cPCWE – Perturbed Convective Wave Equation based on Compressible Flows

Stefan Schoder
Institute of Fundamentals and Theory in Electrical Engineering (IGTE)
Graz University of Technology
8010 Graz, Austria
stefan.schoder@tugraz.at

Abstract

This paper is a short outlook on the derivation of the compressible variant of the perturbed convective wave equation based on the acoustic perturbation equations. In particular, it formulates the relation of Helmholtz's decomposition to the definition of the acoustic potential and the source potential. The details of an implementation are presented algorithmically.

Keywords Aeroacoustics · Fluid dynamics · Acoustics · Helmholtz's decomposition · Flow solver

1 Introduction

Aeroacoustic analogies compute noise radiation efficiently but the obtained joint fluctuating field converges to the acoustic field in steady flow regions only. As first recognized by Phillips [1] and Lilley [2], the source terms responsible for mean flow-acoustics interactions should be part of the wave operator.

Therefore, another approach for computing aeroacoustics is based on a systematic decomposition of the field properties. This circumvents that sources depend on the acoustic solution and provides a rigorous definition of acoustics. Ribner [3] formulated his dilatation equation such that the primary fluctuating pressure variable of Lighthill's analogy is decomposed in a pseudo pressure and an acoustic pressure part $p' = p^0 + p^{\mathrm{a}}$. Hardin and Pope [4] formulated their viscous/acoustic splitting technique expansion about the incompressible flow (EIF), where they introduced a density correction ρ_1. The EIF formulation was modified over the years substantially [5, 6, 7]. For the sake of being more general and starting from the linearized Euler equations (LEE), the field variables $(\rho, \boldsymbol{u}, p)$ are each decomposed in a temporal mean component $\overline{(\star)}$ and a fluctuating component $(\star)'$. Bailly *et al.* [8, 9] indicates significant aeroacoustic source terms on the momentum equation of the LEE. Over the years, the LEE were modified to guarantee that only acoustic waves are propagated [10]. Significant contributions based on the LEE were derived by [11, 12, 13, 14, 15]. Ewert and Schröder [16] proposed a different filtering technique for a uniform flow field leading to the acoustic perturbation equations (APE). Instead of filtering the flow field, the source terms of the wave equations are filtered according to the characteristic properties of the acoustic modes, obtained from LEE. Hüppe [17] derived a computationally efficient reformulation of the APE-2 system and named it perturbed convective wave equation (PCWE). Since then, a number of low Mach number flow applications have been addressed by computational aeroacoustics using the PCWE model sucessfully [18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32]. The PCWE is only valid for incompressible flows. A first attempt to establish a scalar wave equation for intermediate Mach number flows was made by Spieser, who developed the AWE-PO [30, 33]. With a focus on a strict coupling and also relying on a strict definition of the variables, Ewert and Kreuzinger developed a computational workflow for low Mach numbers recently [34]. In this sense, seeking a clear definition and an accurate formulation for a large variety of Mach number flows is still a challenge worth investigating. With the idea of APE and with the use of Helmholtz decomposition, we aim to extend the idea of the efficient PCWE equation for all subsonic Mach number flows (the so-called cPCWE).

2 Theory

Regarding the state-of-the-art analysis, we will derive our wave equation based on the fundamental ideas of [16]. To sum up, the ideas of defining stringent acoustics are already in place for an incompressible flow. Therefore, we aim to generalize (methodologically and physically) the concept and combine it with source filtering. We collect the ideas for incompressible flows, join them to a valid concept (methodological generalization), and further extend the concept to compressible flows (physical generalization). The derived aeroacoustic model should lead to a stable scalar wave equation that solves an (approximated) compressible potential for isothermal/thermal flows. The wave operator should only excite longitudinal wave modes (acoustic modes) and include convection. The physical goal of this new wave equation is that it holds for a wider Mach number range, and in the limit, $\mathrm{Ma} \to 0$ the PCWE [35] is recovered.

Based on the preliminary work, we derive a convective wave equation based on compressible source data (cPCWE). As used during the derivation of the APE system, we first apply linearization and then distinguish between vortical and acoustical perturbations. The cPCWE can be derived efficiently from the APE-1 system [16] with the decomposition introduced by Hardin and Pope [4]

$$p = p_0 + p' = \bar{p} + p_\mathrm{v} + p_\mathrm{a} \,; \qquad \rho = \rho_0 + \rho' = \rho_0 + \rho_1 + \rho_\mathrm{a} \tag{1}$$

$$\boldsymbol{u} = \boldsymbol{u}_0 + \boldsymbol{u}' = \boldsymbol{u}_0 + \boldsymbol{u}_\mathrm{v} + \boldsymbol{u}_\mathrm{a} = \boldsymbol{u}_0 + \nabla \times \boldsymbol{A} + \nabla \psi_\mathrm{a} \,. \tag{2}$$

