Change Point Detection with Stable AR(1) Errors

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1 Introduction and Results

In this paper we are interested to detect possible changes in the location model

$$X_j = c_j + e_j, \quad 1 \leq j \leq n. \tag{1}$$

We wish to test the null hypothesis of stability of the location parameter, i.e.,

$$H_0 : \quad c_1 = c_2 = \ldots = c_n$$

against the one change alternative

$$H_A : \quad \text{there is } \; k^* \text{ such that } \; c_1 = \ldots = c_{k^*} \neq c_{k^*+1} = \ldots = c_n.$$

We say that k^* is the time of change under the alternative. The time of change as well as the location parameters before and after the change are unknown. The most popular methods to test H_0 against H_A are based on the CUSUM process

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$$U_n(x) = \sum_{i=1}^{\lfloor nx \rfloor} X_i - \frac{\lfloor nx \rfloor}{n} \sum_{i=1}^{n} X_i.$$

Clearly, if H_0 is true, then $U_n(t)$ does not depend on the common but unknown location parameter. It is well known that if $X_1, \ldots, X_n$ are independent and identically distributed random variables with a finite second moment, then

$$\frac{1}{(n\mathrm{var}(X_1))^{1/2}} U_n(x) \xrightarrow{\mathscr{D}[0,1]} B(x),$$

where $B(x)$ is a Brownian bridge. Throughout this paper $\mathscr{D}[0, 1]$ denotes the space of right continuous functions on $[0, 1]$ with left limits; $\xrightarrow{\mathscr{D}[0,1]}$ means weak convergence in $\mathscr{D}[0, 1]$ with respect to the Skorohod J_1 topology (cf. Billingsley [10]). Of course, $\mathrm{var}(X_1)$ can be consistently estimated by the sample variance in this case, resulting in

$$\frac{1}{\sigma_n^* n^{1/2}} U_n(x) \quad \xrightarrow{\mathscr{D}[0,1]} \quad B(x) \tag{2}$$

with

$$\sigma_n^* = \left\{ \frac{1}{n} \sum_{i=1}^{n} (X_i - \bar{X}_n)^2 \right\}^{1/2} \quad \text{with} \quad \bar{X}_n = \frac{1}{n} \sum_{i=1}^{n} X_i.$$

Assuming that $X_1, X_2, \ldots, X_n$ are independent and identically distributed random variables in the domain of attraction of a stable law of index $\alpha \in (0, 2)$, Aue et al. [3] showed that

$$\frac{1}{n^{1/\alpha} \hat{L}(n)} U_n(x) \xrightarrow{\mathscr{D}[0,1]} B_\alpha(x),$$

where $\hat{L}$ is a slowly varying function at ∞ and $B_\alpha(x)$ is an α–stable bridge. (The α–stable bridge is defined as $B_\alpha(x) = W_\alpha(x) - xW_\alpha(1)$, where W_α is a Lévy α–stable motion.) Since nothing is known on the distributions of the functionals of α–stable bridges, Berkes et al. [9] suggested the trimmed CUSUM process

$$T_n(x) = \sum_{i=1}^{\lfloor nx \rfloor} X_i I\{|X_i| \le \eta_{n,d}\} - \frac{\lfloor nx \rfloor}{n} \sum_{i=1}^{n} X_i I\{|X_i| \le \eta_{n,d}\},$$

where $\eta_{n,d}$ is the dth largest among $|X_1|, |X_2|, \ldots, |X_n|$. Assuming that the X_i's are independent and identically distributed and are in the domain of attraction of a stable law, they proved

$$\frac{1}{\hat{\sigma}_n n^{1/2}} T_n(x) \xrightarrow{\mathscr{D}[0,1]} B(x),$$

where

$$\hat{\sigma}_n^2 = \frac{1}{n}\sum_{i=1}^{n}\left(X_i I\{|X_i| \leq \eta_{n,d}\} - \frac{1}{n}\sum_{j=1}^{n} X_j I\{|X_j| \leq \eta_{n,d}\}\right)^2,$$

and $B(t)$ is a Brownian bridge. Roughly speaking, the classical CUSUM procedure in (2) can be used on the trimmed variables $X_j I\{|X_j| \leq \eta_{n,d}\}, 1 \leq j \leq n$. The CUSUM process has also been widely used in case of dependent variables, but it is nearly always assumed that the observations have high moments and the dependence in the sequence is weak, i.e. the limit distributions of the proposed statistics are derived from normal approximations. For a review we refer to Aue and Horváth [5]. However, very few papers consider the instability of time series models with heavy tails.

Fama [16] and Mandelbrot [23, 24] pointed out that the distributions of commodity and stock returns are often heavy tailed with possibly infinite variance and they started the investigation of time series models where the marginal distributions have regularly varying tails. Davis and Resnick [14, 15] investigated the properties of moving averages with regularly varying tails and obtained non–Gaussian limits for the sample covariances and correlations. Their results were extended to heavy tailed ARCH by Davis and Mikosch [13]. The empirical periodogram was studied by Mikosch et al. [25]. Andrews et al. [1] estimated the parameters of autoregressive processes with stable innovations.

