OPENCFS-DATA: IMPLEMENTATION OF THE STOCHASTIC NOISE GENERATION AND RADIATION MODEL (SNGR)

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ABSTRACT

Preliminary aeroacoustic investigations in competitive industries require rapid numerical simulation techniques to gain initial insight into the flow and acoustic field. Although there are capabilities to resolve virtually all turbulence length scales, these techniques are often impractical in early stages of component development. Therefore, the flow field is typically assessed by a Reynolds-averaged Navier Stokes Simulation. Building upon the results of that flow simulation, a stochastic approach to reconstruct the turbulent velocity fluctuations. In conjunction with a hybrid aeroacoustic workflow, this approach is useful in early stage virtual prototyping of aeroacoustic applications. In this working paper, we present the SNGR algorithm of CFS-Data, the open-source pre-post-processing part of openCFS, with a focus on the computation of aeroacoustic sources.

Keywords Open Source FEM Software · Multiphysics Simulation · C++ · Acoustics · Aero-Acoustics · openCFS · SNGR

1 Introduction

Within this contribution, we concentrate on the openCFS [32] module *openCFS-Data* [33], the implemented SNGR method called *syntheticTurbulence_SNGR* and potential model applications. The method was initially applied in [46, 47] and more details on a first application can be found there. Stochastic methods constitute a low-cost computational fluid dynamics (CFD) approach to reconstruct the turbulent velocity fluctuations using results Reynolds-averaged Navier-Stokes (RANS) simulations. This approach was introduced by Béchara et al. [2] in 1994 and is known as stochastic noise generation and radiation (SNGR). Regarding the model in [46], it was applied to a cavity noise simulation using a hybrid aeroacoustic workflow [39, 24, 40]. The derivatives of the Lighthill's source term were computed by a radial basis function scheme [23, 36]. Using recently developed equations, the method can potentially be applied to aeroacoustic formulations based on the acoustic potential [26, 19] in combination with a Helmholtz decomposition [35, 34, 25, 30]. Furthermore, this methodology can be valuable for automotive OEMs since it poses a fast estimate on the flow-acoustic properties of broadband noise excitation which can be applied as loading [4, 6, 28, 47, 17]. Furthermore, the hybrid aeroacoustic workflow was found to be useful for fan noise computations [22, 41, 38, 42, 43, 21], the noise emissions of the turbocharger compressor [8, 9, 7], the acoustics of fluid-structure-acoustic-interaction processes [37, 44, 48, 29, 5, 14, 16, 31, 15, 20, 12]. Potential nonphysical behavior generated with the source computation can be identified using [27]. For a literature overview on the developments connecting to SNGR, have a look at the article [13].

2 Stochastic Noise Generation and Radiation

As proposed by Kraichnan [11] and Karweit et al. [10] a spatially stochastic turbulent velocity field can be generated as a finite sum of N statistically independent random Fourier mode

$$\boldsymbol{u}_{\mathrm{t}}(\boldsymbol{x}) = 2\sum_{n=1}^{N} \tilde{u}_n \cos\left(\boldsymbol{k}_n \cdot \boldsymbol{x} + \psi_n\right)\boldsymbol{\sigma}_n, \tag{1}$$

where $\boldsymbol{x}$ is the spatial position, and $\boldsymbol{k}_n, \tilde{u}_n, \psi_n$, and $\boldsymbol{\sigma}_n$ are the wave vector, the amplitude, the phase, and the direction of the n^{th} mode, respectively. The wave vector $\boldsymbol{k}_n$ is randomly chosen on a sphere of radius k_n to ensure isotropy. Additionally, incompressibility of the turbulent flow field implies $\partial u_{ti}/\partial x_i = 0$ and hence

$$\boldsymbol{k}_n \cdot \boldsymbol{\sigma}_n = \text{ for } \quad n = 1, \ldots, N \tag{2}$$