Regarding the definition of Hardin and Pope, we generalize the decomposition into a vortical and compressible acoustic part [36]. We arrive from a general compressible flow at the following perturbation equations

$$\frac{\partial p'}{\partial t} + \boldsymbol{u}_0 \cdot \nabla p' + \rho_0 c_0^2 \nabla \cdot \boldsymbol{u}_\mathrm{a} = 0 \tag{3}$$

$$\rho_0 \frac{\partial \boldsymbol{u}_\mathrm{a}}{\partial t} + \rho_0 \nabla \big(\boldsymbol{u}_0 \cdot \boldsymbol{u}_\mathrm{a}\big) + \nabla p' = \rho_0 \nabla \Phi_\mathrm{p} \tag{4}$$

with spatial uniform mean density ρ_0 (isobaric mean flow for the isothermal case), an incompressible mean velocity $\boldsymbol{u}_0$, and the isentropic speed of sound c_0. We neglect thermal, viscous effects and discard the vorticity mode according to the derivation of APE. We define the compressible acoustic field as irrotational by the acoustic scalar potential ψ_a as a gradient field of the acoustic particle velocity $\boldsymbol{u}_\mathrm{a} = -\nabla\psi_\mathrm{a}$. We rewrite equation (4) and arrive at the definition of the fluctuating pressure

$$p' = \rho_0 \frac{\partial \psi_\mathrm{a}}{\partial t} + \rho_0\, \boldsymbol{u}_0 \cdot \nabla \psi_\mathrm{a} + \rho_0 \Phi_\mathrm{p} = \rho_0 \frac{D\psi_\mathrm{a}}{Dt} + \rho_0 \Phi_\mathrm{p} \,, \tag{5}$$

without further constraints on the mean velocity. The first term accounts for the acoustic pressure and the second term is the vortical pressure of the Laplace filtering [16]

$$\Delta \Phi_\mathrm{p} = -\nabla \cdot \left[((\boldsymbol{u}_\mathrm{v} \cdot \nabla)\boldsymbol{u}_\mathrm{v})' + (\boldsymbol{u}_0 \cdot \nabla)\boldsymbol{u}_\mathrm{v} + (\boldsymbol{u}_\mathrm{v} \cdot \nabla)\boldsymbol{u}_0 \right] \tag{6}$$

The first part includes the self-noise of vortical structures, and the second and third terms the shear noise interactions. Substituting (5) into (3) yields the cPCWE

$$\frac{1}{c_0^2} \frac{D^2 \psi_\mathrm{a}}{Dt^2} - \Delta \psi_\mathrm{a} = -\frac{1}{\rho_0 c_0^2} \frac{D \Phi_\mathrm{p}}{Dt} \,. \tag{7}$$

This convective wave equation fully describes acoustic sources generated by compressible flow structures and their wave propagation through flowing media. In addition, instead of the original unknowns p_a and $\boldsymbol{v}_\mathrm{a}$, we have just one scalar ψ_a. As shown in [33] and consistent with the pressure correction equation, the fluctuating vortical pressure in the overall domain can be recovered by

$$p_\mathrm{v} = \rho_0 \Phi_\mathrm{p} \,. \tag{8}$$

Finally, we have derived a precise scalar formulation that separates the source generation processes of vortical flows and the linear acoustic propagation.

3 Computational workflow

The algorithm 1 can now execute the procedure. In addition to the workload of the AWE-PO, we solve another elliptic problem (Laplace equation) to eliminate the compressible part of the flow velocity. This can be done by the same operator matrix when having no internal wall boundaries. Different from the AWE-PO and from an algorithmic view, we have to do four tasks. Firstly, we are solving the compressible flow equations (requested DNS results from the collaborators). Secondly, we solve the Laplace-filtering to obtain the filtered velocity in every time step

$$\Delta\, s = \nabla \cdot \boldsymbol{u} \quad \text{with} \quad \boldsymbol{u}_\mathrm{v} = \boldsymbol{u} - \nabla s \tag{9}$$

Algorithm 1 Compressible PCWE (cPCWE)