In this paper we study testing H_0 against H_A when the error terms form an autoregressive process of order 1, i.e., e_i is a $\sigma(\varepsilon_j, j \leq i)$ measurable solution of

$$e_i = \rho e_{i-1} + \varepsilon_i \quad -\infty < i < \infty. \tag{3}$$

We assume throughout this paper that

$$\varepsilon_j, -\infty < j < \infty \text{ are independent and identically distributed,} \tag{4}$$

$$\varepsilon_0 \text{ belongs to the domain of attraction of a stable} \tag{5}$$
$$\text{random variable } \xi^{(\alpha)} \text{ with parameter } 0 < \alpha < 2,$$

and

$$\varepsilon_0 \text{ is symmetric when } \alpha = 1. \tag{6}$$

Assumption (5) means that

$$\left(\sum_{j=1}^{n}\varepsilon_j - a_n\right) \bigg/ b_n \xrightarrow{\mathscr{D}} \xi^{(\alpha)} \tag{7}$$

for some numerical sequences a_n and b_n. The necessary and sufficient condition for this is

$$\lim_{t\to\infty}\frac{P\{\varepsilon_0>t\}}{L_*(t)t^{-\alpha}}=p \quad \text{and} \quad \lim_{t\to\infty}\frac{P\{\varepsilon_0\leq -t\}}{L_*(t)t^{-\alpha}}=q \tag{8}$$

for some numbers $p \geq 0, q \geq 0, p+q=1$, where L_* is a slowly varying function at ∞. It is known that (3) has a unique stationary non–anticipative solution if and only if

$$-1<\rho<1. \tag{9}$$

Under assumptions (4), (5), (6), (7), (8), and (9), $\{e_j\}$ is a stationary sequence and $E|e_0|^\kappa<\infty$ for all $0<\kappa<\alpha$ but $E|e_0|^\kappa=\infty$ for all $\kappa>\alpha$. The AR(1) process with stable innovations was considered by Chan and Tran [12], Chan [11], Aue and Horváth [4] and Zhang and Chan [28] who investigated the case when ρ is close to 1 and provided estimates for ρ and the other parameters when the observations do not have finite variances.

The convergence of the finite dimensional distributions of $U_n(x)$ is an immediate consequence of Phillips and Solo [26]. Let $\xrightarrow{fdd}$ denote the convergence of the finite dimensional distributions.

Theorem 1. *If H_0, (3), (4), (5), (6) and (9) hold, then we have that*

$$\frac{1-\rho}{n^{1/\alpha}L_*(n)}U_n(x)\xrightarrow{fdd}B_\alpha(x),$$

where $B_\alpha(x), 0\leq t\leq 1$ is an α–stable bridge.

It has been pointed out by Avram and Taqqu [6, 7] that the convergence of the finite dimensional distributions in Theorem 1 cannot be replaced with weak convergence in $\mathscr{D}[0,1]$. Avram and Taqqu [6, 7] also proved that under further regularity conditions, the convergence of the finite dimensional distributions can be replaced with convergence in $\mathscr{D}[0,1]$ with respect to the M_1 topology. However, the distributions of $\sup_{0\leq x\leq 1}|B_\alpha(x)|dx$ and $\int_0^1 B_\alpha^2(x)dx$ depend on the unknown α and they are unknown for any $0<\alpha<2$.

The statistics used in this paper are based on $T_n(x)$ with a truncation parameter $d=d(n)$ satisfying

$$\lim_{n\to\infty} d(n)/n=0 \tag{10}$$

and

$$d(n)\geq n^\delta \quad \text{with some} \quad 0<\delta<1. \tag{11}$$

Let $F(x) = P\{X_0 \leq x\}$, $H(x) = P\{|X_0| > x\}$ and let $H^{-1}(t)$ be the (generalized) inverse of H. We also assume that ε_0 has a density function $p(t)$ which satisfies

$$\int_{-\infty}^{\infty} |p(t+s) - p(t)|dt \leq C|s| \quad \text{with some} \quad C. \tag{12}$$

Let

$$A_n = d^{1/2} H^{-1}(d/n). \tag{13}$$

The following result was obtained by Bazarova et al. [8]:

Theorem 2. *If H_0, (3), (4), (5), (6) and (9), (10), (11), (12) hold, then we have that*

$$\left(\frac{2-\alpha}{\alpha}\right)^{1/2} \left(\frac{1-\rho}{1+\rho}\right)^{1/2} \frac{T_n(x)}{A_n} \xrightarrow{\mathscr{D}[0,1]} B(x),$$

where $B(x)$ is a Brownian bridge.

The weak convergence in Theorem 2 can be used to construct tests to detect possible changes in the location parameter in model (1). However, the normalizing sequence depends heavily on unknown parameters and they should be replaced with consistent estimators. We discuss this approach in Sect. 2. We show in Sect. 3 that ratio statistics can also be used so we can avoid the estimation of the long run variances.

2 Estimation of the Long Run Variance

The limit result in Theorem 2 is the same as one gets for the CUSUM process in case of weakly dependent stationary variables (cf. Aue and Horváth [5]). Hence we interpret the normalizing sequence as the long run variance of the sum of the trimmed variables. Based on this interpretation we suggest Bartlett type estimators as the normalization.

