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Funded by EC Marie Skłodowska Curie grant agreement No 812638

TENSOR-BASED, CONTACT-MODEL-AGNOSTIC APPROACH TO RECONSTRUCTION OF GRANULAR BULK'S CONTACT FORCES FROM ITS MACRO BEHAVIOR AND CONTACT NETWORK FABRIC

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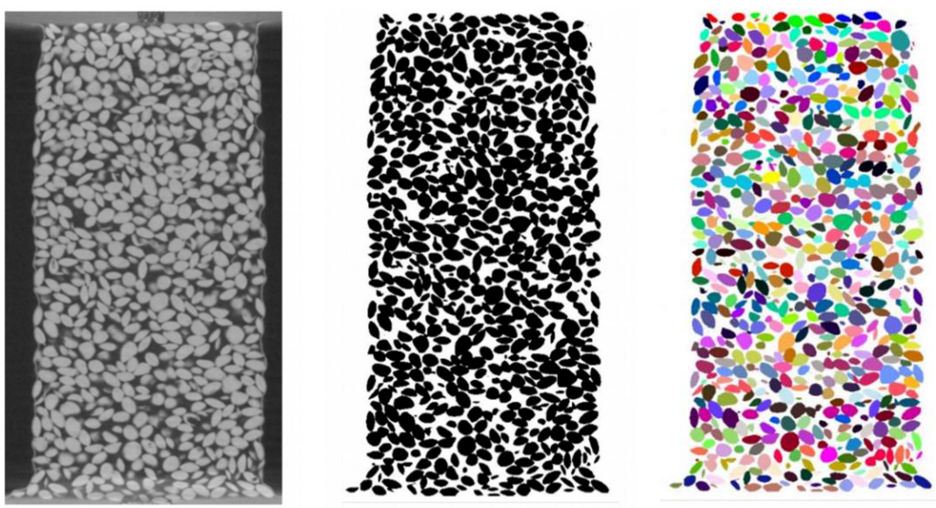
INTRODUCTION

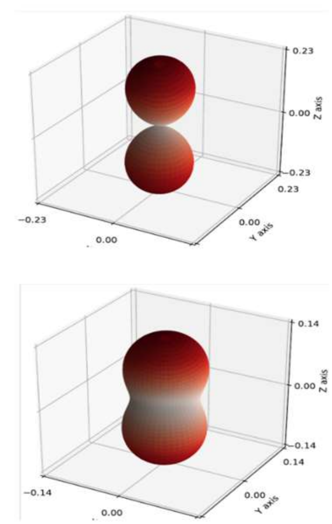
INTRODUCTION

PRACTICAL PROBLEMS - X-RAY TOMOGRAPHY

stages of particle detection process:
grayscale, binary, labelled



contact detection involves assessment
of the ‘amount of contact’



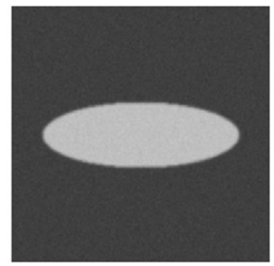
SOURCE: https://wur-yoda.irods.surfsara.nl/research/?dir=%2Fresearchcaliper%2FTraining_Schools%2FTS3%2FTS3_Grenoble%2FLectures
(G. Pinzon, Laboratoire 3SR Grenoble)

INTRODUCTION

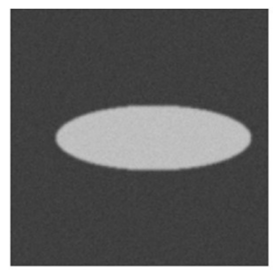
PRACTICAL PROBLEMS - X-RAY TOMOGRAPHY

tracking of the kinematics

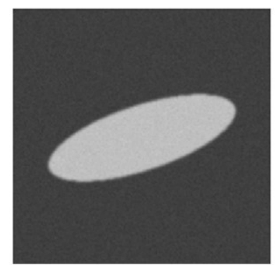
initial state
(*Im0*)



deformed state
(*Im1*)



translation



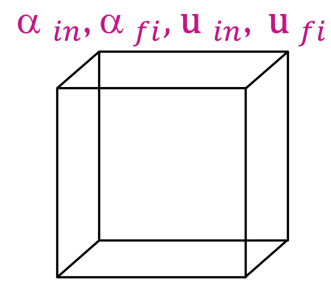
rotation

SOURCE: https://wur-yoda.irods.surfsara.nl/research/?dir=%2Fresearchcaliper%2FTraining_Schools%2FTS3%2FTS3_Grenoble%2FLectures
(G. Pinzon, Laboratoire 3SR Grenoble)

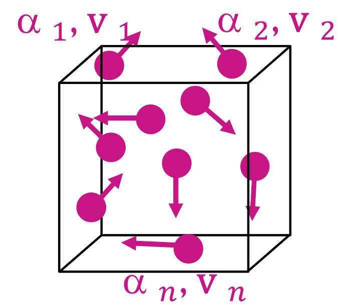
INTRODUCTION

MATERIAL DESCRIPTION

CONTINUUM APPROACH
(uses mathematical models of structureless continuum)

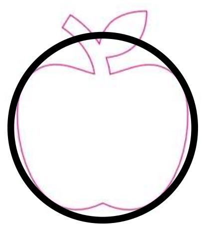


STRUCTURAL (micro-mechanical) APPROACH
(obtaining the mechanics of the specimen is based on interactions among discrete particles)

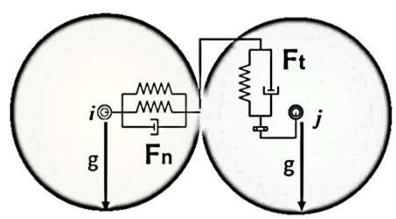


INTRODUCTION

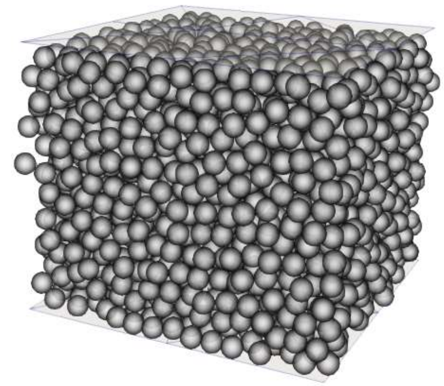
DEM



particle representation
(morphology, material properties)



contact model



simulation

+

calibration

GOAL

completing system's mechanics

microstructural description

without assumptions about interaction

model and the need for calibration – only

from measured (observed) values of

micro and macro variables obtained in

experiments?

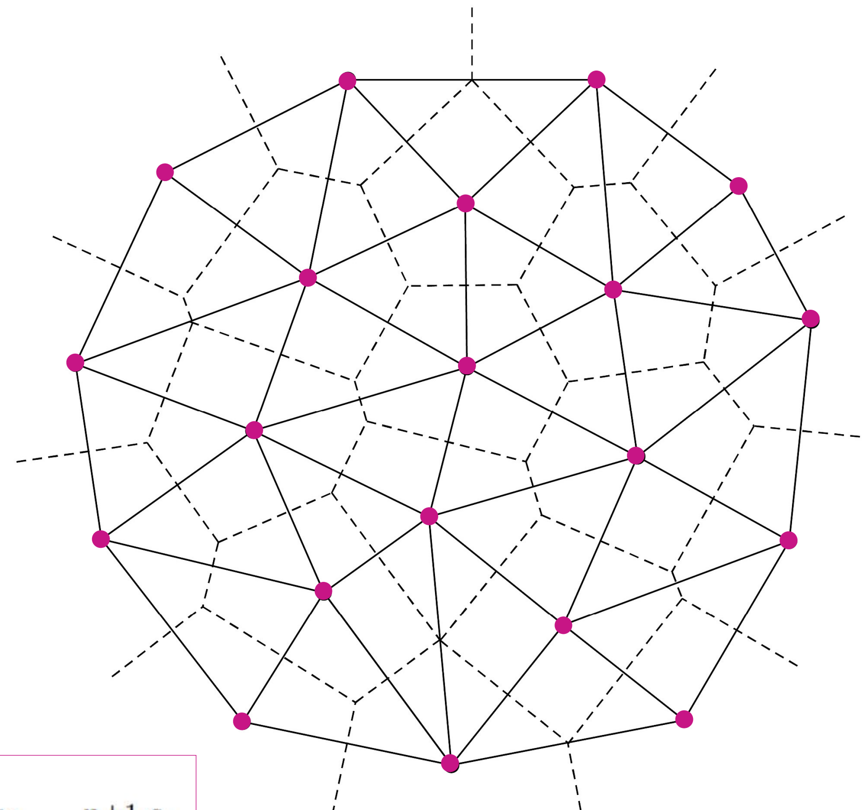
METHOD

METHOD

FABRIC

fabric = microstructure, represented via directional data

$$\{\vec{e}_\iota^{\,n+1},\ \vec{e}_\iota^{\,n+1}, \dots\ \vec{e}_\iota^{\,n+1}\}$$



fabric tensor

$$A_{tot}^{n+1,a_1,..a_k} = \sum_{\iota=1}^{N} \frac{w_\iota^{n+1}}{w_{tot}^{n+1}}\, e_\iota^{n+1,a_1}\ e_\iota^{n+1,a_2} \dots\ e_\iota^{n+1,a_k}$$

$$w_{tot}^{n+1} = \sum_{\iota=1}^{N} w_\iota^{n+1}$$

is a tensorial measure of the structural arrangement of a granular medium.