Require: Compressible Flow Simulation
$(\rho, \boldsymbol{u}, p) = \mathrm{fun}(Setup)$
Require: Source computation
$(\boldsymbol{u}_{\mathrm{v}}) = \mathrm{Laplace-filter}(\boldsymbol{u})$
$(\Phi_{\mathrm{p}}) = \mathrm{Laplace-filter}(\boldsymbol{u}_0, \boldsymbol{u}_{\mathrm{v}})$
$(D_{u_0}\Phi_{\mathrm{p}}) = \mathrm{fun}(\Phi_{\mathrm{p}})$
$(f_e) = \mathrm{integrate}(D_{u_0}\Phi_{\mathrm{p}})$
Require: Acoustic simulation
$(\rho_{\mathrm{a}}, \boldsymbol{u}_{\mathrm{a}}, p_{\mathrm{a}}) = \mathrm{fun}(f_e, Setup)$

but every time step can be processed independently. The filtered velocity is used to assemble the cPCWE source. Thirdly, we must filter this source with the same operator matrix (6) for every time step independently. And finally, we are solving the wave propagation simulation for the time series. To sum up, depending on the number of processors and processes one can submit, the algorithm 1 is of similar complexity than the AWE-PO and the original PCWE. Furthermore, the two filtering stages can be highly parallelized. In the presence of internal wall boundaries near the sources, proper wall boundary conditions must be found for the PDEs of the filtering steps. Furthermore, the velocity filtering step will be the solution to a more complicated curl-curl equation.

4 Conclusion

This short working paper presents a draft derivation of the cPCWE and we are hppy to receive feedback.

References

[1] O. M. Phillips, On the generation of sound by supersonic turbulent shear layers, Journal of Fluid Mechanics 9 (1) (1960) 1–28.

[2] G. M. Lilley, On the noise from jets, AGARD-CP-131, 1974.

[3] H. S. RIBNER, AERODYNAMIC SOUND FROM FLUID DILATATIONS - A Theory of the Sound from Jets and Other Flows, Tech. rep., Institute for Aerospace Studies, University of Toronto (1962).

[4] J. C. Hardin, D. S. Pope, An acoustic/viscous splitting technique for computational aeroacoustics, Theoretical and Computational Fluid Dynamics 6 (5-6) (1994) 323–340.

[5] W. Z. Shen, J. N. Sørensen, Aeroacoustic modelling of low-speed flows, Theoretical and Computational Fluid Dynamics 13 (4) (1999) 271–289.

[6] W. Z. Shen, J. N-oacute, R. S-oacute, R. S-oacute, Comment on the aeroacoustic formulation of Hardin and Pope, AIAA journal 37 (1) (1999) 141–143.

[7] S. A. Slimon, M. C. Soteriou, D. W. Davis, Computational aeroacoustics simulations using the expansion about incompressible flow approach, AIAA journal 37 (4) (1999) 409–416.

[8] C. Bailly, D. Juve, Numerical solution of acoustic propagation problems using linearized Euler equations, AIAA journal 38 (1) (2000) 22–29.

[9] C. Bogey, C. Bailly, D. Juvé, Computation of flow noise using source terms in linearized Euler's equations, AIAA journal 40 (2) (2002) 235–243.

[10] W. De Roeck, G. Rubio, M. Baelmans, W. Desmet, Toward accurate hybrid prediction techniques for cavity flow noise applications, International journal for numerical methods in fluids 61 (12) (2009) 1363–1387.

[11] C.-D. Munz, M. Dumbser, M. Zucchini, The multiple pressure variables method for fluid dynamics and aeroacoustics at low Mach numbers, Numerical methods for hyperbolic and kinetic problems 7 (2003) 335–359.

[12] M. Munz, C.-D.and Dumbser, S. Roller, Linearized acoustic perturbation equations for low Mach number flow with variable density and temperature, Journal of Computational Physics 224 (1) (2007) 352–364.

[13] S. Roller, T. Schwartzkopff, R. Fortenbach, M. Dumbser, C.-D. Munz, Calculation of low Mach number acoustics: a comparison of MPV, EIF and linearized Euler equations, ESAIM: Mathematical Modelling and Numerical Analysis 39 (3) (2005) 561–576.

[14] J. Seo, Y. J. Moon, Perturbed compressible equations for aeroacoustic noise prediction at low mach numbers, AIAA journal 43 (8) (2005) 1716–1724.

[15] J. H. Seo, Y. J. Moon, Linearized perturbed compressible equations for low Mach number aeroacoustics, Journal of Computational Physics 218 (2) (2006) 702–719.

[16] R. Ewert, W. Schröder, Acoustic perturbation equations based on flow decomposition via source filtering, Journal of Computational Physics 188 (2) (2003) 365–398.

[17] A. Hüppe, J. Grabinger, M. Kaltenbacher, A. Reppenhagen, G. Dutzler, W. Kühnel, A Non-Conforming Finite Element Method for Computational Aeroacoustics in Rotating Systems, in: 20th AIAA/CEAS Aeroacoustics Conference, 2014, p. 2739.

