Indirect Measurement of Switch Terms of a Vector Network Analyzer with Reciprocal Devices

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***Abstract*—This paper presents an indirect method for measuring the switch terms of a vector network analyzer (VNA) using at least three reciprocal devices, which do not need to be characterized beforehand. This method is particularly suitable for VNAs that use a three-sampler architecture, which allows for applying first-tier calibration methods based on the error box model. The proposed method was experimentally verified by comparing directly and indirectly measured switch terms and performing a multiline thru-reflect-line (TRL) calibration.**

***Index Terms*—VNA, calibration, microwave measurement**

I. Introduction

Calibration of a vector network analyzer (VNA) is crucial for removing systematic errors between the measured device under test (DUT) and the actual receivers of the VNA. The most common calibration method is the short-open-load-thru (SOLT) method, which is based on the 12-term error model of a two-port VNA. However, this method requires all standards to be fully characterized. In [1], a modification to the SOLT method was introduced where the thru standard is replaced with any transmissive reciprocal device, called the SOLR method. This method is based on the error box model of a two-port VNA. Other advanced self-calibration methods, including thru-reflect-line (TRL), multiline TRL, line-reflect-match (LRM), and line-reflect-reflect-match (LRRM) [2]–[5], also rely on the error box model of a two-port VNA.

A limitation of calibration methods based on the error box model is that it requires a four-sampler VNA to sample all waves, whereas the 12-term error model can still be used in three-sampler VNAs. Fig. 1 illustrates the two VNA sampling architectures. The difference between the two sampling architectures is that in the three-sampler VNA, we do not sample the reflected wave of the termination load of the non-driving port. Although the termination load is generally designed to be matched, in reality, there is always some reflection that needs to be accounted for. This reflection is called the switch term. Since the ports are driven in both forward and reverse directions, there are two switch terms.

Since the terminations of the non-driving ports remain constant, and the switching between the driving ports is often very repeatable with the help of electronic switches, the switch terms introduce a systematic deviation and only need to be measured once to be considered appropriately. These terms can be regarded as part of the calibration coefficients by the

Software code and measurements are available online: https://github.com/ZiadHatab/vna-switch-terms

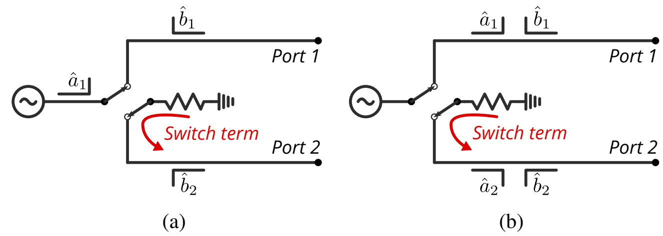


Fig. 1. Illustration of three- (a) and four-sampler (b) architectures of a VNA. Both diagrams depict driving in the forward direction.

conversion relationships between the 12-term and error box models [6]–[9].

In three-sampler VNAs, self-calibration methods based on the error box model cannot be used as a first-tier calibration, as we cannot directly measure the switch terms. However, error box calibration methods can be performed as a second-tier calibration after a SOLT calibration [10]. Such methods require pre-characterized calibration standards, which goes against the purpose of self-calibration methods that use partially defined standards.

This paper aims to introduce a new method to indirectly measure the switch terms using at least three transmissive reciprocal devices, which do not need to be characterized beforehand. The proposed method enables the usage of error box calibration methods in three-sampler VNAs without requiring any first-tier calibration.

II. Mathematical Formulation

A. Problem statement

In a two-port VNA, when all four waves are sampled in both driving directions, the measured S-parameters are described using the following notation [11]:

$$\begin{bmatrix}\hat{b}_{11} & \hat{b}_{12}\\ \hat{b}_{21} & \hat{b}_{22}\end{bmatrix} = \boldsymbol{S}\begin{bmatrix}\hat{a}_{11} & \hat{a}_{12}\\ \hat{a}_{21} & \hat{a}_{22}\end{bmatrix} \tag{1}$$

where $\hat{a}_{ij}$ and $\hat{b}_{ij}$ represent the sampled incident and reflected waves, respectively, at port-i when driven by port-j.