For $\iota = IJ$

$$\boxed{A_{bulk}^{n+1,a_1,..a_k} = \sum_{IJ\in\{IJ\}} \frac{w_{IJ}^{n+1}}{w_{bulk}^{n+1}}\, e_{IJ}^{n+1,a_1}\ e_{IJ}^{n+1,a_2} \dots\ e_{IJ}^{n+1,a_k}}$$

SOURCE: https://s3-eu-west-1.amazonaws.com/ppreviews-plos-725668748/10884671/preview.jpg

METHOD

CONNECTION POSITION - FABRIC - STRESS

fabric tensors created using interaction directional data, are (easily) connectable to stress tensors

ASUMPTION [1] :

$$\sigma_{bulk}^{n+1,ab} = func\left(\kappa_{bulk}^{n+1}, A_{bulk}^{n+1,ab}\right)$$

or

$$\sigma_{bulk}^{n+1,ab} = func\left(\kappa_{bulk}^{n+1,ab}, A_{bulk}^{n+1,ab}\right)$$

according to the equations, relation is valid on different levels of description and…

1. K. Kanatani, Distribution of directional data and fabric tensors, vol. 12, no. 2, pp. 149–164, 1984.

METHOD

FABRIC AND STRESS TENSORS

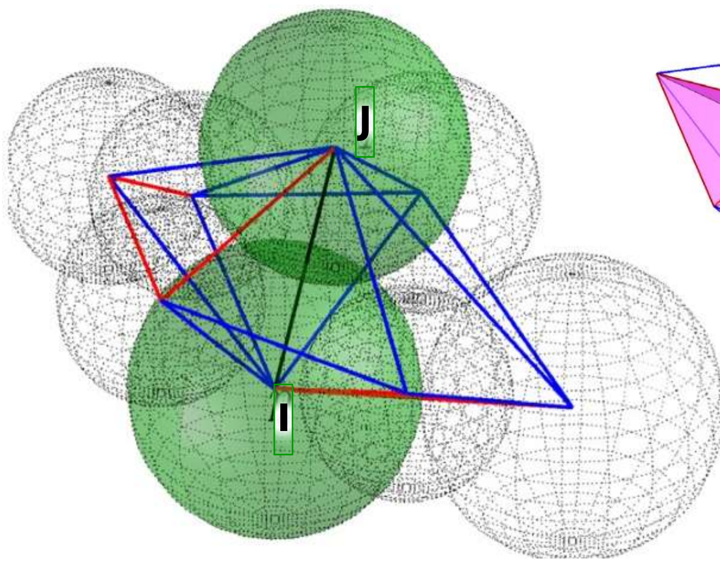
…and different levels are connectable:

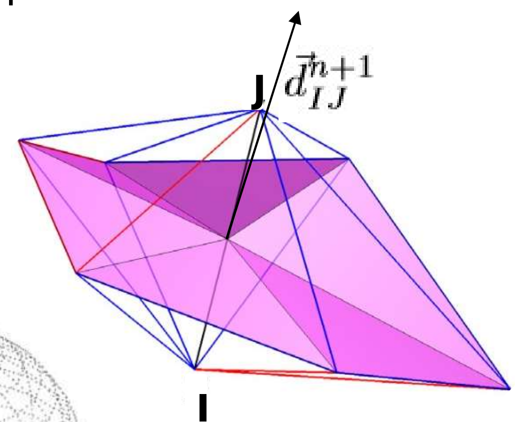
$$
\begin{tikzcd}
 & A^{n+1,ab}_{bulk} \arrow[dr] & & \\
\sigma^{n+1,ab}_{bulk} \arrow[ur] \arrow[dr] & & A^{n+1,ab}_{I} \arrow[dr] & \\
 & \sigma^{n+1,ab}_{I} \arrow[ur] \arrow[dr] & & A^{n+1,ab}_{IJ} \arrow[dl] \\
 & & \sigma^{n+1,ab}_{IJ} &
\end{tikzcd}
$$

METHOD

STRAIN TENSORS – BAGI APPROACH

strain tensor[2]
calculation using
Delaunay network
guarantees validity
of 3^{rd} Newton's law





SOURCE:
O. Durán, N.P. Kruyt*, S. Luding, *Analysis of three-dimensional micro-mechanical strain formulations for granular materials: Evaluation of accuracy*, IJSolStr

complementary area vector

$$\vec{d}_{IJ}^{n+1} = \frac{1}{2D(D+1)} \sum_{(K,L)} \left(\left(\Delta\vec{r}_{IL}^{\,n+1} \times \Delta\vec{r}_{IK}^{\,n+1} \right) - \left(\Delta\vec{r}_{JK}^{\,n+1} \times \Delta\vec{r}_{JL}^{\,n+1} \right) \right)$$

2. Bagi, K., Stress and strain in granular assemblies. Mechanics of Materials, 22(3), 165-177, 1996

METHOD

INTERACTION FORCES

interaction forces
obtained using
contact-force-model-agnostic
approach, without assumptions about
particles' material properties

$$\boxed{\vec{F}_{IJ}^{n+1} = \hat{\sigma}_{IJ}^{n+1} \cdot \vec{d}_{IJ}^{n+1}}$$
$$= -\vec{F}_{JI}^{n+1}$$

complementary
area vector

check stress using Goldhirsch's[3] approach, on the macro level

$$\boxed{\sigma_{bulk}^{(n+1),ab} = \frac{1}{V_{bulk}^{(n+1)}} \sum_{I \in \{I\}} \left\{ \sum_{J \in \{IJ\}} \left(F_{IJ}^{(n+1),a} \left(\vec{r}_{cp,IJ}^{\,(n+1),b} - \vec{r}_{I}^{\,(n+1),b} \right) \right) \right\}}$$

only contact contribution

3. Goldhirsch, I., Stress, stress asymmetry and couple stress: form discrete particles to continuous fields. Granular Matter, 12:239-252, 2010

