This article has been accepted for publication in IEEE Transactions on Industrial Electronics. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TIE.2023.3274855

Discussion on “Semi-Implicit Euler Digital Implementation of Conditioned Super-Twisting Algorithm with Actuation Saturation”

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***Abstract*—Theorem 1 in the discussed paper is shown to be incorrect by means of a simple counterexample. The example shows that the semi-implicit conditioned super-twisting algorithm does not admit a general tuning rule for its parameter κ_1 that can guarantee the same control accuracy as in the absence of actuator saturation.**

***Index Terms*—Counterexample, Input saturation**

THEOREM 1 in [1] is incorrect. The theorem claims boundedness of control error e_k and unsaturated control input u_k by $|e_k| \le h^2 L_1$ and $|u_k| \le F$, respectively, after a finite number of time steps, if the controller parameters κ_1, κ_2 satisfy $\kappa_1 > \sqrt{2\kappa_2 F/(F - L_0)}$ and $\kappa_2 > L_1$, wherein h is the sampling period, L_0 and L_1 are amplitude and slope of a disturbance acting on the plant, and $F > L_0$ is the control saturation level. A counterexample to this claim is now shown.

Consider, for simplicity, sampling period[1] $h = 1$, parameters

$$L_0 = \tfrac{1}{2}, \qquad L_1 = \tfrac{1}{2}, \qquad F = 1, \qquad \kappa_2 = 1 \tag{1}$$

and an *arbitrary* $\kappa_1 > \sqrt{2\kappa_2 F/(F - L_0)} = 2$. Define a periodic disturbance sequence[2] (ε_k) for $k = 0, 1, 2, \dots$ as

$$\varepsilon_k = \begin{cases} 0 & \text{if } k \text{ is even} \\ \frac{1}{2}(-1)^{\frac{k+1}{2}} & \text{if } k \text{ is odd,} \end{cases} \tag{2}$$

which satisfies $|\varepsilon_k| \le L_0$ and $|\Delta_{k+1}| = \frac{1}{h}|\varepsilon_{k+1} - \varepsilon_k| \le L_1$ for all integers k.

Applying (19) or, equivalently, Algorithm 1 from [1] to the plant $e_{k+1} = e_k + h u_k^* + h\varepsilon_{k+1}$ with initial values $e_0 = 0$, $v_0 = -\frac{1}{2}$ then yields periodic sequences

$$(u_k) = (-\frac{1}{2}, \frac{3}{2}, \frac{1}{2}, -\frac{3}{2}, -\frac{1}{2}, \frac{3}{2}, \dots), \tag{3a}$$

$$(u_k^*) = (-\frac{1}{2}, 1, \frac{1}{2}, -1, -\frac{1}{2}, 1, \dots), \tag{3b}$$

$$(v_k) = (-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \dots), \tag{3c}$$

$$(e_k) = (0, -1, 0, 1, 0, -1, \dots) \tag{3d}$$

for unsaturated control input u_k, saturated control input u_k^*, controller state v_k, and control error e_k. Obviously, $|e_k| = 1$ and $|u_k| = \frac{3}{2}$ hold for every odd integer k, contrary to the claim $|e_k| \le h^2 L_1 = \frac{1}{2}$, $|u_k| \le F = 1$ from Theorem 1. Simulation results depicted in Fig. 1 furthermore show that the same effect also occurs with initial condition $v_0 = 0$ and large e_0, as it is typically the case in practical application of the semi-implicit conditioned super-twisting algorithm.

A reason for the invalidity of Theorem 1 is that its proof incorrectly concludes forward invariance of $|u_k| \le F$ from the fact that $|u_k| = F$ implies $|u_{k+1}| - |u_k| \le 0$. In the continuous-time case, which is considered in [2], an analogous reasoning is indeed enough to show forward invariance due to continuity of the trajectory. In the discrete-time case, this reasoning fails, however, because a discrete-time trajectory may skip the case $|u_k| = F$. The counterexample exhibits such a trajectory where this effect additionally deteriorates accuracy.

It is noteworthy that the presented counterexample is independent of the parameter κ_1, because its value becomes relevant in [1, Algorithm 1] only for $|e_k| > h^2\kappa_2 = 1$, which is never the case in (3d). Hence, Theorem 1 stays false also if the condition on the controller gain κ_1 is replaced by any other, more restrictive tuning rule.

References

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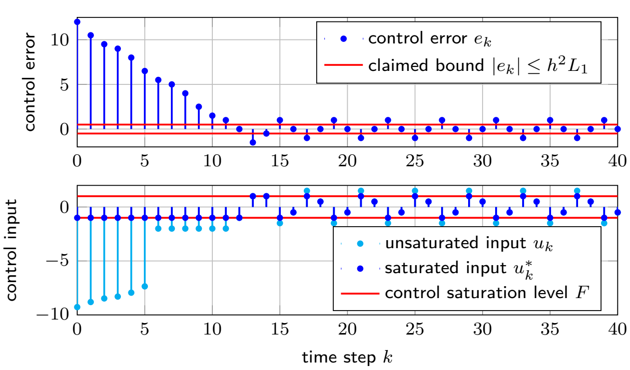

Fig. 1. Simulation results with parameters (1) and $\kappa_1 = 2.1 > 2$, disturbance given by (2), and initial condition $e_0 = 12$, $v_0 = 0$

Work supported by the Christian Doppler Research Association, the Austrian Federal Ministry for Digital and Economic Affairs and the National Foundation for Research, Technology and Development.

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[1] Analogous examples for arbitrary other sampling periods may be obtained by means of time-scaling.

[2] Note that $\varepsilon_k = \varepsilon(kh)$ holds with a continuous-time sawtooth signal ε with period $4h$ satisfying $|\dot{\varepsilon}| \le L_1$ almost everywhere and $|\varepsilon| \le L_0$.