In a three-sampler VNA, the waves $\hat{a}_{12}$ and $\hat{a}_{21}$ are not measured due to a lack of dedicated receivers. To address this, the measured incident waves in (1) can be split into two matrices as follows:

$$\begin{bmatrix}\hat{b}_{11} & \hat{b}_{12}\\ \hat{b}_{21} & \hat{b}_{22}\end{bmatrix} = \boldsymbol{S}\begin{bmatrix}1 & \frac{\hat{a}_{12}}{\hat{a}_{22}}\\ \frac{\hat{a}_{21}}{\hat{a}_{11}} & 1\end{bmatrix}\begin{bmatrix}\hat{a}_{11} & 0\\ 0 & \hat{a}_{22}\end{bmatrix} \tag{2}$$

By taking the inverse of the diagonal matrix on the right-hand side of (2), we obtain the conventionally measured ratios.

$$\begin{bmatrix} \frac{\hat{b}_{11}}{\hat{a}_{11}} & \frac{\hat{b}_{12}}{\hat{a}_{22}} \\ \frac{\hat{b}_{21}}{\hat{a}_{11}} & \frac{\hat{b}_{22}}{\hat{a}_{22}} \end{bmatrix} = \boldsymbol{S} \begin{bmatrix} 1 & \frac{\hat{a}_{12}}{\hat{a}_{22}} \\ \frac{\hat{a}_{21}}{\hat{a}_{11}} & 1 \end{bmatrix} \tag{3}$$

If we define the ratios on the left-hand side of (3) as the measured S-parameters, we can then rewrite the remaining ratios on the right-hand side as follows:

$$\begin{bmatrix} \overline{S}_{11} & \overline{S}_{12} \\ \overline{S}_{21} & \overline{S}_{22} \end{bmatrix} = \boldsymbol{S} \begin{bmatrix} 1 & \overline{S}_{12}\Gamma_{12} \\ \overline{S}_{21}\Gamma_{21} & 1 \end{bmatrix} \tag{4}$$

where $\overline{S}_{ij}$ represents the measured S-parameters and Γ_{ij} represents the switch terms of the VNA:

$$\overline{S}_{ij} = \frac{\hat{b}_{ij}}{\hat{a}_{jj}}, \qquad \Gamma_{ij} = \frac{\hat{a}_{ij}}{\hat{b}_{ij}} \tag{5}$$

The switch terms are formed by the ratios of the receivers of the non-driving port. Therefore, they are independent of the measured DUT, as any influence introduced by the DUT will be seen equally by both waves $\hat{a}_{ij}$ and $\hat{b}_{ij}$. In general, the switch term corrected S-parameters are given as follows:

$$\boldsymbol{S} = \begin{bmatrix} \overline{S}_{11} & \overline{S}_{12} \\ \overline{S}_{21} & \overline{S}_{22} \end{bmatrix} \begin{bmatrix} 1 & \overline{S}_{12}\Gamma_{12} \\ \overline{S}_{21}\Gamma_{21} & 1 \end{bmatrix}^{-1} \tag{6}$$

In the special case where the measured two-port device is transmissionless, the switch terms Γ_{ij} do not influence the measurements as $\overline{S}_{21} = \overline{S}_{12} = 0$.

Using a four-sampler VNA, we can directly measure Γ_{ij} by connecting any transmissive device and calculating the ratios according to the definition in (5). Furthermore, we can measure the S-parameters directly using (1). In contrast to the four-sampler VNA, a three-sampler one can only measure $\overline{S}_{ij}$. Therefore, it is advantageous for three-sampler VNAs to find a way to measure Γ_{ij} without measuring the waves $\hat{a}_{12}$ and $\hat{a}_{21}$, and using only $\overline{S}_{ij}$ measurements.