The turbulent kinetic energy K is computed as the statistical mean of $1/2 u_{ti} u_{ti}$ and may thus be written with Eq. (1) as

$$K = \sum_{n=1}^{N} \tilde{u}_n^2. \tag{3}$$

A homogeneous isotropic turbulence is characterized by a three-dimensional energy spectrum $E(k)$ which allows to compute the turbulent kinetic energy by

$$\int_0^\infty E(k)\mathrm{d}k = K. \tag{4}$$

Moreover, the spectrum $E(k)$ allows computing the rate of dissipation of turbulence energy ϵ by

$$2\nu \int_0^\infty k^2 E(k)\mathrm{d}k = \epsilon, \tag{5}$$

where ν is the kinematic viscosity. Discretizing Eq. (4) and combining it with Eq. (3) yields

$$\tilde{u}_n = \sqrt{E\left(k_n\right)\Delta k_n}. \tag{6}$$

The spectrum $E(k)$ used to simulate the complete spectral range is a von Kármán-Pao spectrum [45, 18]

$$E(k) = \alpha \frac{u'^2}{k_{\mathrm{e}}} \frac{\left(k/k_{\mathrm{e}}\right)^4}{\left[1 + \left(k/k_{\mathrm{e}}\right)^2\right]^{17/6}} \exp\left[-2\left(\frac{k}{k_\eta}\right)^2\right], \tag{7}$$

where $k_\eta = \left(\epsilon/\nu^3\right)^{1/4}$ is the Kolmogorov wave number, and $u' = \sqrt{2K/3}$ is the root-mean-square value of the velocity fluctuations. Assuming that the turbulent kinetic energy K and the rate of dissipation of turbulence energy ϵ are known from the computational fluid dynamics (CFD) solution, and inserting Eq. (8) into Eq. (4) yields

$$\alpha = \frac{55}{9\sqrt{\pi}} \frac{\Gamma\left(\frac{5}{6}\right)}{\Gamma\left(\frac{1}{3}\right)} \approx 1.45276. \tag{8}$$

To compute the wave number k_{e} of the most energetic eddies corresponding to the maximum of the energy spectrum $E(k)$, Eq. (8) is inserted into Eq. (5), which leads to

$$\frac{u'^3}{\epsilon} = \frac{2}{k_{\mathrm{e}}}\left[\alpha \Gamma\left(\frac{2}{3}\right)\right]^{-\frac{3}{2}} \approx 0.725 \frac{1}{k_{\mathrm{e}}} \tag{9}$$

when $k_{\mathrm{e}}/k_\eta \to 0$. To achieve a better discretization of the power in the lower wave number range corresponding to the larger energy-containing eddies, a logarithmic distribution of the N wave numbers is used. A logarithmic step of such a distribution reads

$$\Delta k_1 = \frac{\log k_N - \log k_1}{N - 1} \tag{10}$$

and hence the n^{th} wave number k_n is given by

$$k_n = \exp\left[\log k_1 + (n-1)\log \Delta k_{\mathrm{l}}\right]. \tag{11}$$

Since this method only models spatial correlation but not temporal correlation, Bailly et al. [1] included time-dependence in Eq. (1) to arrive at reasonable statistical properties. Hence, the turbulent velocity field is now computed as a sum of unsteady random Fourier modes

$$\boldsymbol{u}_{\mathrm{t}}(\boldsymbol{x}, t) = 2 \sum_{n=1}^{N} \tilde{u}_n \cos\left(\boldsymbol{k}_n \cdot (\boldsymbol{x} - t\boldsymbol{u}_{\mathrm{c}}) + \psi_n + \omega_n t\right) \boldsymbol{\sigma}_n, \tag{12}$$

where $\boldsymbol{u}_{\mathrm{c}}$ is the convection velocity and ω_n is the angular frequency of the n^{th} mode. As opposed to the convection velocity $\boldsymbol{u}_{\mathrm{c}}$, which is a function of the known local mean flow, the angular frequency ω_n is a random variable. Kraichnan [11] treated ω_n and k_n as independent variables; however, aerodynamic noise generation cannot be treated satisfactorily with this supposition. Therefore, Bailly et al. [1] proposed to draw ω_n from a distribution associated to a Gaussian probability density function

$$p_n(\omega) = \frac{1}{\omega_{0,n}\sqrt{2\pi}} \exp\left(-\frac{(\omega - \omega_{0,n})^2}{2\omega_{0,n}^2}\right), \tag{13}$$

where the mean angular frequency of the n^{th} mode $\omega_{0,n}$ is connected to the wave number k by $\omega_{0,n} = u' k_n$. Additionally, the authors propose to use the Heisenberg time $\tau_{\mathrm{H}} \sim (u'k)^{-1}$ as the spectrally local characteristic time. Billson et al. [3] compute the wave number k_{e} of the most energetic eddies starting from the turbulence length scale of the RANS solution

$$\Lambda = C_\mu \frac{K^{3/2}}{\epsilon}, \tag{14}$$

where C_μ is a closure coefficient of the $k - \epsilon$ turbulence model. Under the assumption that this length scale is the equal to the integral length scale for isotropic turbulence [13], the following relation holds

$$\Lambda = \frac{\pi}{2u'^2} \int_0^\infty \frac{E(k)}{k} \,\mathrm{d}k. \tag{15}$$

