Straight Skeletons and Mitered Offsets of Nonconvex Polytopes

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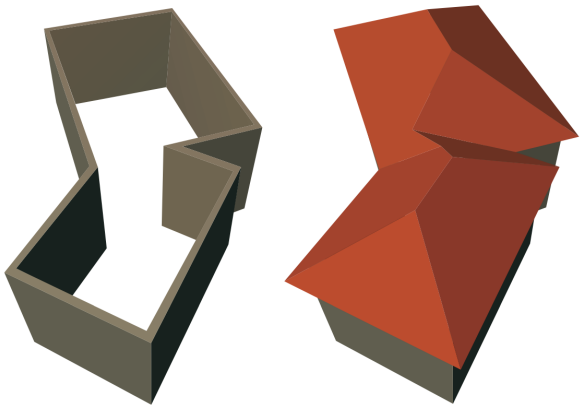
Introduction

- Skeletal structures for geometric objects are widely used concept
- We are interested in **straight skeletons of polytopes**
- Straight skeleton structure is defined by a mitered boundary offsetting process
- Shrinking polytope in a self-parallel way till it vanishes

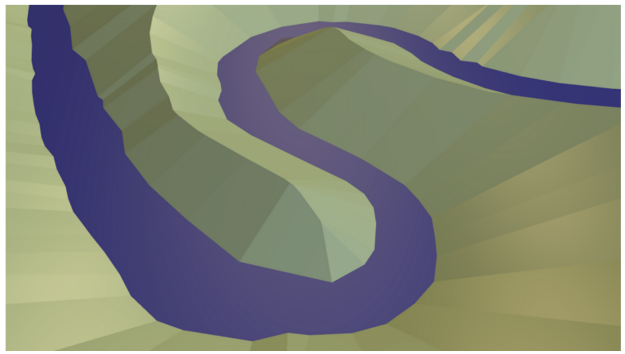
Motivation

- For straight skeletons there are numerous applications:
  - Offset calculation and path generation (e.g. in CAD)
  - Shape comparison and manipulation (e.g. in image processing)
  - Automatic roof design in architecture
  - Terrain and city modelling (e.g. in GIS)

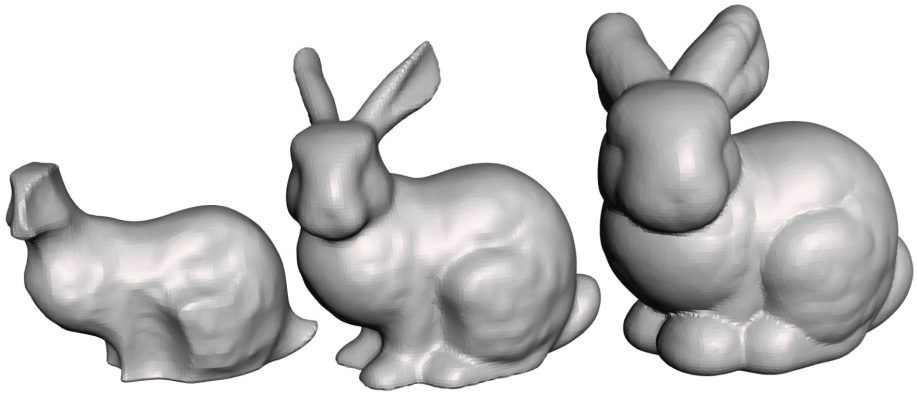
Motivation



[D. Eppstein. A Subquadratic Algorithm for the Straight Skeleton]



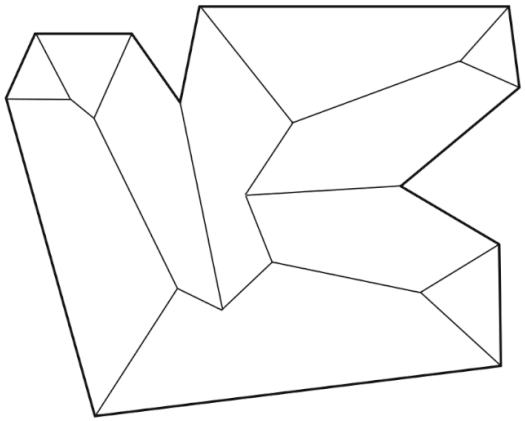
[S. Huber. Computing Straight Skeletons and Motorcycle Graphs: Theory and Practice]