B. Proposed indirect measurement of the switch terms

Fig. 2 shows the error box model of a two-port VNA. Using T-parameters, the measured DUT is given in terms of wave-parameter as follows [11]:

$$\begin{bmatrix} \hat{a}_{11} & \hat{a}_{12} \\ \hat{b}_{11} & \hat{b}_{12} \end{bmatrix} = \boldsymbol{E}_\mathrm{L}\boldsymbol{T}_\mathrm{D}\boldsymbol{E}_\mathrm{R} \begin{bmatrix} \hat{a}_{21} & \hat{a}_{22} \\ \hat{b}_{21} & \hat{b}_{22} \end{bmatrix} \tag{7}$$

where $\boldsymbol{E}_\mathrm{L}$ and $\boldsymbol{E}_\mathrm{R}$ are the left and right error boxes, and $\boldsymbol{T}_\mathrm{D}$ is the actual DUT.

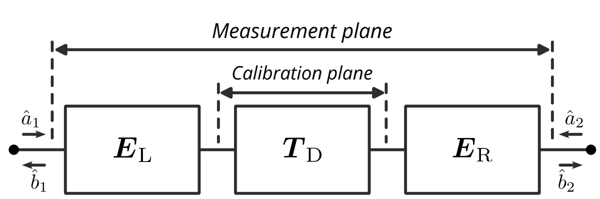


Fig. 2. Two-port VNA error box model.

We split the wave-parameter matrices in (7) into two matrices as follows:

$$\begin{bmatrix} 1 & \frac{\hat{a}_{12}}{\hat{b}_{12}} \\ \frac{\hat{b}_{11}}{\hat{a}_{11}} & 1 \end{bmatrix} \begin{bmatrix} \hat{a}_{11} & 0 \\ 0 & \hat{b}_{12} \end{bmatrix} = \boldsymbol{E}_\mathrm{L}\boldsymbol{T}_\mathrm{D}\boldsymbol{E}_\mathrm{R} \begin{bmatrix} \frac{\hat{a}_{21}}{\hat{b}_{21}} & 1 \\ 1 & \frac{\hat{b}_{22}}{\hat{a}_{22}} \end{bmatrix} \begin{bmatrix} \hat{b}_{21} & 0 \\ 0 & \hat{a}_{22} \end{bmatrix} \tag{8}$$

The above expression can be simplified by multiplying the inverse of the diagonal matrix at the right-hand side. This step reduces all wave parameters into ratios as follows:

$$\begin{bmatrix} 1 & \frac{\hat{a}_{12}}{\hat{b}_{12}} \\ \frac{\hat{b}_{11}}{\hat{a}_{11}} & 1 \end{bmatrix} \begin{bmatrix} \frac{\hat{a}_{11}}{\hat{b}_{21}} & 0 \\ 0 & \frac{\hat{b}_{12}}{\hat{a}_{22}} \end{bmatrix} = \boldsymbol{E}_\mathrm{L}\boldsymbol{T}_\mathrm{D}\boldsymbol{E}_\mathrm{R} \begin{bmatrix} \frac{\hat{a}_{21}}{\hat{b}_{21}} & 1 \\ 1 & \frac{\hat{b}_{22}}{\hat{a}_{22}} \end{bmatrix} \tag{9}$$

The final simplification is to replace the ratios with the definitions established in (5). The rearranged expression is presented in (10).

$$\begin{bmatrix} 1 & \Gamma_{12} \\ \overline{S}_{11} & 1 \end{bmatrix} \begin{bmatrix} 1/\overline{S}_{21} & 0 \\ 0 & \overline{S}_{12} \end{bmatrix} = \boldsymbol{E}_\mathrm{L}\boldsymbol{T}_\mathrm{D}\boldsymbol{E}_\mathrm{R} \begin{bmatrix} \Gamma_{21} & 1 \\ 1 & \overline{S}_{22} \end{bmatrix} \tag{10}$$

Our goal is to extract Γ_{21} and Γ_{12} without prior knowledge of the error boxes or the DUT. We can do this by assuming that the DUT is a reciprocal device, i.e., $\det(\boldsymbol{T}_\mathrm{D}) = 1$. By applying the determinate operator to (10) and using the property that $\det(\boldsymbol{AB}) = \det(\boldsymbol{A})\det(\boldsymbol{B})$, we can derive the following:

$$(1 - \overline{S}_{11}\Gamma_{12})\frac{\overline{S}_{12}}{\overline{S}_{21}} = \underbrace{\det(\boldsymbol{E}_\mathrm{L})\det(\boldsymbol{E}_\mathrm{R})}_{=c \text{ (constant)}}(\Gamma_{21}\overline{S}_{22} - 1) \tag{11}$$