METHOD

DEGREE OF ANISOTROPY

fabric tensor is of the 1st kind, rank 2

$$\hat{A}^{n+1}_{bulk} = \hat{N}^{n+1}_{bulk}$$

$$\hat{A}^{n+1}_{I} = \hat{N}^{n+1}_{I}$$

$$\hat{A}^{n+1}_{IJ} = \hat{N}^{n+1}_{IJ}$$

degree of anisotropy

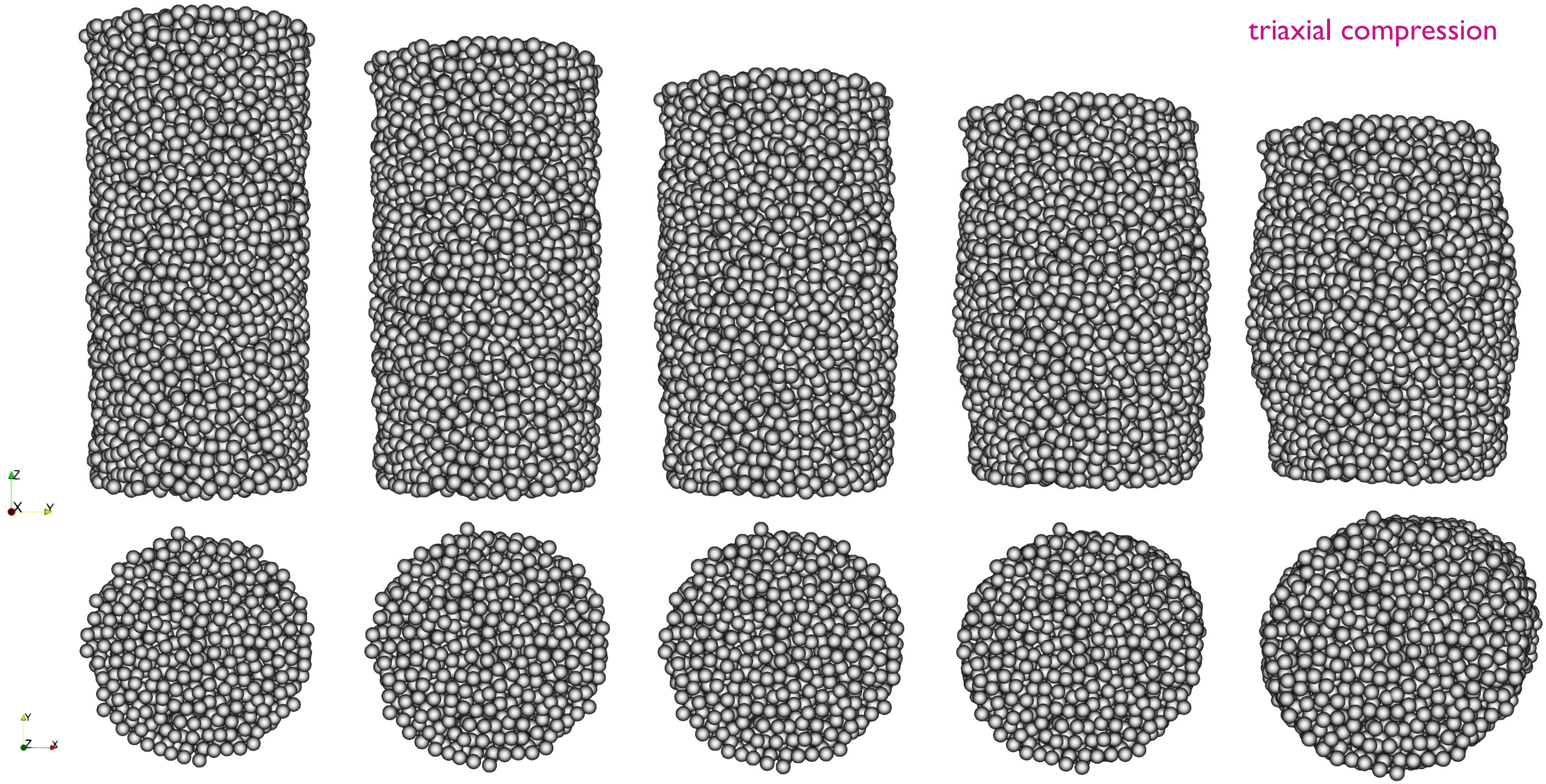
$$\zeta\left(\hat{N}^{n+1}\right) = \sqrt{\frac{1}{2}\Big((N_1^{n+1} - N_2^{n+1})^2 + (N_2^{n+1} - N_3^{n+1})^2 + (N_3^{n+1} - N_1^{n+1})^2\Big)}$$

$$\boxed{\zeta\left(\frac{\hat{\sigma}^{n+1}}{tr\hat{\sigma}^{n+1}}\right) = \sqrt{\frac{1}{2}\Big((\frac{\sigma_1^{n+1}}{tr\hat{\sigma}^{n+1}} - \frac{\sigma_2^{n+1}}{tr\hat{\sigma}^{n+1}})^2 + (\frac{\sigma_2^{n+1}}{tr\hat{\sigma}^{n+1}} - \frac{\sigma_3^{n+1}}{tr\hat{\sigma}^{n+1}})^2 + (\frac{\sigma_3^{n+1}}{tr\hat{\sigma}^{n+1}} - \frac{\sigma_1^{n+1}}{tr\hat{\sigma}^{n+1}})^2\Big)}}$$

1, 2, 3 are the tensor eigen values

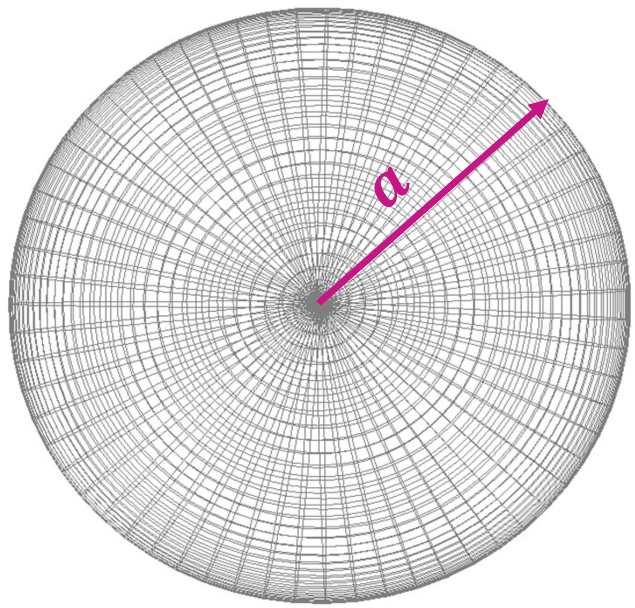
EXPERIMENT OVERVIEW

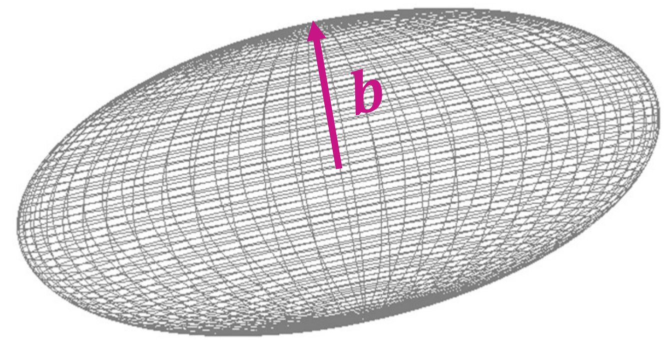
EXPERIMENT OVERVIEW



SOURCE: G. Pinzon, Laboratoire 3SR Grenoble

SIMULATION OVERVIEW





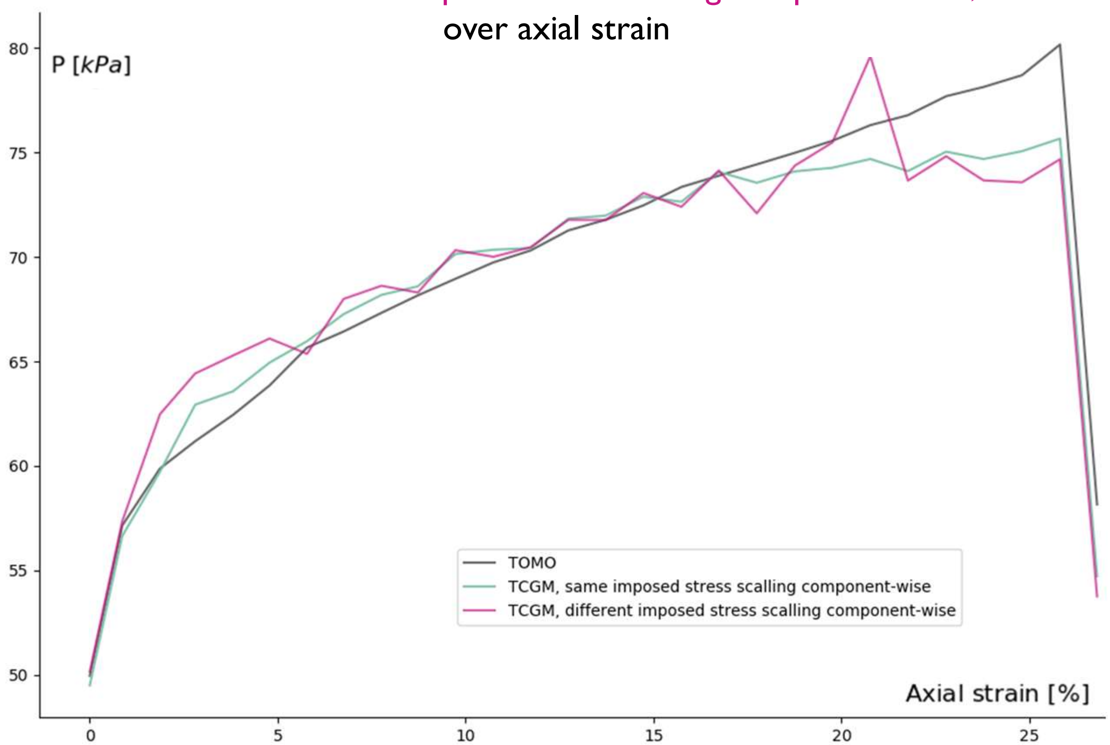
green lentils are approximately oblate spheroids with a = 2.33 mm mm and b = 1.8 mm