The Bartlett estimator computed from the trimmed variables $X_i^* = X_i I\{|X_i| \leq \eta_{n,d}\}$ is given by

$$\hat{s}_n^2 = \hat{\gamma}_0 + 2\sum_{j=1}^{n-1} \omega\left(\frac{j}{h(n)}\right)\hat{\gamma}_j,$$

where

$$\hat{\gamma}_j = \frac{1}{n}\sum_{i=1}^{n-j}(X_i^* - \bar{X}_n^*)(X_{i+j}^* - \bar{X}_n^*), \quad \bar{X}_n^* = \frac{1}{n}\sum_{i=1}^{n} X_i^*,$$

$\omega(\cdot)$ is the kernel and $h(\cdot)$ is the length of the window. We assume that $\omega(\cdot)$ and $h(\cdot)$ satisfy the following standard assumptions:

$$\omega(0) = 1, \tag{14}$$

$$\omega(t) = 0 \quad \text{if} \quad t > a \quad \text{with some} \quad a > 0, \tag{15}$$

$$\omega(\cdot) \quad \text{is a Lipschitz function,} \tag{16}$$

$$\hat{\omega}(\cdot), \text{ the Fourier transform of } \omega(\cdot), \text{ is also Lipschitz and integrable} \tag{17}$$

and

$$h(n) \to \infty \quad \text{and} \quad h(n)/n \to \infty \quad \text{as} \quad n \to \infty. \tag{18}$$

For functions satisfying (14), (15), (16), and (17) we refer to Taniguchi and Kakizawa [27]. Following the methods in Liu and Wu [22] and Horváth and Reeder [18], the following weak law of large numbers can be established under H_0:

$$\frac{n\hat{s}_n^2}{A_n^2(1+\rho)\alpha/((1-\rho)(2-\alpha))} \xrightarrow{P} 1, \quad \text{as} \quad n \to \infty. \tag{19}$$

The next result is an immediate consequence of Theorem 2 and (19).

Corollary 1. *If H_0, (3), (4), (5), (6), (9), (10), (11), (12) and* (19) *hold, then we have that*

$$\frac{T_n(x)}{n^{1/2}\hat{s}_n} \xrightarrow{\mathscr{D}[0,1]} B(x),$$

where $B(x)$ is a Brownian bridge.

It follows immediately that under the no change null hypothesis

$$\hat{\mathscr{Q}}_n = \sup_{0 \le x \le 1} \frac{|T_n(x)|}{n^{1/2}\hat{s}_n} \xrightarrow{\mathscr{D}} \sup_{0 \le x \le 1} |B(x)|.$$

Simulations show that $\hat{s}_n$ performs well under H_0 but it overestimates the norming sequence under the alternative. Hence $\hat{\mathscr{Q}}_n$ has little power. The estimation of the long–run variance when a change occurs has been addressed in the literature. We follow the approach of Antoch et al. [2], who provided estimators for the long run variance which are asymptotically consistent under H_0 as well as under the one change alternative. Let x_0 denote the smallest value in $[0, 1]$ where $|T_n(x)|$ reaches its maximum and let $\tilde{k} = \lfloor x_0 n \rfloor$. The modified Bartlett estimator is defined as

$$\tilde{s}_n^2 = \hat{\gamma}_0' + 2\sum_{j=1}^{n-1} \omega\left(\frac{j}{h(n)}\right)\tilde{\gamma}_j,$$

where

$$\tilde{\gamma}_j = \frac{1}{n-j}\sum_{\ell=1}^{n-j} \iota_\ell \iota_{\ell+j}, \qquad \iota_\ell = X_\ell^* - \frac{1}{\hat{k}}\sum_{\ell=1}^{\hat{k}} X_\ell^*, \quad \ell = 1, \dots, \hat{k},$$

$$\iota_\ell = X_\ell^* - \frac{1}{n-\hat{k}}\sum_{\ell=\hat{k}+1}^{n} X_\ell^*, \quad \ell = \hat{k}+1, \dots, n.$$

Combining the proofs in Antoch et al. [2] with Liu and Wu [22] and Horváth and Reeder [18] one can verify that

$$\frac{n\tilde{s}_n^2}{A_n^2(1+\rho)\alpha/((1-\rho)(2-\alpha))} \xrightarrow{P} 1, \quad \text{as} \quad n \to \infty \tag{20}$$

under H_0 as well as under the one change alternative H_A. Due to (20) we immediately have the following result:

Corollary 2. *If* H_0, *(3), (4), (5), (6), (9), (10), (11), (12) and* (20) *hold, then we have that*

$$\frac{T_n(x)}{n^{1/2}\tilde{s}_n} \xrightarrow{\mathscr{D}[0,1]} B(x),$$

where $B(x)$ *is a Brownian bridge.*

We suggest testing procedures based on

$$\tilde{\mathscr{Q}}_n = \frac{1}{n^{1/2}\tilde{s}_n} \sup_{0 \le x \le 1} |T_n(x)|.$$

It follows immediately from Corollary 2 that under H_0

$$\tilde{\mathscr{Q}}_n \xrightarrow{\mathscr{D}} \sup_{0 \le x \le 1} |B(x)|. \tag{21}$$

First we study experimentally the rate of convergence in Theorem 2. In this section we assume that the innovations ε_i in (3), (4), (5), (6), and (7) have the common distribution function

$$F(t) = \begin{cases} q(1-t)^{-3/2}, & \text{if} \quad -\infty < t \le 0, \\ 1 - p(1-t)^{-3/2}, & \text{if} \quad 0 < t < \infty, \end{cases}$$

where $p \ge 0$, $q \ge 0$ and $p + q = 1$. We present the results for the case of $\rho = p = q = 1/2$ based on 10^5 repetitions. We simulated the elements of an autoregressive sample $(e_1, \dots, e_n)$ from the recursion (3) starting with some initial value and with a burn in period of 500, i.e. the first 500 generated variables were discarded and