The above expression can be simplified as follows:

$$\frac{\overline{S}_{12}}{\overline{S}_{21}} - \overline{S}_{11}\frac{\overline{S}_{12}}{\overline{S}_{21}}\Gamma_{12} - \overline{S}_{22}c\Gamma_{21} + c = 0 \tag{12}$$

From (12), we can recognize that we have a linear equation in three unknowns: Γ_{12}, $c\Gamma_{21}$, and c. Therefore, if we measure at least three unique transmissive reciprocal devices, we can solve for these unknowns by solving the following linear system of equations:

$$\begin{bmatrix} -\overline{S}_{11}^{(1)}\frac{\overline{S}_{12}^{(1)}}{\overline{S}_{21}^{(1)}} & -\overline{S}_{22}^{(1)} & 1 & \frac{\overline{S}_{12}^{(1)}}{\overline{S}_{21}^{(1)}} \\ \vdots & \vdots & \vdots & \vdots \\ -\overline{S}_{11}^{(M)}\frac{\overline{S}_{12}^{(M)}}{\overline{S}_{21}^{(M)}} & -\overline{S}_{22}^{(M)} & 1 & \frac{\overline{S}_{12}^{(M)}}{\overline{S}_{21}^{(M)}} \end{bmatrix} \begin{bmatrix} \Gamma_{12} \\ c\Gamma_{21} \\ c \\ 1 \end{bmatrix} = \boldsymbol{0} \tag{13}$$

where $M \geq 3$ is the number of measured reciprocal devices. We need at least three distinct measurements to determine the unknowns, since the system matrix must have a rank of 3 to be solved. In general, the uniqueness of the reciprocal standards affects the conditioning of the system matrix.

To solve for the unknowns, we need to find the nullspace of the system matrix. We can estimate the nullspace by applying the singular value decomposition (SVD) to (13). In this case, the best approximate of the nullspace corresponds to the right singular vector that is associated to the smallest singular value [12]. Since the nullspace is only unique up to a scalar multiple, we can solve for the switch terms by taking the ratio of the elements of the nullspace vector as follows:

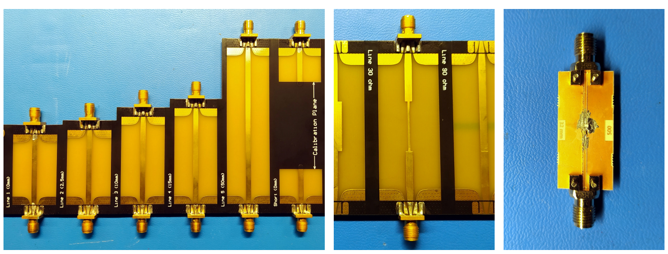
$$\Gamma_{12} = \frac{v_{41}}{v_{44}}, \qquad \Gamma_{21} = \frac{v_{42}}{v_{43}} \tag{14}$$

where $\boldsymbol{v}_4 = [v_{41}, v_{42}, v_{43}, v_{44}]^T$ is the nullspace vector found through the SVD.

III. Experiment

The experiment consists of two parts. In the first step, we tested our proposed method for extracting switch terms using only three of the four available receivers on a VNA. We compared the obtained results to the switch terms computed directly using the fourth receiver. As the second step, we performed a multiline TRL calibration using both switch terms computed directly and indirectly. The results were compared by calibrating a stepped impedance line.

The reciprocal devices that we used consisted of a line standard from the multiline TRL kit (50 mm line) and a series-shunt (L-circuit) of $100\,\Omega$ resistors, which were measured twice by flipping the ports, as it is an asymmetric device ($S_{11} \neq S_{22}$). For the multiline TRL kit, we implemented microstrip lines on an FR4 substrate with a trace width of $3\,\mathrm{mm}$ and a substrate height of $1.55\,\mathrm{mm}$. The lengths of the lines (referenced to the first line) are as follows: $\{0, 2.5, 10, 15, 50\}\,\mathrm{mm}$, with the reflect standard implemented as a short. The standards are shown in Fig. 3.