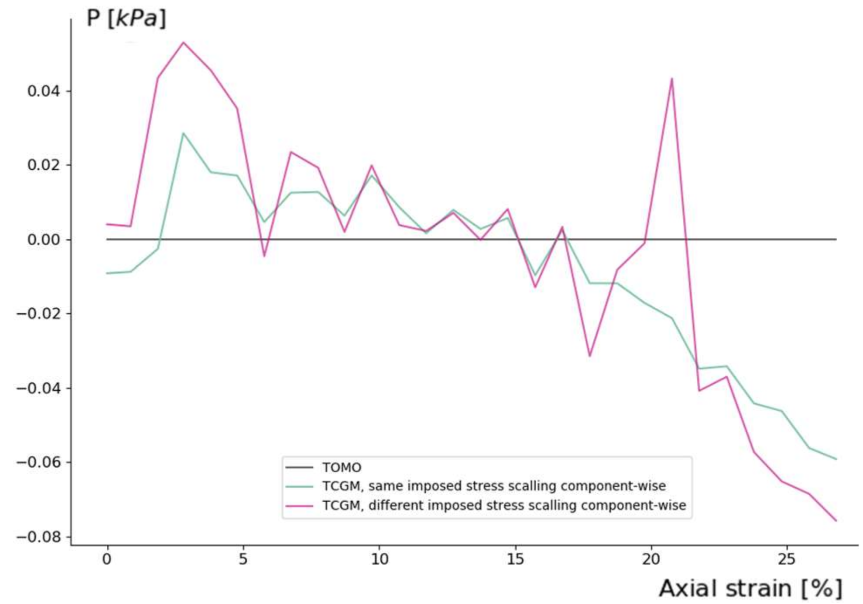
RESULTS

RESULTS

AVERAGE BULK STRESSES

Comparison of TOMO and TCGM **pressures** for trials with the same/different imposed stress scaling component-wise, over axial strain



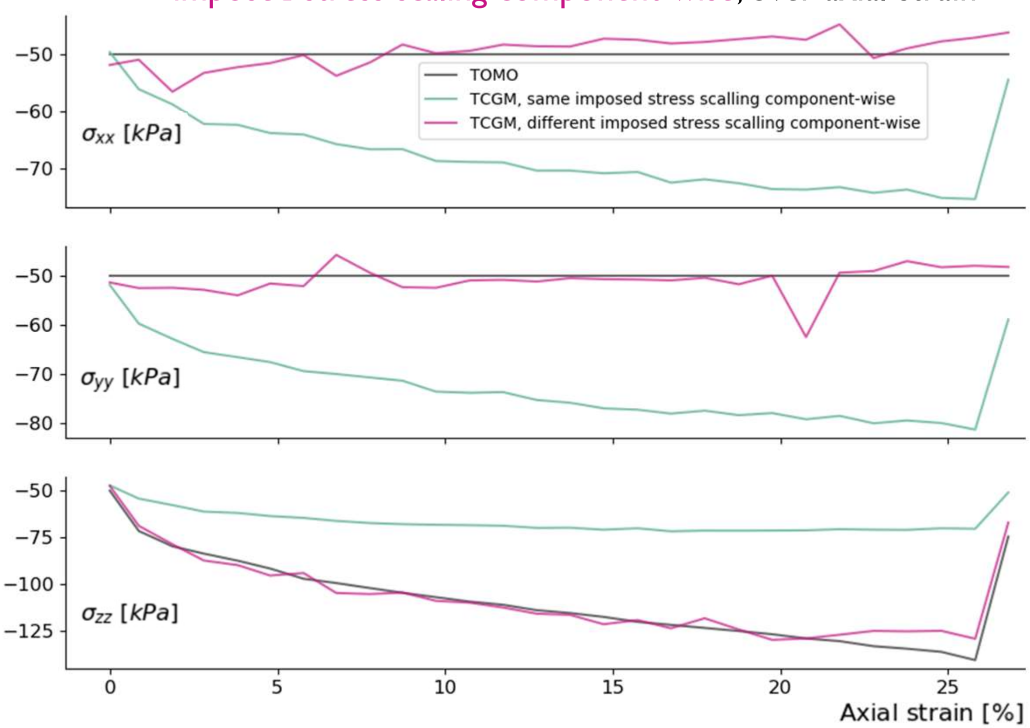


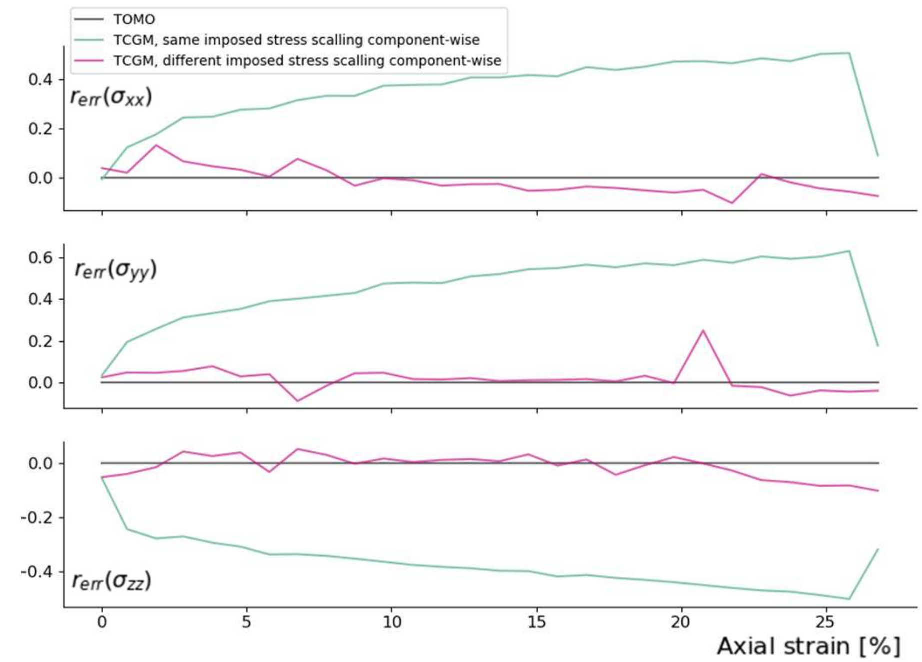
Normalized relative error of TCGM pressure for trials with the same/different imposed stress scaling component-wise, over axial strain

RESULTS

AVERAGE BULK STRESSES

Comparison of TOMO and TCGM **diagonal average bulk stress components** for trials with the same/different imposed stress scaling component-wise, over axial strain



