PAMM · Proc. Appl. Math. Mech. **2**, 404–405 (2003) / **DOI** 10.1002/pamm.200310186

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Material Modelling of Porous Media for Wave Propagation Problems

Under the assumption of a linear geometry description and linear constitutive equations, the governing equations are derived for two poroelastic theories, Biot's theory and Theory of Porous Media (TPM), using solid displacements and pore pressure as unknowns. In both theories, this is only possible in the Laplace domain. Comparing the sets of differential equations of Biot's theory and of TPM, they show different constant coefficients but the same structure of coupled differential equations. Identifying these coefficients with the material data and correlating them leads to the known problem with Biot's 'apparent mass density'. Further, in trying to find a correlation between Biot's stress coefficient to parameters used in TPM yet unsolved inconsistencies are found. For studying the numerical effect of these differences, wave propagation results of a one-dimensional poroelastic column are analysed. Differences between both theories are resolved only for compressible constituents.

1. Governing equations

Nowadays, two theories are used to describe the constitutive behaviour of two-phase porous media. Based on the work of von Terzaghi, the so-called Biot theory and, based on the work of Fillunger, the Theory of Porous Media (TPM) was developed. Assuming a linear geometry description and a linear elastic constitutive law for the solid skeleton, the set of governing differential equations is given for both theories.

For the linearised version of TPM, the linearised balance of momentum for the mixture as well as for the fluid is given ($()'$ denotes the material time derivative)

$$n_0^S \varrho_0^{SR} u_i'' + n_0^F \varrho_0^{FR} (u_i'' + w_i') = G u_{i,jj} + \left(K + \frac{1}{3}G\right) u_{j,ji} - n_0^F \left(1 + n_0^S\right) p_{,i} + \varrho b_i \tag{1}$$

$$n_0^F \varrho_0^{FR} [u_i'' + w_i'] = -n_0^F p_{,i} - \frac{\left(n_0^F\right)^2 \gamma^{FR}}{k^F} w_i + n_0^F \varrho^{FR} b_i^F \tag{2}$$

with the solid displacement u_i, the seepage velocity w_i, and the pore pressure p as unknowns. Due to the linearisation, a constant density for the solid ϱ_0^{SR} and for the fluid ϱ_0^{FR} is assumed besides the body force terms ϱb_i and $n_0^F \varrho^{FR} b_i^F$. In these terms, a linear approximation of the density can be inserted. The constant volume fractions are denoted by n_0^S and n_0^F ($n_0^F + n_0^S = 1$), respectively. The solid skeleton is described by the bulk compression modulus K and the shear modulus G. The second term on the right hand side of Eq. (2) represents the interaction force between the solid and the fluid characterised by the permeability k^F and the specific weight γ^{FR} of the fluid. The behaviour of the fluid itself is governed by the continuity equation

$$n_0^F \frac{p'}{R\vartheta} + \varrho_0^{FR} n_0^F \left(1 + n_0^S\right) u_{i,i}' + n_0^F \varrho_0^{FR} w_{i,i} = 0 \tag{3}$$

with reference to the solid coordinate system. In Eq. (3), the continuity equation of the solid and the ideal gas equation is used with the absolute temperature ϑ and the gas constant R.

In 3-d, Eqs. (1-3) are seven equations for seven unknowns: solid displacement u_i, seepage velocity w_i, and pore pressure p. However, from a physical point of view, it is sufficient to describe the problem with four unknowns, e.g., the solid displacement u_i and the pore pressure p, because the fluid is modelled without any shear components and, therefore, can be governed by the scalar pressure. Hence, Eq. (2) is taken to eliminate the seepage velocity in both other equations. Since w_i is given as time derivative in Eq. (2), this is only possible in Laplace domain. Denoting the Laplace transformed functions with $\hat{()}$ a coupled system of partial differential equations for the unknowns solid displacement $\hat{u}_i$ and pore pressure $\hat{p}$ is established ($\varrho_0 = n_0^S \varrho_0^{SR} + n_0^F \varrho_0^{FR}$)

$$G\hat{u}_{i,jj} + \left(K + \frac{1}{3}G\right) \hat{u}_{j,ji} - \left(n_0^F \left(1 + n_0^S\right) - \beta\right) \hat{p}_{,i} - s^2 \left(\varrho_0 - \beta \varrho_0^{FR}\right) \hat{u}_i = \beta \varrho^{FR} \hat{b}_i^F - \varrho \hat{b}_i \tag{4}$$

$$\hat{p}_{,ii} - n_0^F \frac{s^2}{\beta R \vartheta} \hat{p} - \frac{s^2 \varrho_0^{FR}}{\beta} \left(n_0^F \left(1 + n_0^S\right) - \beta\right) \hat{u}_{i,i} = \varrho^{FR} \hat{b}_{i,i}^F \, . \tag{5}$$

In Eqs. (4) and (5), β is an abbreviation dependent on material parameters and the Laplace variable s.