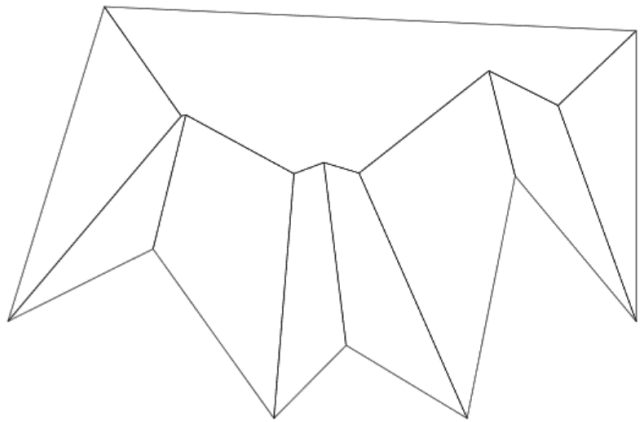
[gradientspace.com]

Straight skeletons in $\mathbb{R}^2$

- Encoding shape of polygon in suitable manner
- Unique graph whose leaf nodes are polygon vertices and arcs stem from angle bisectors



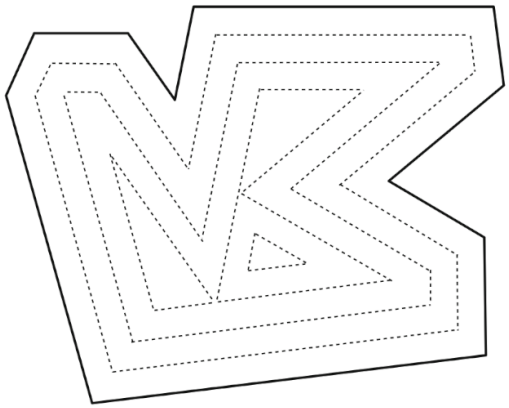
[F. Aurenhammer, G. Walzl. Straight Skeletons and Mitered Offsets of Nonconvex Polytopes]



[F. Cacciola. A CGAL implementation of the Straight Skeleton of a Simple 2D Polygon with Holes]

Mitered offsets of polytopes in $\mathbb{R}^2$

- Boundary of polygon is translated inwards at unit speed
- Shrinking process in $\mathbb{R}^2$ is combinatorically trivial



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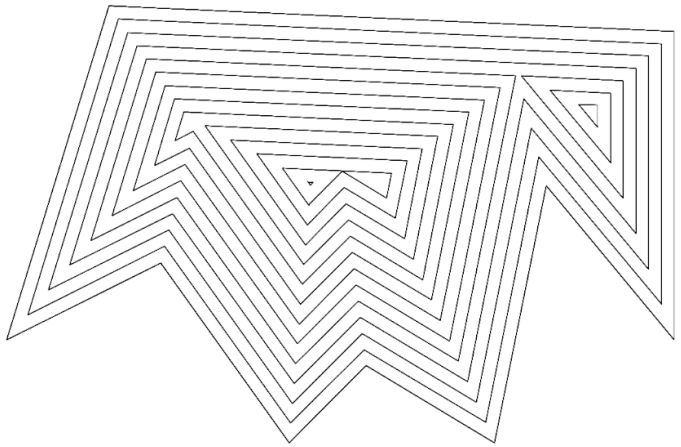
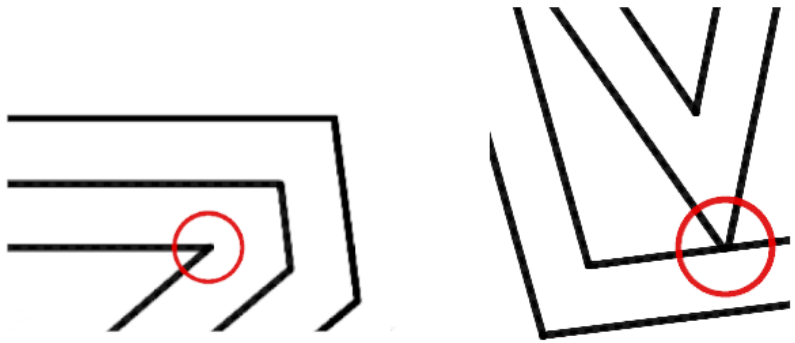