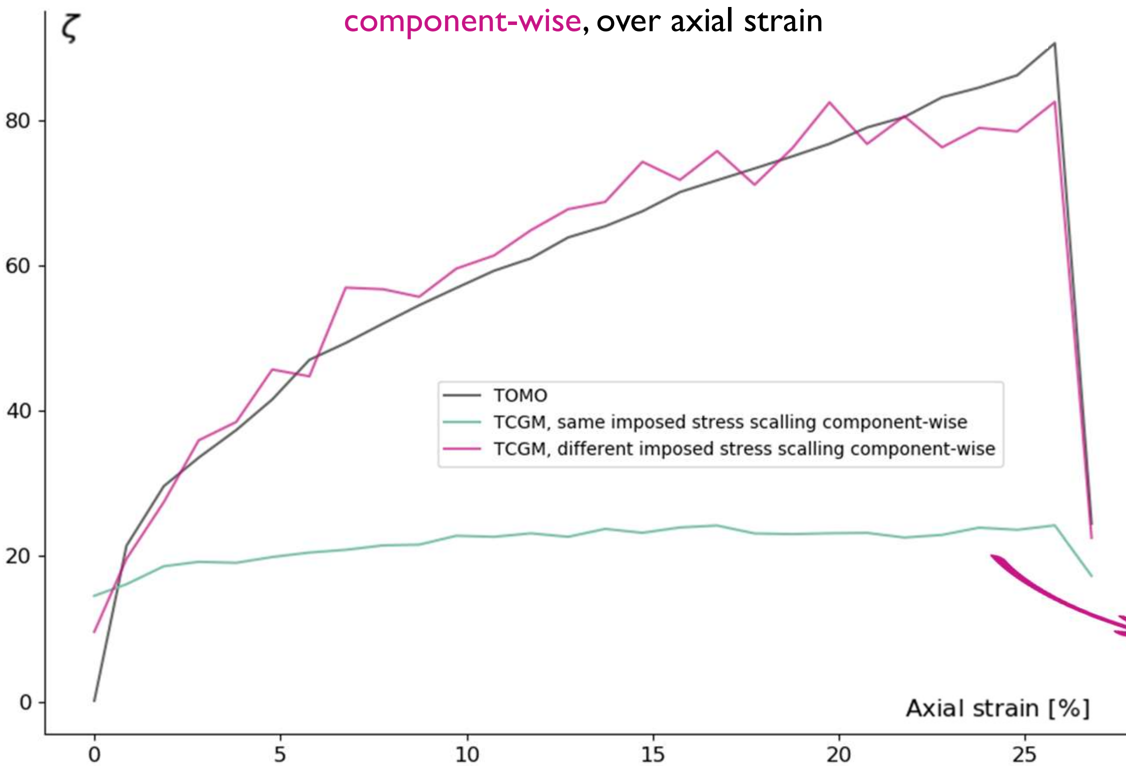


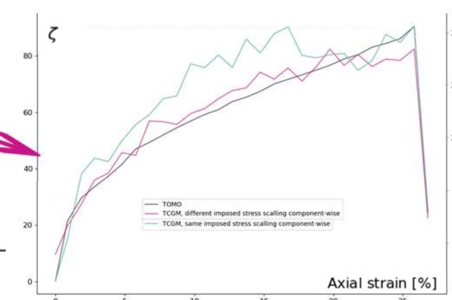
Normalized relative error of TCGM diagonal average bulk stress components for trials with the same/different imposed stress scaling component-wise, over axial strain

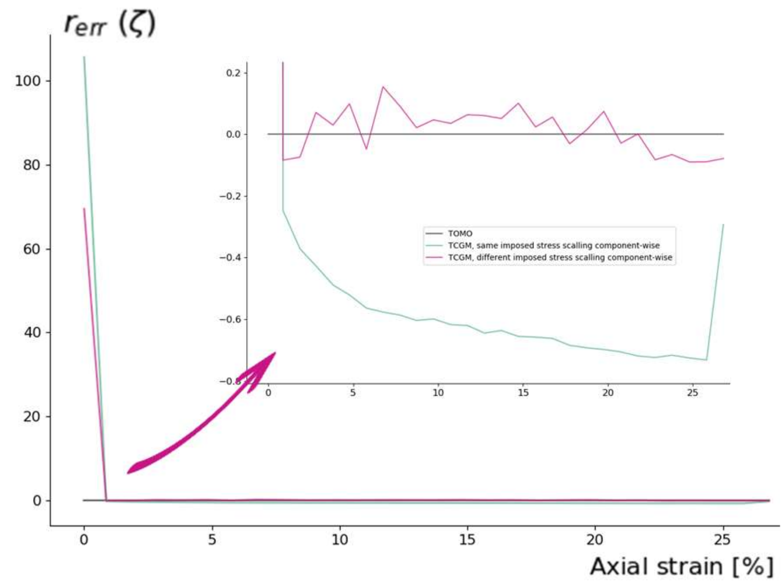
RESULTS

AVERAGE BULK STRESSES

Comparison of TOMO and TCGM **degree of anisotropy** for trials with the same/different imposed stress scaling component-wise, over axial strain





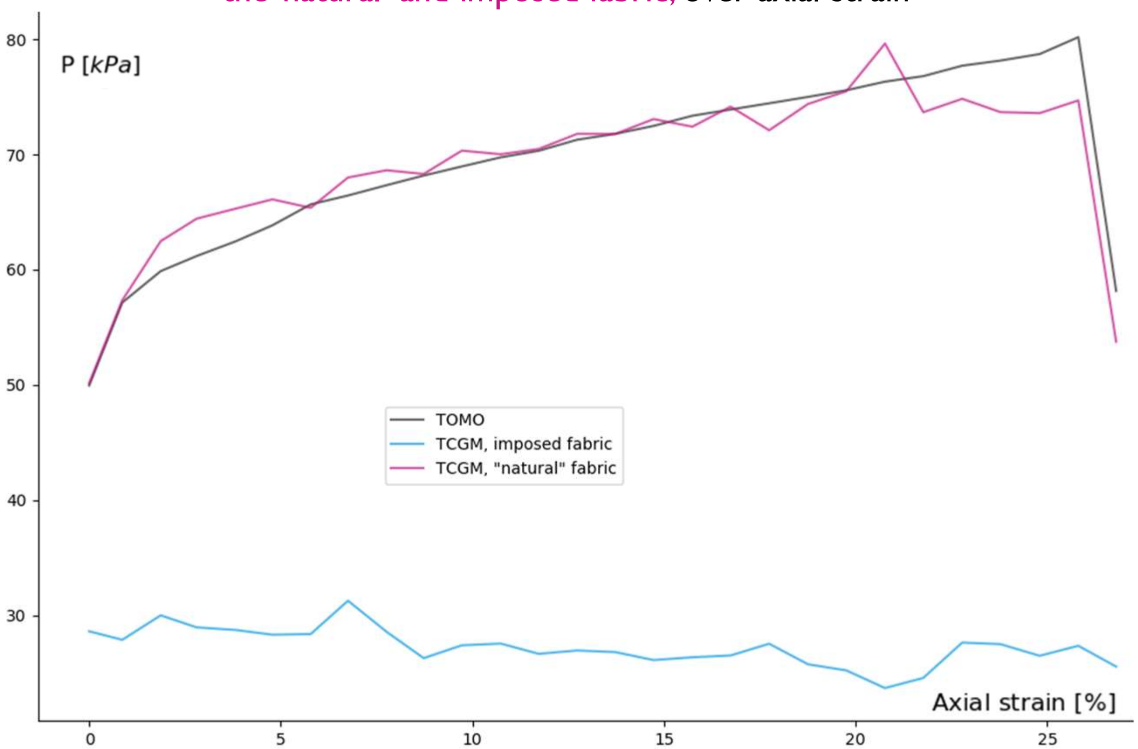


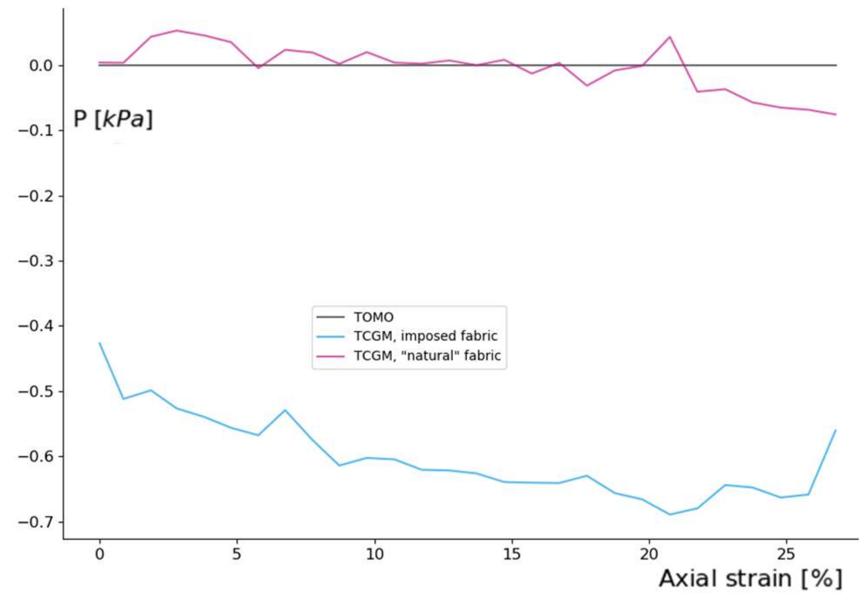
Normalized relative error of TCGM degree of anisotropy for trials with the same/different imposed stress scaling component-wise, over axial strain

RESULTS

AVERAGE BULK STRESSES

Comparison of TOMO and TCGM **pressures** for trials with the 'natural' and imposed fabric, over axial strain