Table 1 Simulated 95 % percentiles of the distribution of $\mathcal{Q}_n$ under H_0

n	400	600	800	1,000	∞
	1.29	1.32	1.33	1.34	1.36

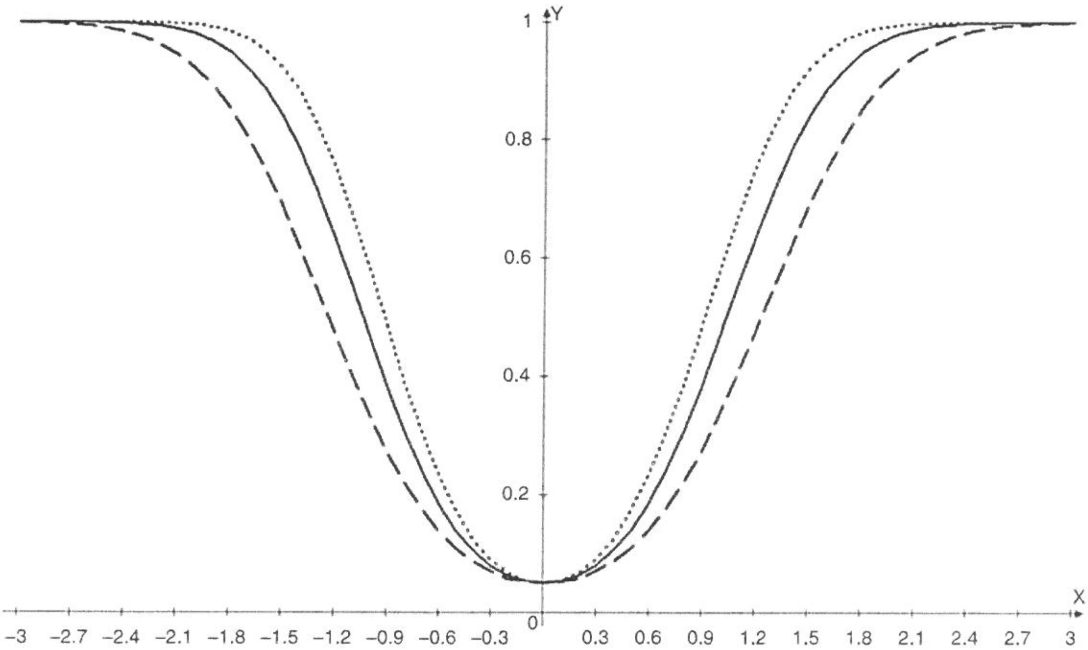

Fig. 1 Empirical power for $\mathcal{Q}_n$ with significance level 0.05, $n = 400$ (*dashed*), $n = 600$ (*solid*) and $n = 800$ (*dotted*) with $k_1 = n/2$

the next n give the sample $(e_1, \ldots, e_n)$. Thus $(e_1, \ldots, e_n)$ are from the stationary solution of (3). We trimmed the sample using $d(n) = \lfloor n^{0.45} \rfloor$ and computed

$$\mathcal{Q}_n = \left(\frac{2-\alpha}{\alpha}\right)^{1/2} \left(\frac{1-\rho}{1+\rho}\right)^{1/2} \frac{1}{A_n} \sup_{0 \le x \le 1} |T_n(x)|.$$

Under H_0 we have

$$\mathcal{Q}_n \xrightarrow{\mathcal{D}} \sup_{0 \le x \le 1} |B(x)|.$$

The critical values in Table 1 provide information on the rate of convergence in Theorem 2.

Figures 1 and 2 show the empirical power of the test for H_0 against H_A based on the statistic $\mathcal{Q}_n$ for a change at time $k^* = n/4$ and $n/2$ and when the location changes from 0 to $c \in \{-3, -2.9, \ldots, 2.9, 3\}$ and the level of significance is 0.05. We used the asymptotic critical value 1.36. Comparing Figs. 1 and 2 we see that we have higher power when the change occurs in the middle of the data at $k^* = n/2$. We provided these results to illustrate the behaviour of functionals of T_n without introducing further noise due to the estimation of the norming sequence.

