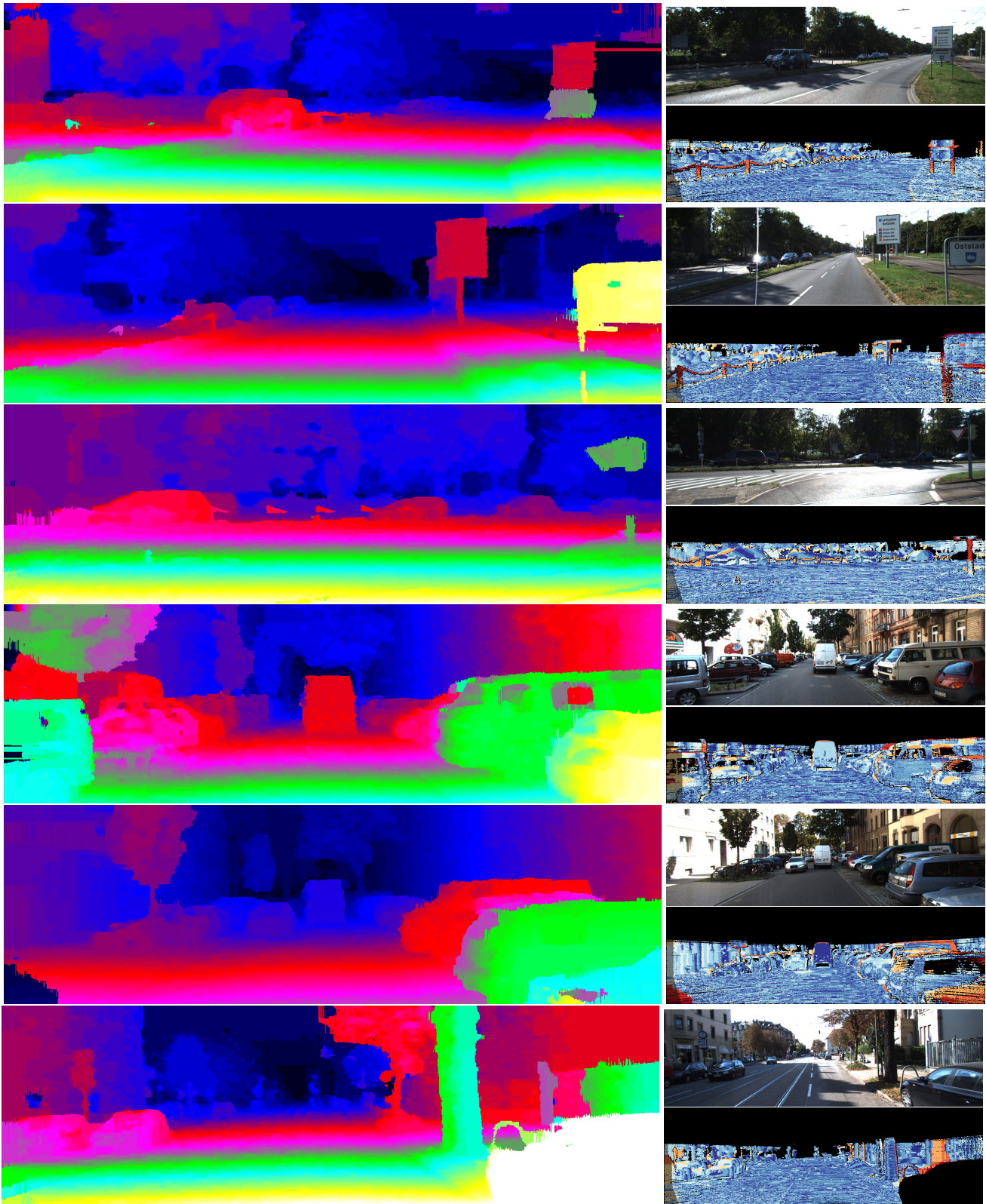


Figure B.4: Qualitative comparison on the test set of KITTI 2015.