[18] S. Schoder, M. Kaltenbacher, Hybrid aeroacoustic computations: State of art and new achievements, Journal of Theoretical and Computational Acoustics 27 (04) (2019) 1950020.

[19] S. Schoder, M. Weitz, P. Maurerlehner, A. Hauser, S. Falk, S. Kniesburges, M. Döllinger, M. Kaltenbacher, Hybrid aeroacoustic approach for the efficient numerical simulation of human phonation, The Journal of the Acoustical Society of America 147 (2) (2020) 1179–1194.

[20] S. Schoder, C. Junger, M. Kaltenbacher, Computational aeroacoustics of the eaa benchmark case of an axial fan, Acta Acustica 4 (5) (2020) 22.

[21] S. Schoder, A. Wurzinger, C. Junger, M. Weitz, C. Freidhager, K. Roppert, M. Kaltenbacher, Application limits of conservative source interpolation methods using a low mach number hybrid aeroacoustic workflow, Journal of Theoretical and Computational Acoustics 29 (01) (2021) 2050032.

[22] S. Schoder, P. Maurerlehner, A. Wurzinger, A. Hauser, S. Falk, S. Kniesburges, M. Döllinger, M. Kaltenbacher, Aeroacoustic sound source characterization of the human voice production-perturbed convective wave equation, Applied Sciences 11 (6) (2021) 2614.

[23] S. Schoder, C. Junger, M. Weitz, M. Kaltenbacher, Conservative source term interpolation for hybrid aeroacoustic computations, in: 25th AIAA/CEAS aeroacoustics conference, 2019, p. 2538.

[24] S. Falk, S. Kniesburges, S. Schoder, B. Jakubaß, P. Maurerlehner, M. Echternach, M. Kaltenbacher, M. Döllinger, 3d-fv-fe aeroacoustic larynx model for investigation of functional based voice disorders, Frontiers in physiology 12 (2021) 616985.

[25] M. Lasota, P. Šidlof, M. Kaltenbacher, S. Schoder, Impact of the sub-grid scale turbulence model in aeroacoustic simulation of human voice, Applied Sciences 11 (4) (2021) 1970.

[26] P. Maurerlehner, S. Schoder, C. Freidhager, A. Wurzinger, A. Hauser, F. Kraxberger, S. Falk, S. Kniesburges, M. Echternach, M. Döllinger, et al., Efficient numerical simulation of the human voice, e & i Elektrotechnik und Informationstechnik 138 (3) (2021) 219–228.

[27] S. Schoder, K. Roppert, opencfs: Open source finite element software for coupled field simulation–part acoustics, arXiv preprint arXiv:2207.04443 (2022).

[28] S. Schoder, F. Kraxberger, S. Falk, A. Wurzinger, K. Roppert, S. Kniesburges, M. Döllinger, M. Kaltenbacher, Error detection and filtering of incompressible flow simulations for aeroacoustic predictions of human voice, The Journal of the Acoustical Society of America 152 (3) (2022) 1425–1436.

[29] L. Tieghi, S. Becker, A. Corsini, G. Delibra, S. Schoder, F. Czwielong, Machine-learning clustering methods applied to detection of noise sources in low-speed axial fan, Journal of Engineering for Gas Turbines and Power (2022).

[30] S. Schoder, M. Kaltenbacher, É. Spieser, H. Vincent, C. Bogey, C. Bailly, Aeroacoustic wave equation based on pierce's operator applied to the sound generated by a mixing layer, in: 28th AIAA/CEAS Aeroacoustics 2022 Conference, 2022, p. 2896.

[31] M. Tautz, Aeroacoustic Noise Prediction of Automotive HVAC Systems, FAU University Press, 2019.

[32] J. Valášek, M. Kaltenbacher, P. Sváček, On the application of acoustic analogies in the numerical simulation of human phonation process, Flow, Turbulence and Combustion 102 (1) (2019) 129–143. doi:10.1007/s10494-018-9900-z.

[33] E. Spieser, Modélisation de la propagation du bruit de jet par une méthode adjointe formulée pour l'acoustique potentielle, Ph.D. thesis, University Lyon (2020).

[34] R. Ewert, J. Kreuzinger, Hydrodynamic/acoustic splitting approach with flow-acoustic feedback for universal subsonic noise computation, Journal of Computational Physics 444 (2021) 110548. doi:10.1016/j.jcp.2021.110548.

[35] M. Kaltenbacher, A. Hüppe, A. Reppenhagen, F. Zenger, S. Becker, Computational aeroacoustics for rotating systems with application to an axial fan, AIAA journal (2017) 3831–3838.

[36] S. Schoder, Aeroacoustic analogies based on compressible flow data, Ph.D. thesis, TU Wien (2019). doi:10.34726/hss.2018.40525.