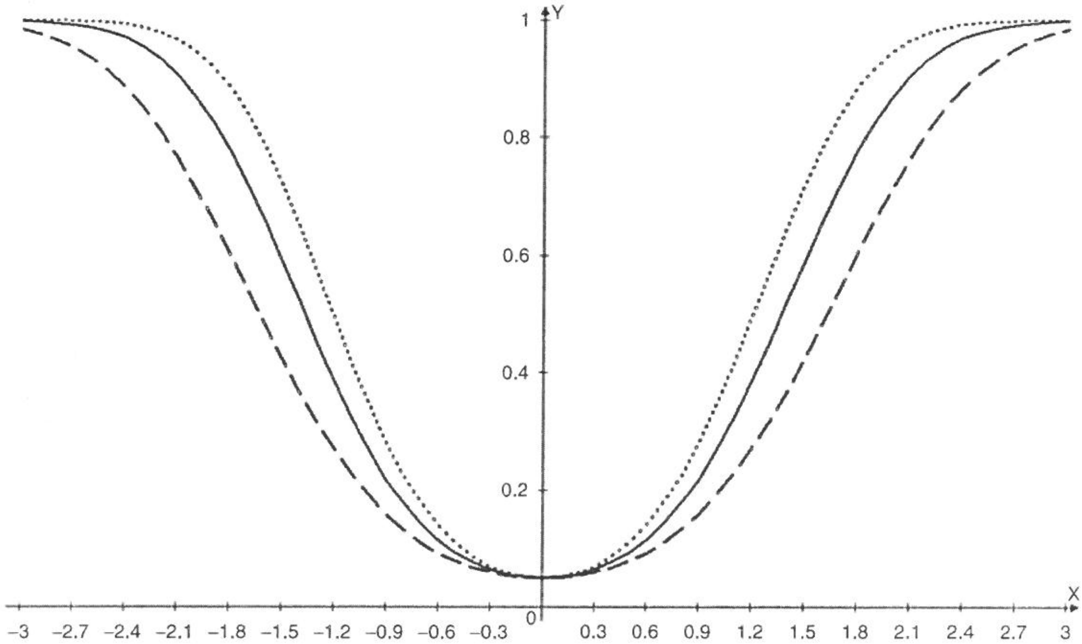

Fig. 2 Empirical power for $\mathscr{Q}_n$ with significance level 0.05, $n = 400$ (*dashed*), $n = 600$ (*solid*) and $n = 800$ (*dotted*) with $k_1 = n/4$

Table 2 Simulated 95 % percentiles of the distribution of $\tilde{\mathscr{Q}}_n$ under H_0

n	400	600	800	1,000	∞
	1.57	1.52	1.50	1.49	1.36

Next we study the applicability of (21) in case of small and moderate sample sizes. We used $h(n) = n^{1/2}$ as the window and the flat top kernel

$$\omega(t) = \begin{cases} 1 & 0 \le t \le .1 \\ 1.1 - |t| & .1 \le t \le 1.1 \\ 0 & t \ge 1.1 \end{cases}$$

Figures 3 and 4 show the empirical power of the test for H_0 against H_A based on the statistic $\tilde{\mathscr{Q}}_n$ for a change at time $k^* = n/4$ and $n/2$ and when the location changes from 0 to $c \in \{-3, -2.9, \ldots, 2.9, 3\}$ and the level of significance is 0.05. We used the asymptotic critical value 1.36 (Table 2). Comparing Figs. 3 and 4 we see that we have again higher power when the change occurs in the middle of the data at $k_1 = n/2$.

Figure 5 shows how the power of the test behaves depending on the value of $d = n^{\epsilon}$, $\epsilon \in \{0.3, 0.35, 0.42, 0.45, 0.5\}$ for $n = 400$. The bigger the d is, the better is the power curve.

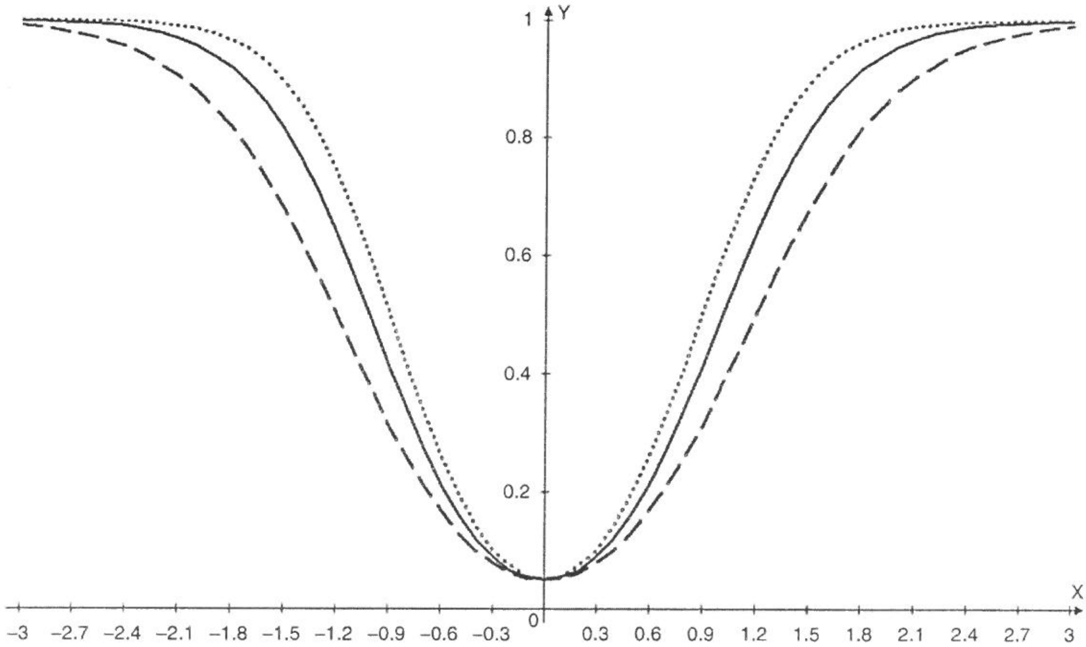

Fig. 3 Empirical power for $\widetilde{\mathscr{D}}_n$ with significance level 0.05, $n = 400$ (*dashed*), $n = 600$ (*solid*) and $n = 800$ (*dotted*) with $k_1 = n/2$

