

MATCHING 2D POINT MAPS OF PAPER PROPERTIES FOR MODELLING OF LOCAL PRINT DENSITY

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1. INTRODUCTION

Matching 2D-maps of paper properties is a useful approach to investigate influence factors on print mottle. Fig. 1 shows such an investigation for the influence of local paper brightness on gravure print unevenness for SC paper. The unprinted paper is marked with a corner shaped piece of adhesive tape (Fig. 1 left), scanned in a desktop scanner, printed in a laboratory printing press and scanned again. The images before and after print are matched using the corner mark as coordinate system axes (Fig. 1 middle). Matching of two images consists of translation and rotation of the images, if they have different pixel size they are also rescaled. In each position on the paper local brightness before print and print reflectance after print is extracted from the matched images. After that one can analyze the spatial interrelationships of paper properties (Fig. 1 right). Each point in the scatter plot stands for a small area in a specific position on the paper. A relationship can be observed, regions with higher paper brightness also tend to be brighter after printing ($r^2=0.12$).

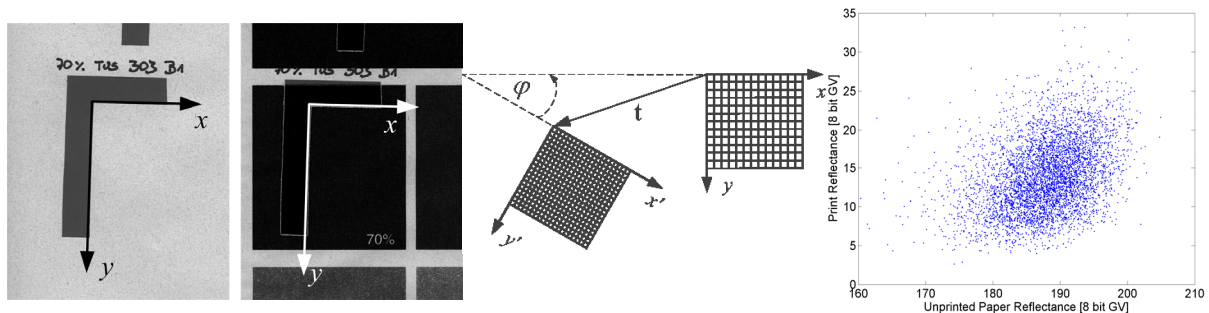


Fig. 1 Image Registration. A corner shaped mark (left) is used to define a coordinate system on the paper before and after printing. Registering two images consists of rotation and translation of the image coordinates (middle). After registration local values of paper brightness before print are correlated to print reflectance after print (right), $r^2=0.12$.

In this paper we will use synthetic images of stochastic structures of defined size to examine three aspects of image matching and image correlation. First it is demonstrated that for correlating images a minimum area has to be evaluated to prevent accidental correlations. Then the effect of matching error on the image correlation is investigated. Finally it is shown, that similarity maximization based image matching techniques may also produce misleading correlations between images.

2. SYNTHETIC IMAGES OF STOCHASTIC STRUCTURES WITH DEFINED SIZE

In order to have images with exactly defined structure size synthetic images were created. The images were generated from Gaussian distributed random numbers. For the smallest structure size a base image is created where each image pixel is assigned one random value (Fig. 2 left). For larger structure sizes a small part of the base image is extracted and rescaled to full image size. Essentially larger structures are obtained by magnifying a small part of the base image using linear interpolation. Example images with varying structure size are shown in Fig. 2. Whereas *absolute structure size* refers to the size in length units (i.e. pixels), *relative structure size* is defined as the absolute structure size divided by image size. The results in the following sections were computed from about 30.000 pairs of such synthetic images for each structure size and image size.

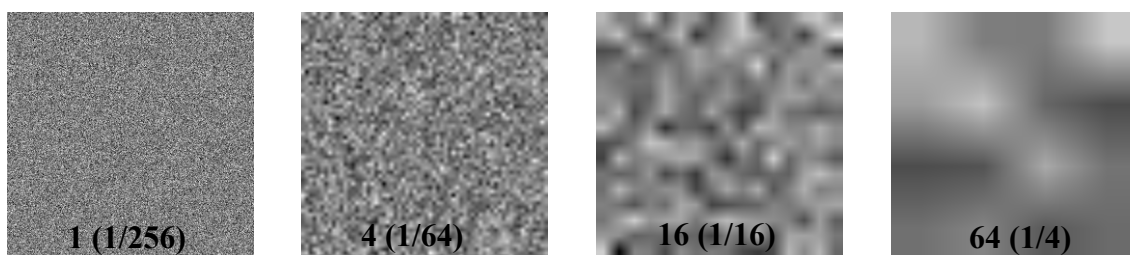


Fig. 2 Synthetic Images with varying structure size. Absolute structure size is varied between 1(left) and 64 pixels. Relative structure size is absolute structure size divided by image size, it varies between 1/256 and 1/4 (in parenthesis).