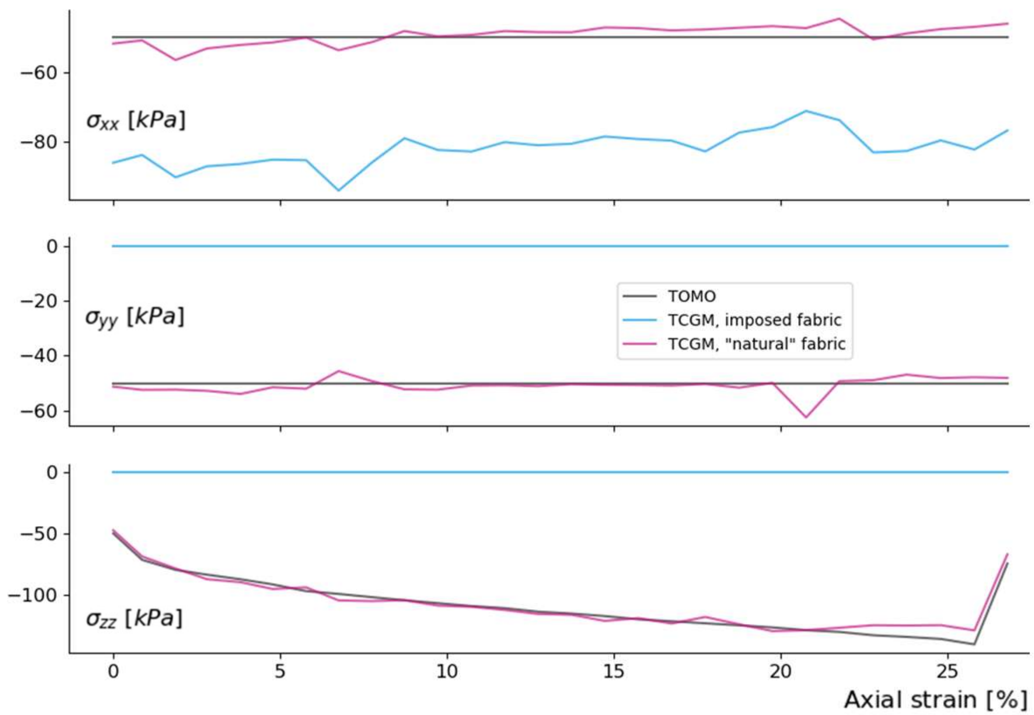


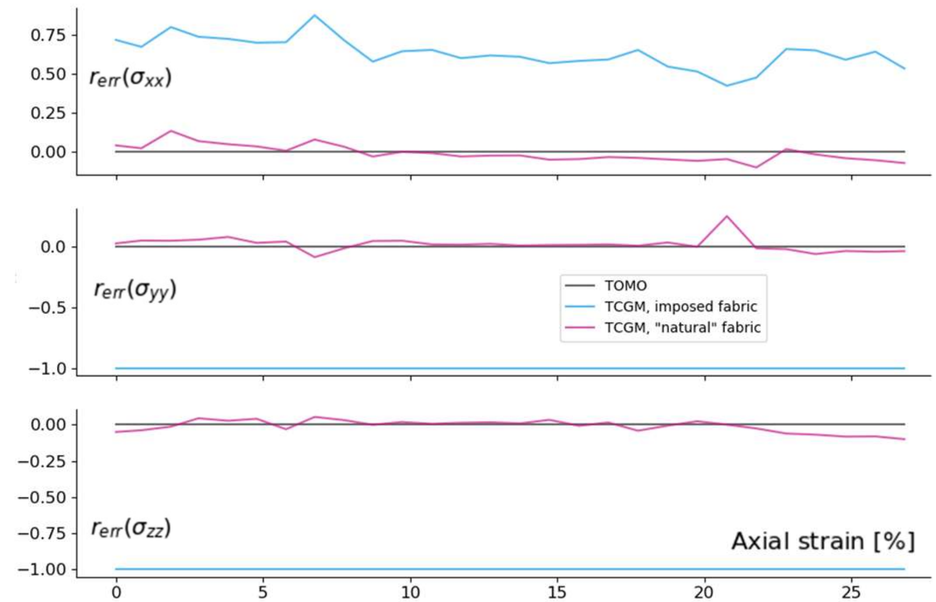
Normalized relative error of TCGM pressure for trials with the 'natural' and imposed fabric, over axial strain

RESULTS

AVERAGE BULK STRESSES

Comparison of TOMO and TCGM **diagonal average bulk stress components** for trials with the 'natural' and imposed fabric, over axial strain



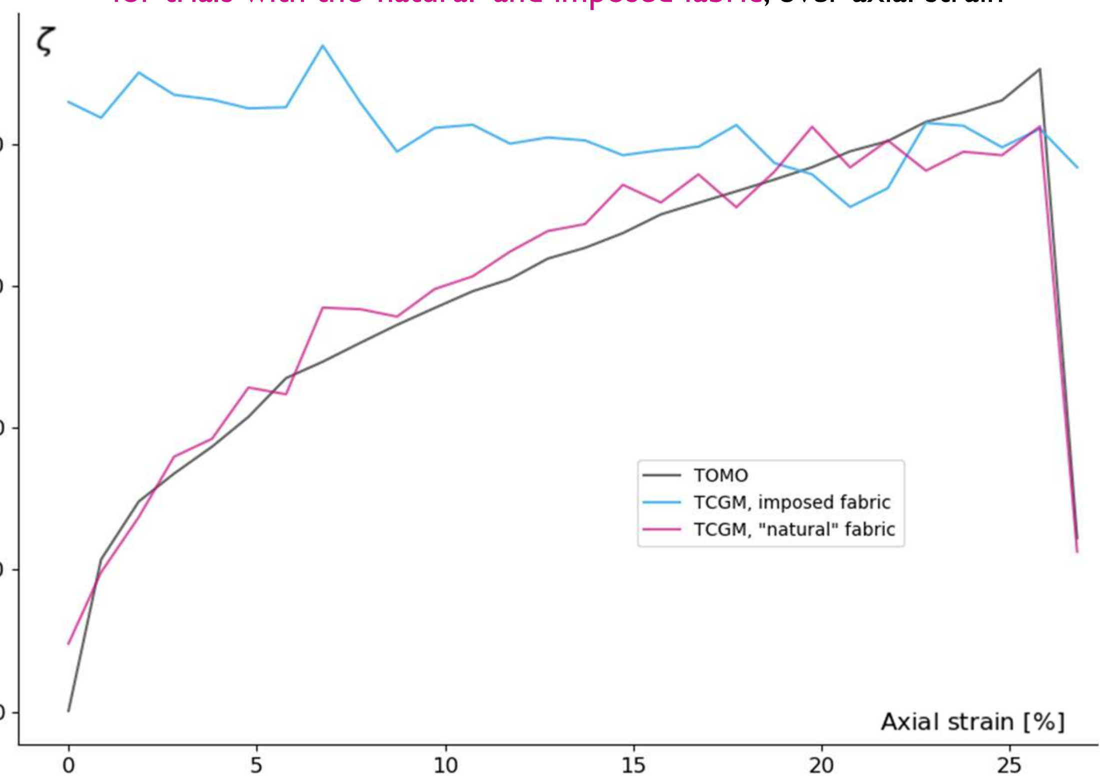


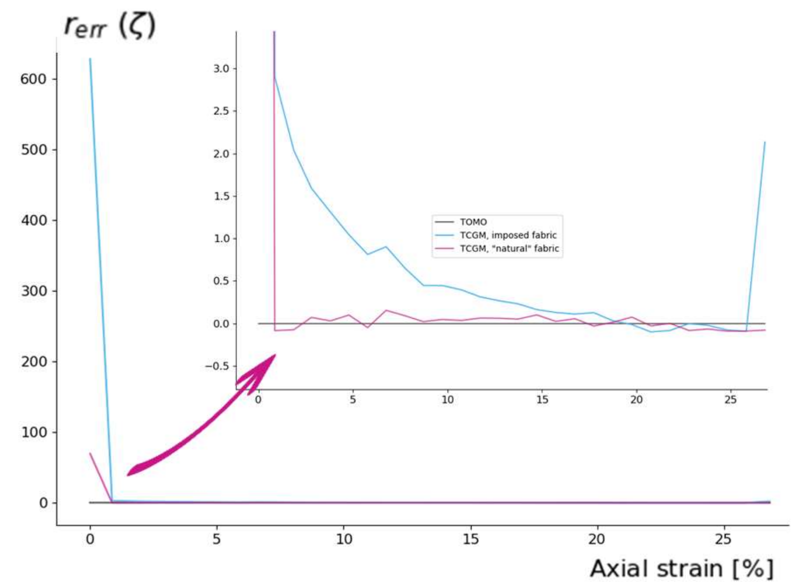
Normalized relative error of TCGM diagonal average bulk stress components for trials with the 'natural' and imposed fabric, over axial strain

RESULTS

AVERAGE BULK STRESSES

Comparison of TOMO and TCGM **degree of anisotropy** for trials with the 'natural' and imposed fabric, over axial strain





Normalized relative error of TCGM degree of anisotropy for trials with the 'natural' and imposed fabric, over axial strain

CONCLUSION

CONCLUSION

- Both imposed stress scaling and bulk fabric have influence on trends of degree of anisotropy and stress components curves:
  - fabric influences the curve shape of the progression of the degree of anisotropy and stress scaling scales it, whereas for macro stress response the influence cannot be separated

- Only together both influences yield proper outcome:
  - Correct fabric and wrong imposed stress scaling yield wrong outcome
  - Wrong fabric and correct imposed stress scaling yield wrong outcome
  - Correct fabric and correct imposed stress scaling yield best results

- Choice of contact directional data to construct the fabric influences curve trends of both average bulk stress components and degree of anisotropy not too significantly www.caliper-itn.org

Funded by EC Marie Skłodowska Curie grant agreement No 812638

THANKS FOR YOUR TIME!