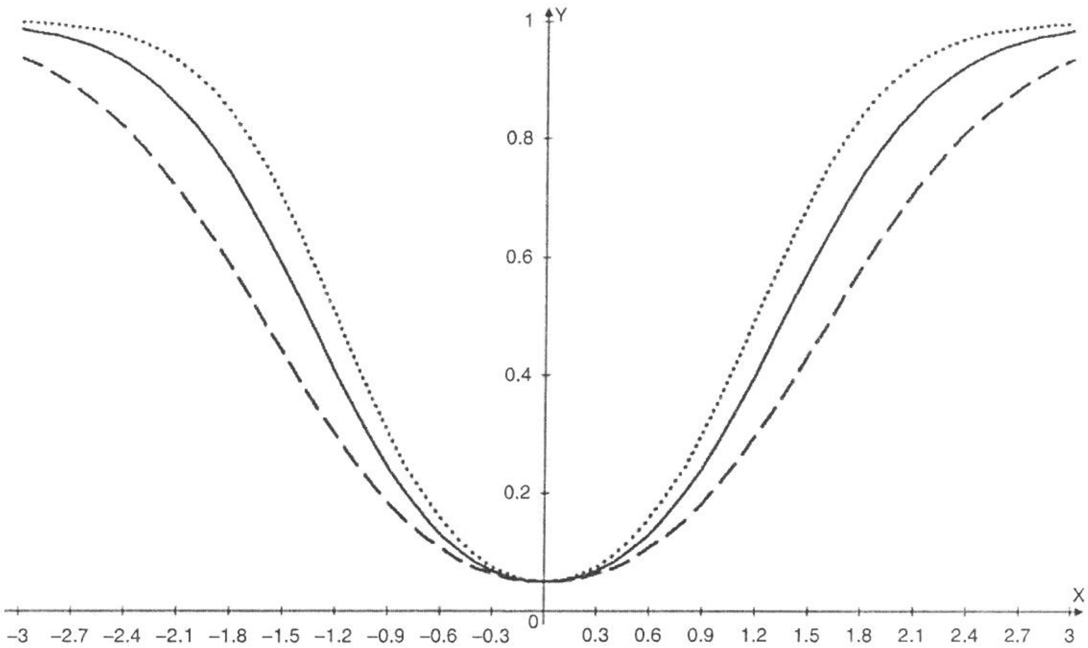

Fig. 4 Empirical power for $\widetilde{\mathscr{D}}_n$ with significance level 0.05, $n = 400$ (*dashed*), $n = 600$ (*solid*) and $n = 800$ (*dotted*) with $k_1 = n/4$

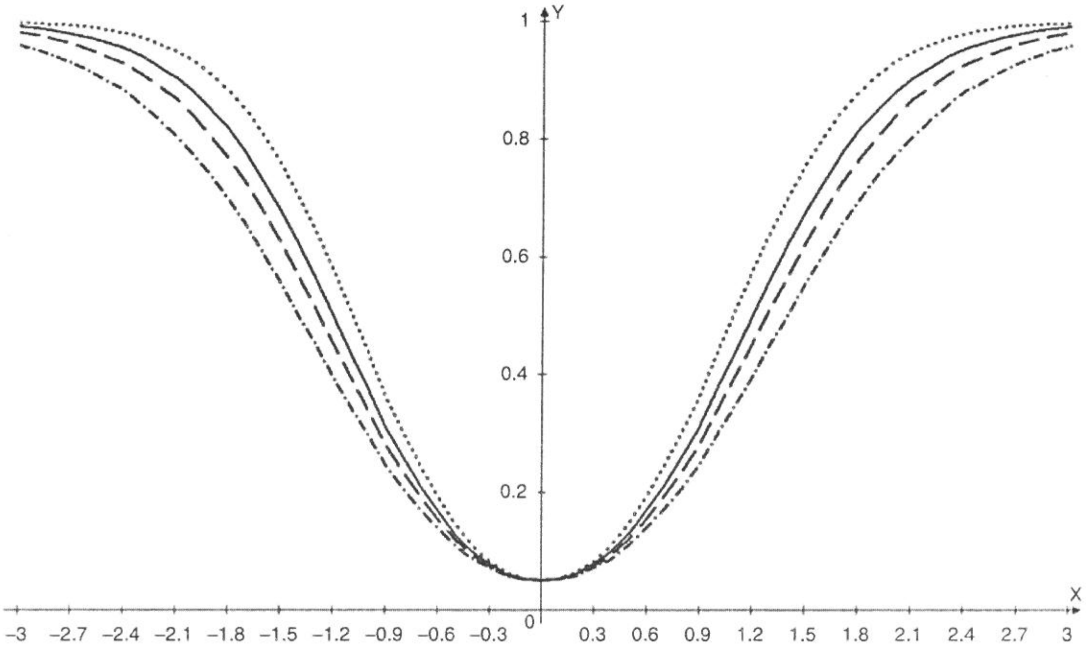

Fig. 5 Empirical power curves for $\tilde{\mathscr{Q}}_n$ with significance level 0.05 for $d = n^{\epsilon}$, $\epsilon = 0.35$ (*dash-dotted*), $\epsilon = 0.42$ (*dashed*), $\epsilon = 0.45$ (*solid*), $\epsilon = 0.5$ (*dotted*) with $n = 400$, $k_1 = n/2$