3. THE INFLUENCE OF MATCHING ERROR ON THE CORRELATION OF IMAGES

An interrelation between different local paper properties can only be detected by property map matching and subsequent correlation if the matching process is 'sufficiently precise'. The effect of matching error has been investigated quantitatively using the stochastic images introduced section 2. If two identical images are matched and correlated (i.e. the grayvalues of the image pixels in the corresponding image positions are correlated, equivalent to Fig. 1, right) the correlation coefficient $r^2=1$. Increasingly displacing the images to each other simulates a matching error. The larger the displacement (matching error) is, the more drops the correlation between the images, Fig. 3. For images with small structure sizes only a minor displacement causes a total collapse of the correlation whereas for larger structure sizes the correlation remains fairly stable even for larger displacements (Fig. 3, left). Redrawing the curves from the left figure with displacement d expressed in multiples of the structure size (instead of the absolute values in pixels) the displacement curves from the left image converge to one single curve, the black bold line in Fig. 3 (right). This demonstrates that the effect of matching error can be directly expressed in terms of absolute structure size. If the matching error (the displacement between images) exceeds structure size ($d>1$) an existing correlation almost disappears. In contrast to that 95% of the correlation between images is preserved if the matching error is below 13.6% of structure size ($d<0.136$).

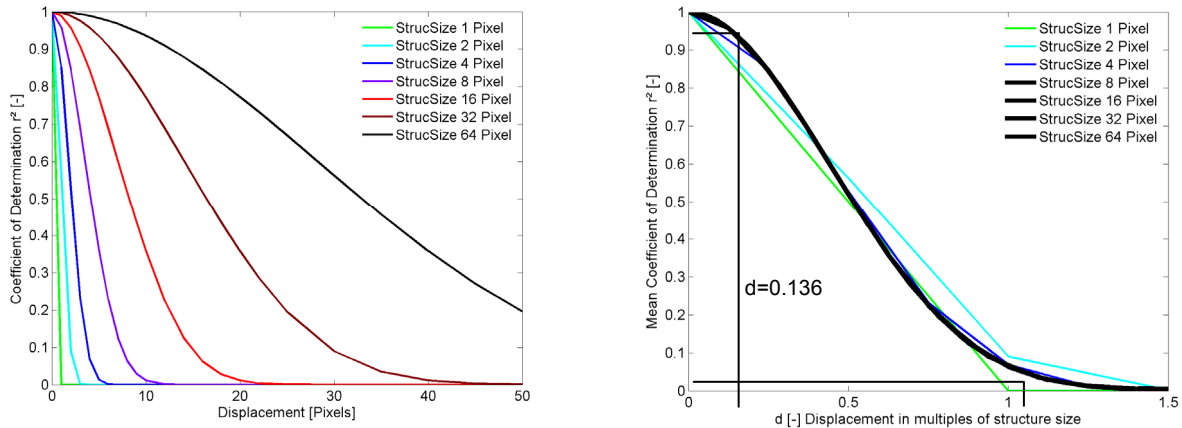


Fig. 3 Influence of matching error on image correlation. With increasing displacement (i.e. matching error) between identical images the correlation decreases. For images with larger structures some of the correlation remains even for considerable displacement (left). The tolerable matching error depends on the size of the structures to be matched (right). If the matching error (image displacement) exceeds structure size ($d > 1$) an existing correlation has almost disappeared. 95% of the correlation between images is preserved if the matching error is below 13.6% of structure size ($d < 0.136$).

These results demonstrate that the accuracy of image matching must be specified depending on the structure of the images. The tolerable matching error is a multiple of the absolute structure size in the image. If the matching error exceeds the structure size an existing correlation between images can not be found any more. Furthermore Fig. 3 (right) gives a quantitative relation between the tolerable matching error and the preserved correlation between structures. For a displacement smaller $d < 0.136$ at least 95% of the correlation between images is preserved ($r^2 > 0.95$). Other important values are: $r^2 > 0.99$ for $d < 0.059$ and $r^2 > 0.90$ for $d < 0.194$.