Hence, k_{e} can be computed as

$$k_{\mathrm{e}} = \frac{9\pi}{55} \frac{\alpha}{\Lambda}. \tag{16}$$

2.1 Filter definition

The following XML-snippet illustrates a typical setting of the SNGR filter.

```xml
<syntheticTurbulence_SNGR id="synthTurb" tkeCriterion="0.1"
numOfModes="30" inputFilterIds="cfdEnsight" lengthScaleFactor="1.0"
timeScaleFactor="1.0" angularFreqFactor="1.0">
   <incrementModes>equidistant</incrementModes>
   <waveNumBounds minWN="0.1" maxWN="10"/>
   <frequencyBounds minFreq="200" maxFreq="12000"/>
   <TKE resultName="fluidMechTKE"/>
   <TEF resultName="fluidMechTEF"/>
   <localDensity resultName="fluiMechDensity"/>
   <localTemp resultName="fluidMechTemp"/>
   <meanVelocity resultName="fluidMechVelocity"/>
   <output resultName="syntheticTurbVelocity"/>
</syntheticTurbulence_SNGR>
```

- tkeCriterion: only use turbulent kinetic energy (TKE) values when larger than 10%.
- numOfModes: N
- lengthScaleFactor, timeScaleFactor, angularFreqFactor scalars to stretch the space, time, or frequency respectively
- incrementModes: equidistant or logarithmic
- waveNumBounds: minimum and maximum resolve wave number
- frequencyBounds: minimum and maximum resolve frequency
- TKE, TEF (turbulent eddy frequency), localDensity, localTemp (field Temperature), meanVelocity are the field variables that must be supplied by the input.
- output: output field definition.

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References

[1] C. Bailly and D. Juve. A stochastic approach to compute subsonic noise using linearized euler's equations. In *5th AIAA/CEAS aeroacoustics conference and exhibit*, page 1872, 1999.

[2] W. Bechara, C. Bailly, P. Lafon, and S. M. Candel. Stochastic approach to noise modeling for free turbulent flows. *AIAA journal*, 32(3):455–463, 1994.

[3] M. Billson, L.-E. Eriksson, L. Davidson, and P. Jordan. Modeling of synthesized anisotropic turbulence and its sound emission. In *10th AIAA/CEAS aeroacoustics conference*, page 2857, 2004.

[4] R. Engelmann, C. Gabriel, S. Schoder, and M. Kaltenbacher. A generic testbody for low-frequency aeroacoustic buffeting. Technical report, SAE Technical Paper, 2020.

[5] S. Falk, S. Kniesburges, S. Schoder, B. Jakubaß, P. Maurerlehner, M. Echternach, M. Kaltenbacher, and M. Döllinger. 3d-fv-fe aeroacoustic larynx model for investigation of functional based voice disorders. *Frontiers in physiology*, 12:616985, 2021.

[6] C. Freidhager, P. Maurerlehner, K. Roppert, A. Wurzinger, A. Hauser, M. Heinisch, S. Schoder, and M. Kaltenbacher. Simulationen von strömungsakustik in rotierenden bauteilen zur entwicklung von antriebskonzepten der autos der zukunft. *e & i Elektrotechnik und Informationstechnik*, 138(3):212–218, 2021.

[7] C. Freidhager, S. Schoder, and M. Kaltenbacher. The influences of spatial and temporal discretization in flow simulation on lighthill's aeroacoustic source terms applied to a turbocharger. In *AIAA AVIATION 2020 FORUM*, page 2546, 2020.

[8] C. Freidhager, S. Schoder, P. Maurerlehner, A. Renz, S. Becker, and M. Kaltenbacher. Applicability of two hybrid sound prediction methods for assessing in-duct sound absorbers of turbocharger compressors. *Acta Acustica*, 6:37, 2022.

[9] M. Kaltenbacher, C. Freidhager, and S. Schoder. Modelling and numerical simulation of the noise generated by automotive turbocharger compressor. Technical report, SAE Technical Paper, 2020.