3 Ratio Statistics

The statistics $\hat{Q}_n$ as well as $\tilde{Q}_n$ are very sensitive to the behaviour of $\hat{s}_n$ and $\tilde{s}_n$. As we pointed out, $\hat{s}_n$ is the right norming only under H_0. The sequence $\tilde{Q}_n$ works under H_0 and under the one change alternative, but it could break down if multiple changes occur under the alternative. Even if the Bartlett type estimator is the asymptotically correct norming factor, the rate of convergence can be slow. Also, these estimators are very sensitive to the choice of the window $h = h(n)$. Following the work of Kim [19] (cf. also Kim et al. [20]) and Leybourne and Taylor [21], Horváth et al. [17] proposed ratio type statistics of functionals of CUSUM processes. We adapt their approach to the trimmed CUSUM process. Let $0 < \delta < 1$ and define

$$Z_n = \max_{n\delta \le k \le n - n\delta} \frac{Z_{n,1}(k)}{Z_{n,2}(k)},$$

where

$$Z_{n,1}(k) = \max_{1 \le i \le k} \left| \sum_{j=1}^{i} (X_j I\{|X_j| \le \eta_{n,d}\} - (1/k) \sum_{j=1}^{k} (X_j I\{|X_j| \le \eta_{n,d})\}) \right|$$

and

$$Z_{n,2}(k) = \max_{k<i\leq n} \left| \sum_{j=i}^{n} (X_j I\{|X_j| \leq \eta_{n,d}\} - (1/(n-k)) \sum_{j=k+1}^{n} (X_j I\{|X_j| \leq \eta_{n,d})\}) \right|.$$

Roughly speaking, we split the data into two subsets at k, compute the maximum of the CUSUM in both subsamples and compare these maxima. To state the limit distribution of Z_n under the null hypothesis, we need to introduce

$$z_1(t) = \sup_{0\leq s\leq t} |W(s) - (s/t)W(t)|$$

and

$$z_2(t) = \sup_{t\leq s\leq 1} |W^*(s) - ((1-s)/(1-t))W^*(t)|,$$

where $W^*(t) = W(1) - W(t)$. The following result is an immediate consequence of Theorem 2.

Theorem 3. *If H_0, (3), (4), (5), (6) and (9), (10), (11), (12) hold, then we have that*

$$Z_n \xrightarrow{\mathscr{D}} \sup_{\delta\leq t\leq 1-\delta} \frac{z_1(t)}{z_2(t)}. \tag{22}$$

We reject the no change null hypothesis if Z_n is large. Using Monte Carlo simulations, it is easy to obtain the distribution function of the limit in (22). Selected critical values can be found in Horváth et al. [17], where some probabilistic properties of the limit are also discussed.

Below we study the finite sample behaviour of Z_n. Table 3 contains simulated significance levels when $\delta = .2$, $n = 400, 600, 800, 1{,}000$ and $n = 5{,}000$. (Since the distribution function of the limit in (22) is unknown, we used $n = 5{,}000$ for the limit distribution.)

Figures 6 and 7 contain the empirical power curves of the test for H_0 against H_A based on the statistic Z_n for a change at time $k^* = n/4$ and $n/2$ and when the location changes from 0 to $c \in \{-5, -4.9, \ldots, 4.9, 5\}$ and the level of significance is 0.05. We used critical values from Table 3. Figure 8 shows how the power of the test behaves depending on the value of $d = n^{\epsilon}$, $\epsilon \in \{0.3, 0.35, 0.42, 0.45, 0.5\}$ for $n = 400$. The bigger the d is, the better is the power curve.

Table 3 Simulated 95 % percentiles of the distribution of Z_n under H_0

n	400	600	800	1,000	5,000
	5.90	5.67	5.49	5.43	5.03

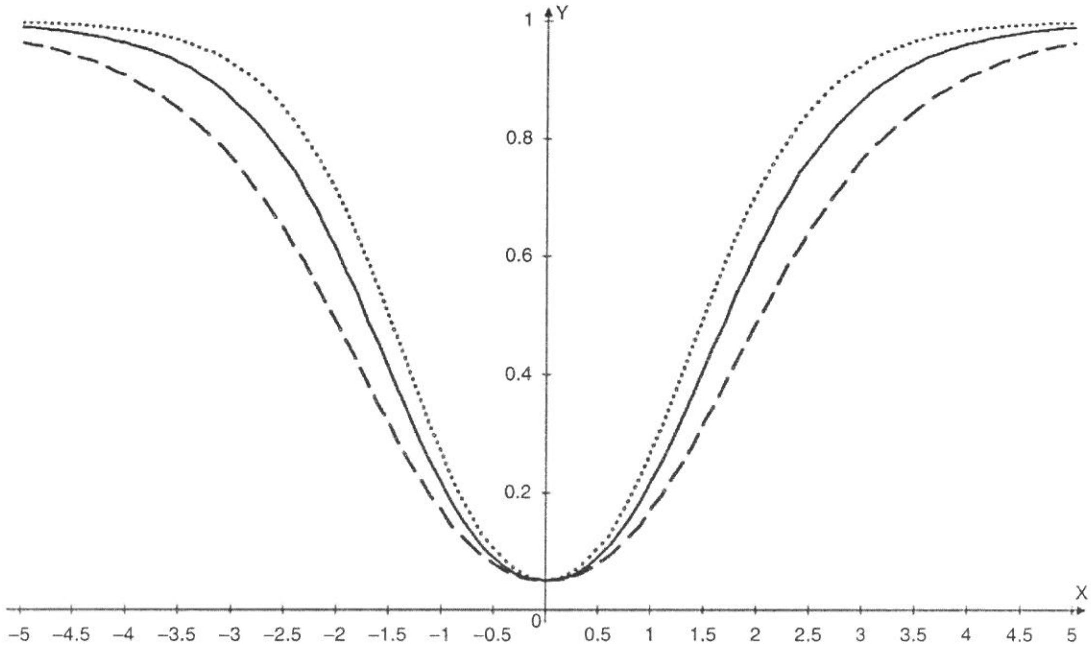

Fig. 6 Empirical power curves for Z_n with significance level 0.05, $n = 400$ (*dashed*), $n = 600$ (*solid*) and $n = 800$ (*dotted*) with $k_1 = n/2$