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ADDITION

METHOD

FABRIC AND STRESS TENSORS

…and different levels are connectable:

$\kappa_{bulk}^{n+1,ab}$

$$\sigma_{bulk}^{n+1,ab} = func\left(\kappa_{bulk}^{n+1}, A_{bulk}^{n+1,ab}\right)$$

$$A_{bulk}^{n+1,ab} = func\left(w_{I}^{n+1}, w_{bulk}^{n+1}, A_{I}^{n+1,ab}\right), \quad I \in \{I\}$$

$$A_{bulk}^{n+1,ab} = func\left(w_{IJ}^{n+1}, w_{bulk}^{n+1}, A_{IJ}^{n+1,ab}\right)$$
$$IJ \in \{IJ\}$$

$$\sigma_{bulk}^{n+1,ab} = func\left(\kappa_{I}^{n+1}, \kappa_{bulk}^{n+1}, A_{I}^{n+1,ab}\right)$$

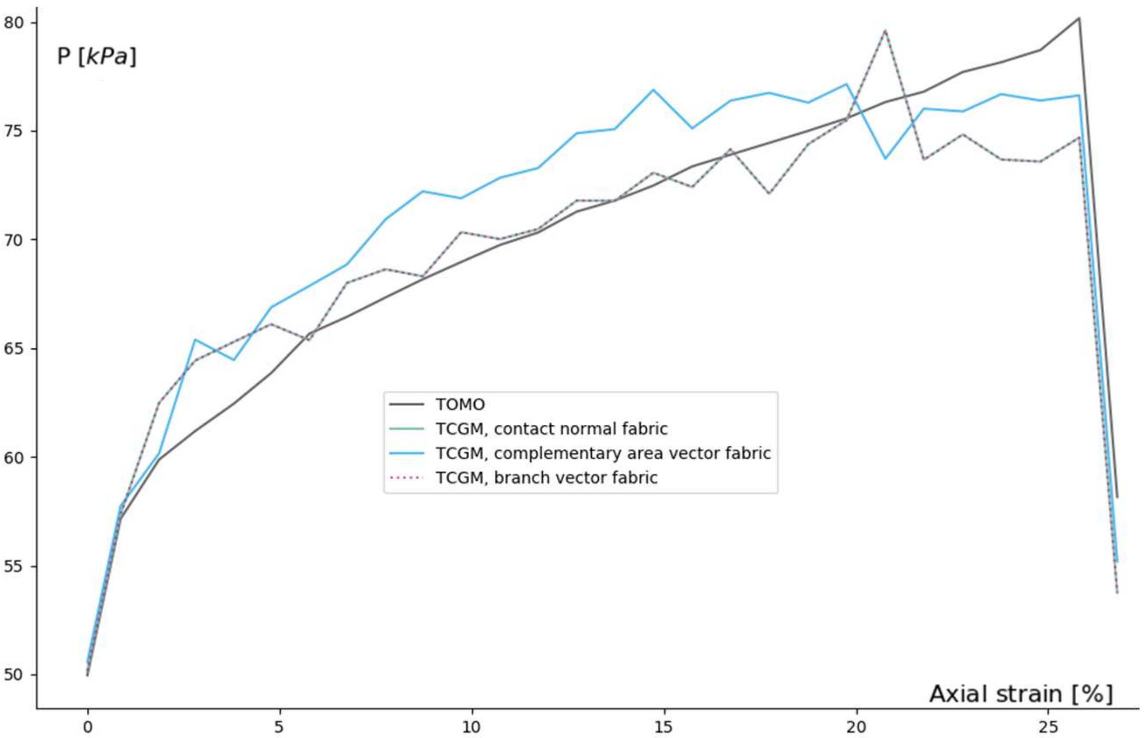
$$\sigma_{bulk}^{n+1,ab} = func\left(\kappa_{IJ}^{n+1}, \kappa_{bulk}^{n+1}, A_{IJ}^{n+1,ab}\right)$$

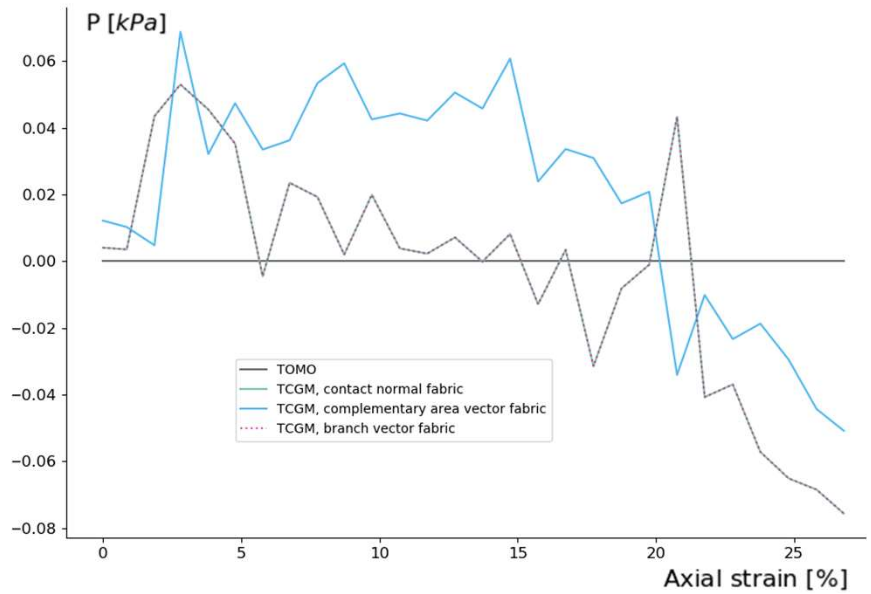
$$\begin{tikzcd}
 & A_{bulk}^{n+1,ab} \arrow[dr] & & \\
\sigma_{bulk}^{n+1,ab} \arrow[ur] \arrow[dr] & & A_{I}^{n+1,ab} \arrow[dr] & \\
 & \sigma_{I}^{n+1,ab} \arrow[ur] \arrow[dr] & & A_{IJ}^{n+1,ab} \arrow[dl] \\
 & & \sigma_{IJ}^{n+1,ab} &
\end{tikzcd}$$

RESULTS

AVERAGE BULK STRESSES

Comparison of TOMO and TCGM **pressures** for trials with different contact fabric, over axial strain