4. ACCIDENTAL CORRELATION BETWEEN UNRELATED STRUCTURES

Synthetic images were generated, matched to each other *at random positions* and correlated. Random matching of two different images simulates the situation when two unrelated paper properties are measured, matched and correlated. The structures are independent from each other, no correlation is to be expected. The result from these computations is depicted in Fig. 4 (a), it shows the probability density distribution of the correlations between the images. As expected the *mean* correlation is zero, the probability distributions are symmetric to the center value of zero. Still there are some accidental correlations, negative and positive. The broader the distribution is, the higher were the accidental correlations between the random images.

The key observation is that higher correlations appeared for larger relative structure sizes. For random images with small structures (e.g. $\text{Imgsize}/\text{StructureSize}=128$, yellow curve) all correlations were below $r^2=0.01$. For images with larger relative structure size (e.g. $\text{Imgsize}/\text{StructureSize}=8$, purple curve) some correlations already approach $r^2=0.1$. For

even larger relative structure size a correlation coefficient of $r^2=0.25$ or higher is not uncommon. Please note that this effect is independent from the *absolute* structure size, it is controlled by *relative structure size*, see section 0.

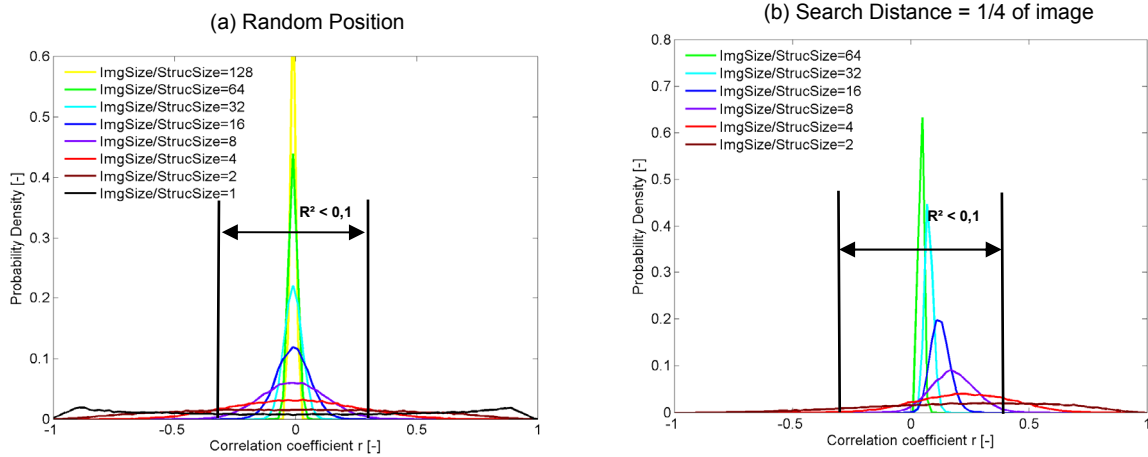


Fig. 4 Correlation of stochastic structures. Figure (a) gives the probability distribution for correlation between random images of increasing structure size. Structure size is given in relation to image size, it varies between 1/1 and 1/128. While it is very unlikely that random images with small structures (e.g. $ImgSize/StructureSize=128$) are correlated, images with larger structures are more likely to be correlated. If random images are matched using cross correlation maximization (b) a systematic correlation between random images is introduced, see section 0. The curves are systematically shifted to the right. This effect is smaller for images with small relative structure size.

From these findings recommendations regarding the sample area to be examined using image matching and correlation can be derived. When correlating images with large relative structure size a larger total area has to be examined to prevent misleading correlations, because it is more likely that an accidental correlation occurs. Fig. 4 (b) is discussed in the following section.

Integrating the probability density functions from Fig. 4 one can compute the probability that a correlation is found although the images are random, Fig. 5. Integrating the curves from Fig. 4 (a) one obtains the bold black curves in Fig. 5. If the image size (i.e. the examined area) is 8 times larger than the structure size it is very unlikely ($p<0.02$) that an accidental correlation larger than $r^2>0.09$ (i.e. $r=\pm 0.3$) occurs, Fig. 5 left. If image size is 16 times larger than structure size the probability for an accidental correlation $r^2>0.04$ is less than 0.5% ($p<0.001$), Fig. 5 right. It can be concluded that if one wants to avoid accidental correlations between matched images it is advisable to evaluate a measurement area between 8 and 16 times the image structure size.