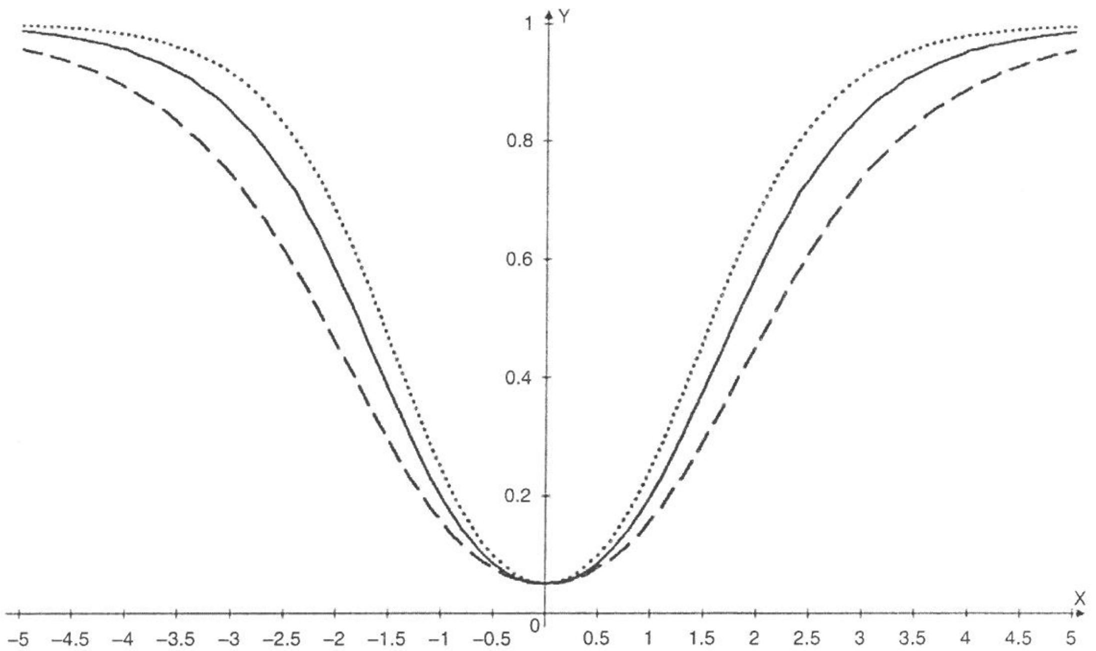

Fig. 7 Empirical power curves for Z_n with significance level 0.05, $n = 400$ (*dashed*), $n = 600$ (*solid*) and $n = 800$ (*dotted*) with $k_1 = n/4$

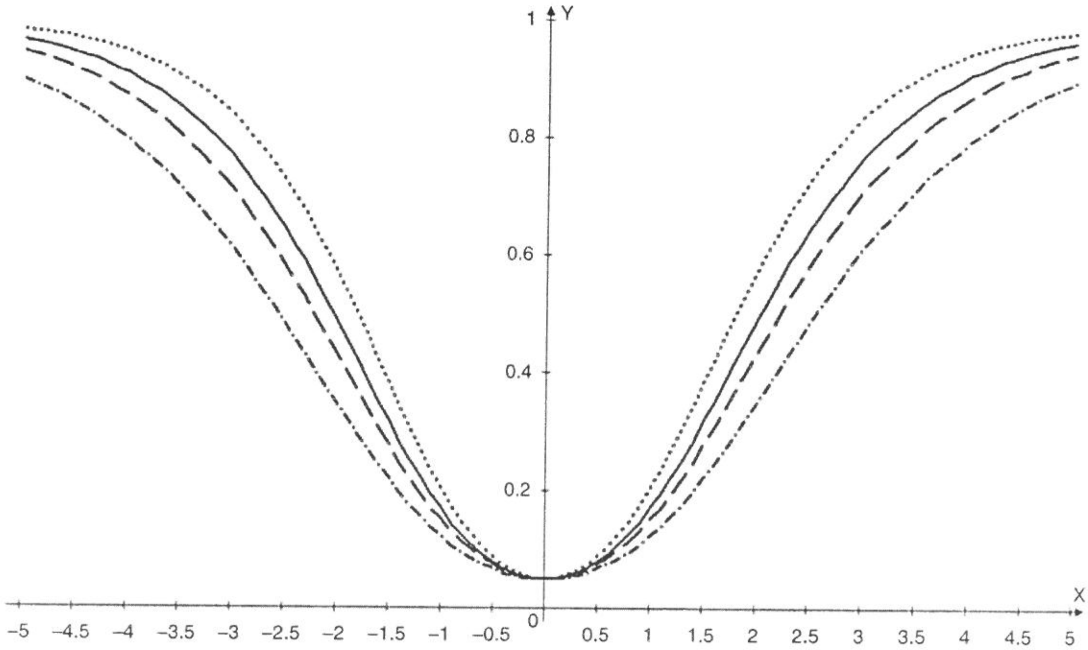

Fig. 8 Empirical power curves for Z_n with significance level 0.05 for $d = n^{\epsilon}$, $\epsilon = 0.35$ *(dash-dotted)*, $\epsilon = 0.42$ *(dashed)*, $\epsilon = 0.45$ *(solid)*, $\epsilon = 0.5$ *(dotted)* with $n = 400$, $k_1 = n/2$

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