For Biot's theory, the same set of differential equations is achieved when the solid displacement and the pore pressure are again chosen as unknowns. Both theories are essentially different in their material parameters used, i.e., the constant coefficients in Eqs. (4) and (5). Besides the equivalence of obvious material data a direct comparison of the differential equations for both theories yields the identification

$$\frac{\varrho_0^{FR} n_0^F}{R^B} = \frac{1}{R\vartheta} \qquad \text{and} \quad \alpha = n_0^F \left(1 + n_0^S\right) \qquad \text{and} \quad \varrho_a \equiv 0 \, , \tag{6}$$

with Biot's effective stress coefficient α and R^B. Further, as already reported in [1], it is required that the apparent mass density ϱ_a introduced by Biot vanishes. In the special case of incompressible constituents, the factor in front of $\hat{p}$ in Eq. (5) tends to zero and $\alpha = 1$ and acoordingly the term $n_0^F \left(1 + n_0^S\right) = 1$ has to be introduced in Eqs. (4) and (5). More details may be found in [2].

2. Numerical comparison of both theories

A one-dimensional column of length ℓ is considered. It is assumed that the side walls and the bottom are rigid, frictionless, and impermeable. Hence, the displacements normal to the surface are blocked and the column is otherwise free to slide parallel to the wall. At the top, the stress is prescribed and the pore pressure vanishes. At the bottom, the column is fixed and impermeable. Due to these restrictions, only the longitudinal displacement and the pore pressure remain as degrees of freedom. Hence, the governing set of differential Eqs. (4) and (5) is reduced to two scalar coupled ordinary differential equations which are solved using exponential ansatz functions. Subsequently, an Eigenvalue problem yields the final solution in Laplace domain. The response in time domain is determined using the convolution quadrature method.

In Figs. 1 and 2, the displacement at the top of a soil column are depicted versus time for incompressible constituents and for compressible constituents, respectively. In the latter case, large differences between the results

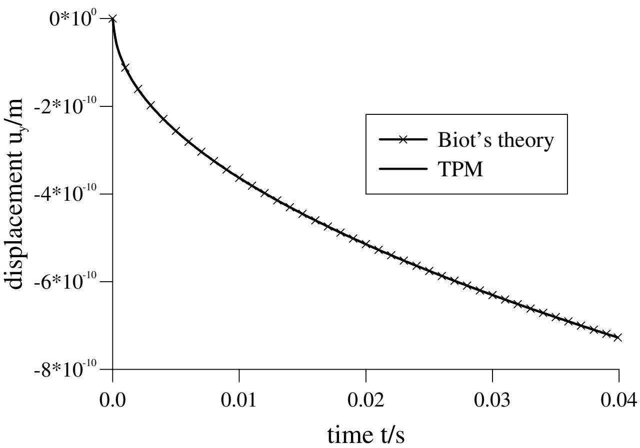

Figure 1: Top displacement $u\left(\ell, t\right)$ versus time for incompressible constituents

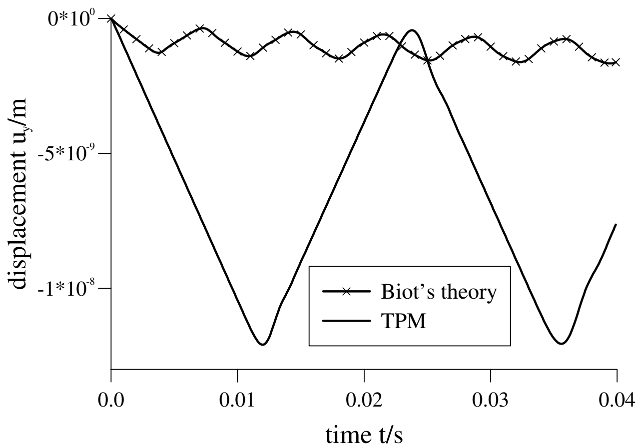

Figure 2: Top displacement $u\left(\ell, t\right)$ versus time for compressible constituents

for TPM and Biot's theory are observed contrary to the incompressible constituents case where no difference is visible. This is caused by different models for the fluid in both theories which becomes evident from the identification $\frac{\varrho_0^{FR}\phi}{R^B} = \frac{1}{R\vartheta}$. Clearly, by changing the value of $R\vartheta$ to unrealistic ones, the same displacement amplitudes as for Biot can be achieved. A more detailed study of this fact is found in [2].

3. References

1 Ehlers, W.; Kubik, J.: On Finite Dynamic Equations for Fluid-Saturated Porous Media. Acta Mechanica. **105** (1994), 101–117.

2 Schanz, M.; Diebels, S.: A Comparative Study of Biot's Theory and the Linear Theory of Porous Media for Wave Propagation Problems. Submitted to Acta Mechanica. (2002).

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