5. INFLUENCE OF MATCHING METHOD ON RESULTING IMAGE CORRELATIONS

For combination of different measurement methods it is usually recommended to use distinct features in the images, so called landmarks, for image matching [1,2,3]. The main advantage of this method is, that the matching works even if the structures in the image are not similar. We use landmark registration with the corner shaped marks as landmarks, see Fig. 1. Other methods align the image by maximization of their similarity using cross correlation [1] or mutual information [2,3] within a given search region. The key point is, that these methods *maximize* similarity i.e. they try to find similarity even if the images are random. As an effect they systematically introduce correlations even when the images are not interrelated. Taking independent random images and applying similarity matching the probability distributions for finding a correlation between the images are systematically shifted to the right, Fig. 4 (b). On average a correlation is measured although the images are random. This effect is more pronounced for images with larger structures, e.g. for a relative structure size being 1/8th of image size (Fig. 4, right, purple curve) correlations between $r=-0.05$ and $r=0.45$ are found, the mean correlation is $r=0.25$.

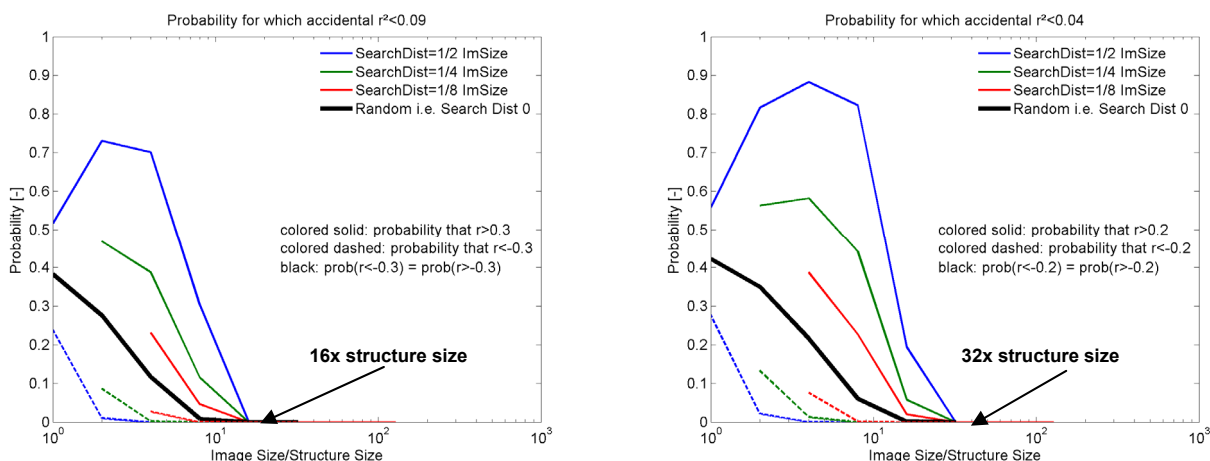


Fig. 5 Probability that two independent stochastic images are correlated. The figures show the probability to obtain misleading correlations due to undersized sample area (black lines) or similarity image matching (colored lines). They give the probability that a correlation of $r>0.3$ is found (left) and the probability that a correlation of $r>0.2$ is found (right). The probability to obtain a fallacious correlation increases for smaller images and for a larger matching search distance.

Also for similarity matching the probability density functions from Fig. 4 (b) can be integrated to obtain probabilities of misleading correlations between random images. The green lines in Fig. 5 give the probability for a correlation using similarity matching within a search region of 1/4 image size, it is integrated from the curves in Fig. 4 (b). The solid green lines stand for the probability that $r>0.3$ (left) respectively that $r>0.2$ (right), the dashed lines give $r<0.3$ (left) and $r<0.2$ (right). The red and blue lines represent other search distances. Similarity maximization matching increases the occurrence of accidental correlations, especially if images are small and the search distance during similarity matching is large, Fig. 5. However similarity matching still may be performed, provided

that the search distance and image size are within certain bounds. If image size is larger than 32 times structure size and search distance is smaller than $\frac{1}{2}$ of image size accidental correlations $r^2 > 0.04$ hardly appear ($p < 0.001$, Fig. 5 right). If image size is larger 16 times structure size and search distance is smaller than $\frac{1}{2}$ of image size accidental correlations $r^2 > 0.09$ hardly appear ($p < 0.001$, Fig. 5 left).

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