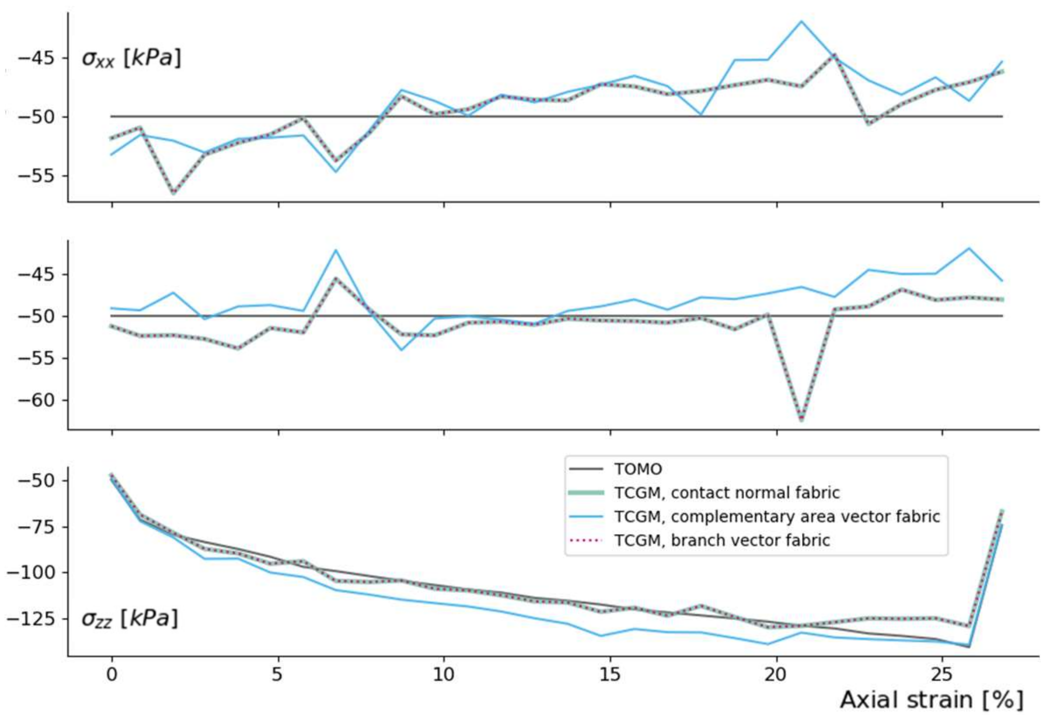


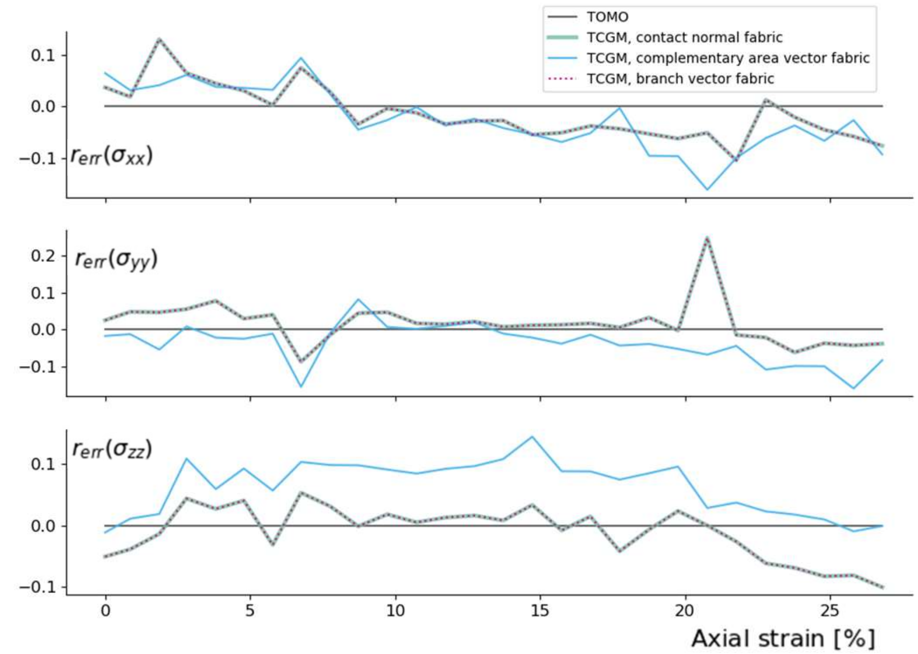
Normalized relative error of TCGM pressure for trials with different contact fabric, over axial strain

RESULTS

AVERAGE BULK STRESSES

Comparison of TOMO and TCGM **diagonal average bulk stress components** for trials with different contact fabric, over axial strain



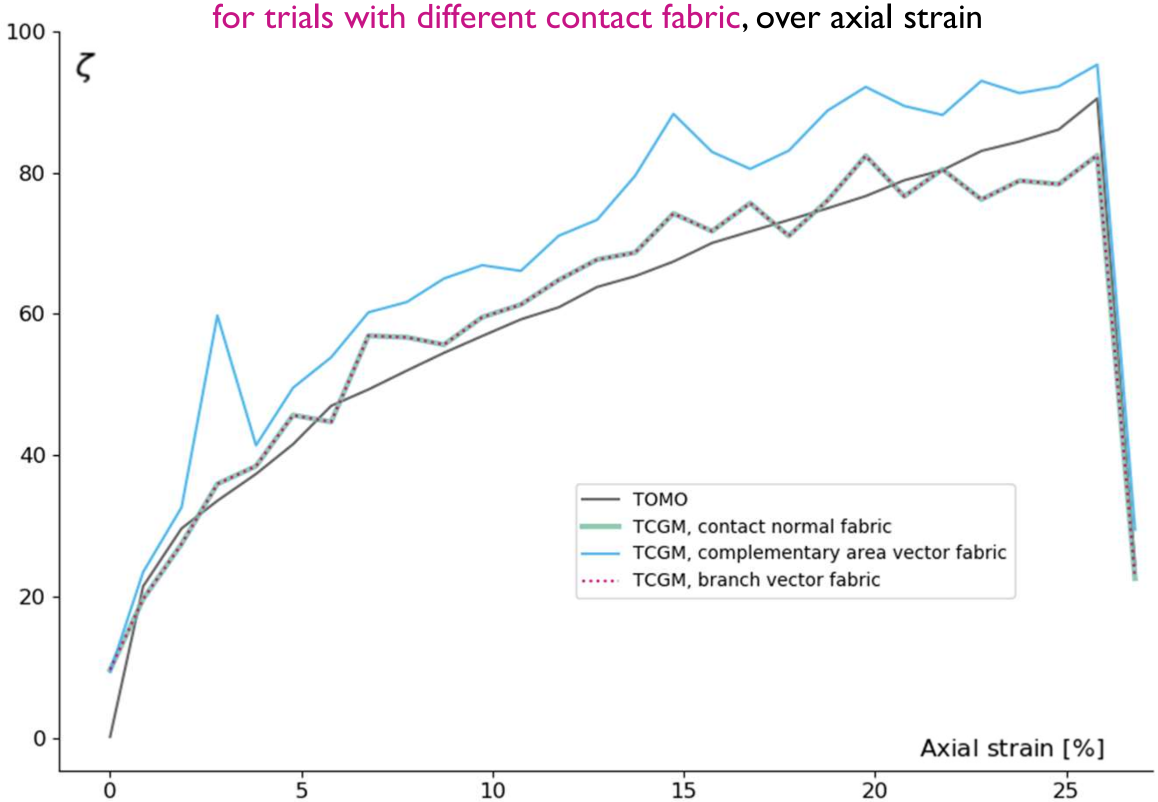


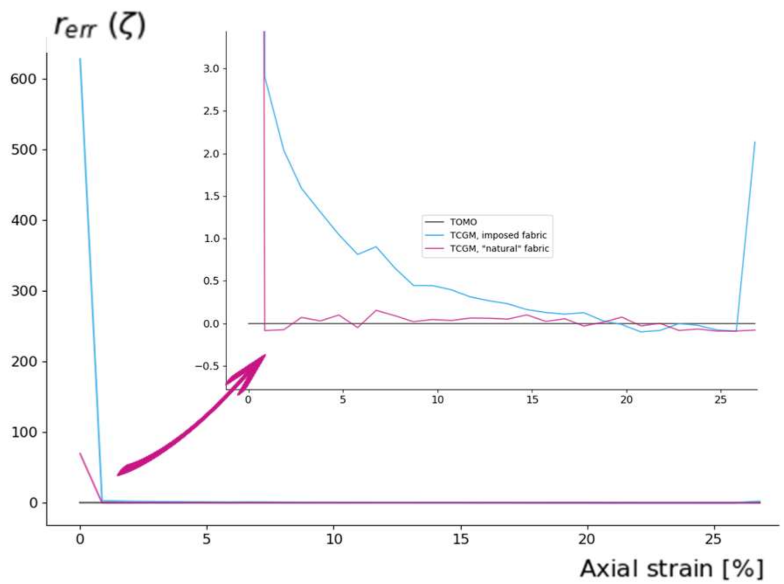
Normalized relative error of TCGM diagonal average bulk stress components for trials with different contact fabric, over axial strain

RESULTS

AVERAGE BULK STRESSES

Comparison of TOMO and TCGM **degree of anisotropy** for trials with different contact fabric, over axial strain





Normalized relative error of TCGM degree of anisotropy for trials with different contact fabric, over axial strain