[10] M. Karweit, P. Blanc-Benon, D. Juvé, and G. Comte-Bellot. Simulation of the propagation of an acoustic wave through a turbulent velocity field: A study of phase variance. *The Journal of the Acoustical Society of America*, 89(1):52–62, 1991.

[11] R. H. Kraichnan. Diffusion by a random velocity field. *The physics of fluids*, 13(1):22–31, 1970.

[12] F. Kraxberger, A. Wurzinger, and S. Schoder. Machine-learning applied to classify flow-induced sound parameters from simulated human voice. *arXiv preprint arXiv:2207.09265*, 2022.

[13] A. Lafitte, T. L. Garrec, C. Bailly, and E. Laurendeau. Turbulence generation from a sweeping-based stochastic model. *AIAA journal*, 52(2):281–292, 2014.

[14] M. Lasota, P. Šidlof, M. Kaltenbacher, and S. Schoder. Impact of the sub-grid scale turbulence model in aeroacoustic simulation of human voice. *Applied Sciences*, 11(4):1970, 2021.

[15] M. Lasota, P. Šidlof, P. Maurerlehner, M. Kaltenbacher, and S. Schoder. Anisotropic minimum dissipation subgrid-scale model in hybrid aeroacoustic simulations of human phonation. *arXiv preprint arXiv:2301.00606*, 2023.

[16] P. Maurerlehner, S. Schoder, C. Freidhager, A. Wurzinger, A. Hauser, F. Kraxberger, S. Falk, S. Kniesburges, M. Echternach, M. Döllinger, et al. Efficient numerical simulation of the human voice. *e & i Elektrotechnik und Informationstechnik*, 138(3):219–228, 2021.

[17] P. Maurerlehner, S. Schoder, J. Tieber, C. Freidhager, H. Steiner, G. Brenn, K.-H. Schäfer, A. Ennemoser, and M. Kaltenbacher. Aeroacoustic formulations for confined flows based on incompressible flow data. *Acta Acustica*, 6:45, 2022.

[18] Y.-H. Pao. Structure of turbulent velocity and scalar fields at large wavenumbers. *The Physics of Fluids*, 8(6):1063–1075, 1965.

[19] S. Schoder. cpcwe–perturbed convective wave equation based on compressible flows. *arXiv preprint arXiv:2209.11410*, 2022.

[20] S. Schoder. Pcwe for fsai–derivation of scalar wave equations for fluid-structure-acoustics interaction of low mach number flows. *arXiv preprint arXiv:2211.07490*, 2022.

[21] S. Schoder and F. Czwielong. Dataset fan-01: Revisiting the eaa benchmark for a low-pressure axial fan. *arXiv preprint arXiv:2211.12014*, 2022.

[22] S. Schoder, C. Junger, and M. Kaltenbacher. Computational aeroacoustics of the eaa benchmark case of an axial fan. *Acta Acustica*, 4(5):22, 2020.

[23] S. Schoder, C. Junger, K. Roppert, and M. Kaltenbacher. Radial basis function interpolation for computational aeroacoustics. In *AIAA AVIATION 2020 FORUM*, page 2511, 2020.

[24] S. Schoder and M. Kaltenbacher. Hybrid aeroacoustic computations: State of art and new achievements. *Journal of Theoretical and Computational Acoustics*, 27(04):1950020, 2019.

[25] S. Schoder, M. Kaltenbacher, and K. Roppert. Helmholtz's decomposition applied to aeroacoustics. In *25th AIAA/CEAS Aeroacoustics Conference*, 2019-2561.

[26] S. Schoder, M. Kaltenbacher, É. Spieser, H. Vincent, C. Bogey, and C. Bailly. Aeroacoustic wave equation based on pierce's operator applied to the sound generated by a mixing layer. In *28th AIAA/CEAS Aeroacoustics 2022 Conference*, page 2896, 2022.

[27] S. Schoder, F. Kraxberger, S. Falk, A. Wurzinger, K. Roppert, S. Kniesburges, M. Döllinger, and M. Kaltenbacher. Error detection and filtering of incompressible flow simulations for aeroacoustic predictions of human voice. *The Journal of the Acoustical Society of America*, 152(3):1425–1436, 2022.

[28] S. Schoder, I. Lazarov, and M. Kaltenbacher. Numerical investigation of a deep cavity with an overhanging lip considering aeroacoustic feedback mechanism. *arXiv preprint arXiv:2006.03279*, 2020.