(a) (b) (c)

Fig. 3. Measured structures. (a) microstrip line multiline TRL kit ($50\,\Omega$), (b) stepped impedance line ($90\,\Omega$), and (c) series-shunt $100\,\Omega$ circuit.

The R&S ZVA is the four-sampler VNA used in this experiment. To extract the switch terms, we measured the aforementioned reciprocal devices and processed the data offline with the help of the *scikit-rf* package in Python [13]. The results, along with those obtained by direct wave ratio computation with the fourth receiver, are shown in Fig. 4. The presented results highlight that both results overlap in magnitude and phase.

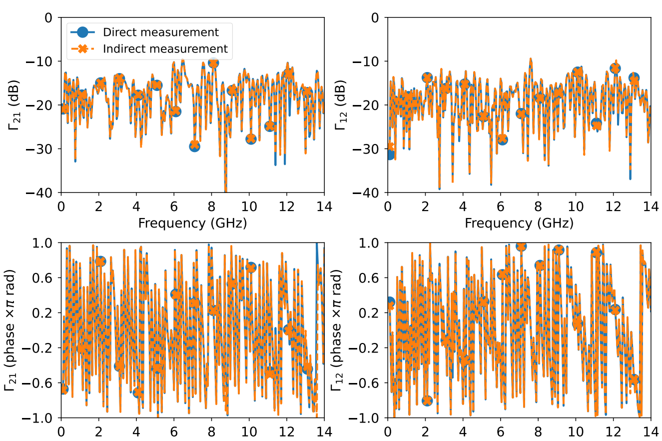


Fig. 4. Comparison of direct and indirect measurements of the switch terms.

Finally, we performed a multiline TRL calibration using the algorithm in reference [14]. In Fig. 5, we present the calibrated results of a stepped impedance line in different scenarios: ignoring the switch terms ($\Gamma_{21} = \Gamma_{12} = 0$), directly measuring the switch terms, and indirectly measuring the switch terms. The results in Fig. 5 show that both directly and indirectly measured switch terms deliver the same results. However, ignoring the switch terms results in noise-like behavior on the traces.

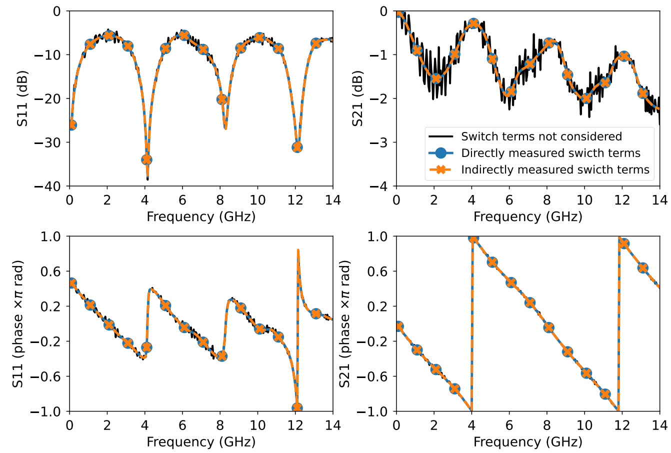


Fig. 5. Results of the multiline TRL calibrated stepped impedance line.

IV. Conclusion

In this paper, we presented an innovative method that utilizes a minimum of three transmissive reciprocal devices to measure the switch terms of a VNA, relying on just three receivers. We applied our method to compute the switch terms using only three receivers through practical measurements conducted on a four-sampler VNA. To validate the accuracy of our results, we compared them with direct measurements obtained using the fourth receiver, revealing comparable outcomes. Furthermore, we successfully demonstrated the implementation of a first-tier multiline TRL calibration, utilizing only three receivers of the VNA.

One significant advantage of the proposed method is that it does not require prior knowledge of the reciprocal devices. For example, one could use electronically controlled resistors at the test ports to quickly obtain the switch terms. This approach is particularly useful for error box calibration methods in multiport VNAs. By using this method, $N+1$ samplers can replace a full-reflectometry architecture with $2N$ samplers, where N is the number of ports. Such a simplification in the setup of a multiport VNA significantly reduces the complexity and cost of the device.

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