[29] S. Schoder, P. Maurerlehner, A. Wurzinger, A. Hauser, S. Falk, S. Kniesburges, M. Döllinger, and M. Kaltenbacher. Aeroacoustic sound source characterization of the human voice production-perturbed convective wave equation. *Applied Sciences*, 11(6):2614, 2021.

[30] S. Schoder, E. Museljic, F. Kraxberger, and A. Wurzinger. Post-processing subsonic flows using physics-informed neural networks. In *2023 AIAA AVIATION Forum*, 2022.

[31] S. Schoder and K. Roppert. Learning expertise actively to model domain knowledge (lead) with application to human phonation. *arXiv*, 2022.

[32] S. Schoder and K. Roppert. opencfs: Open source finite element software for coupled field simulation–part acoustics. *arXiv preprint arXiv:2207.04443*, 2022.

[33] S. Schoder and K. Roppert. opencfs-data: Data pre-post-processing tool for opencfs–aeroacoustics source filters. *arXiv preprint arXiv:2302.03637*, 2023.

[34] S. Schoder, K. Roppert, and M. Kaltenbacher. Helmholtz's decomposition for compressible flows and its application to computational aeroacoustics. *SN Partial Differ. Equ. Appl.*, pages 1–20, 2020.

[35] S. Schoder, K. Roppert, and M. Kaltenbacher. Postprocessing of direct aeroacoustic simulations using helmholtz decomposition. *AIAA Journal*, pages 1–9, 2020.

[36] S. Schoder, K. Roppert, M. Weitz, C. Junger, and M. Kaltenbacher. Aeroacoustic source term computation based on radial basis functions. *International Journal for Numerical Methods in Engineering*, 121(9):2051–2067, 2020.

[37] S. Schoder, M. Weitz, P. Maurerlehner, A. Hauser, S. Falk, S. Kniesburges, M. Döllinger, and M. Kaltenbacher. Hybrid aeroacoustic approach for the efficient numerical simulation of human phonation. *The Journal of the Acoustical Society of America*, 147(2):1179–1194, 2020.

[38] S. Schoder, A. Wurzinger, C. Junger, M. Weitz, C. Freidhager, K. Roppert, and M. Kaltenbacher. Application limits of conservative source interpolation methods using a low mach number hybrid aeroacoustic workflow. *Journal of Theoretical and Computational Acoustics*, 29(01):2050032, 2021.

[39] S. J. Schoder. *Aeroacoustic analogies based on compressible flow data*. PhD thesis, Wien, 2018.

[40] S. J. Schoder, C. Junger, M. Weitz, and M. Kaltenbacher. Conservative interpolation of aeroacoustic sources in a hybrid workflow applied to fan. *arXiv preprint arXiv:2009.02341*, 2020.

[41] M. Tautz, K. Besserer, S. Becker, and M. Kaltenbacher. Source formulations and boundary treatments for lighthill's analogy applied to incompressible flows. *AIAA Journal*, 56(7):2769–2781, 2018.

[42] L. Tieghi, S. Becker, A. Corsini, G. Delibra, S. Schoder, and F. Czwielong. Machine-learning clustering methods applied to detection of noise sources in low-speed axial fan. In *2022 Turbomachinery Technical Conference & Exposition: ASME Turbo Expo 2022*, 2022.

[43] L. Tieghi, S. Becker, A. Corsini, G. Delibra, S. Schoder, and F. Czwielong. Machine-learning clustering methods applied to detection of noise sources in low-speed axial fan. *Journal of Engineering for Gas Turbines and Power*, 145(3):031020, 2023.

[44] J. Valášek, M. Kaltenbacher, and P. Sváček. On the application of acoustic analogies in the numerical simulation of human phonation process. *Flow, Turbulence and Combustion*, 102(1):129–143, 2019.

[45] T. Von Karman. Progress in the statistical theory of turbulence. *Proceedings of the National Academy of Sciences*, 34(11):530–539, 1948.

[46] M. Weitz. *An approach to compute cavity noise using stochastic noise generation and radiation*. PhD thesis, Wien, 2019.

[47] M. Weitz, S. Schoder, and M. Kaltenbacher. Numerical investigation of the resonance behavior of flow-excited helmholtz resonators. *PAMM*, 19(1):e201900033, 2019.

[48] S. Zörner, P. Šidlof, A. Hüppe, and M. Kaltenbacher. Flow and acoustic effects in the larynx for varying geometries. *Acta Acustica united with Acustica*, 102(2):257–267, 2016.