[F. Cacciola. A CGAL implementation of the Straight Skeleton of a Simple 2D Polygon with Holes]

Event detection in $\mathbb{R}^2$

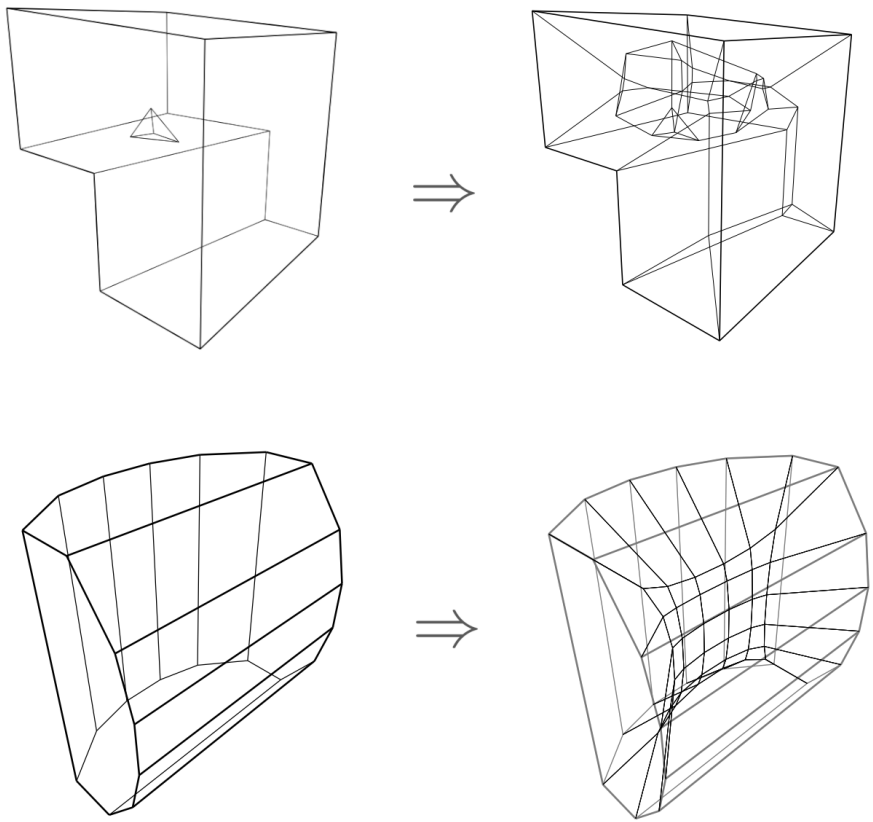
- There are only two kinds of events that alter the polygon boundary combinatorically:
  - *Edge Event*: Polygon edge shrinks to length zero
  - *Split Event*: Polygon vertex runs into a non-incident polygon edge



Straight skeletons in $\mathbb{R}^3$

- Like in $\mathbb{R}^2$, skeleton construction in $\mathbb{R}^3$ complies with shrinking process
- Each component of the polytope traces out a certain component of the skeleton:
  - Facet $\Rightarrow$ *cell*
  - Edge $\Rightarrow$ *sheet*
  - Vertex $\Rightarrow$ *spoke*
- In an event, spokes get an endpoint which we call *corners*

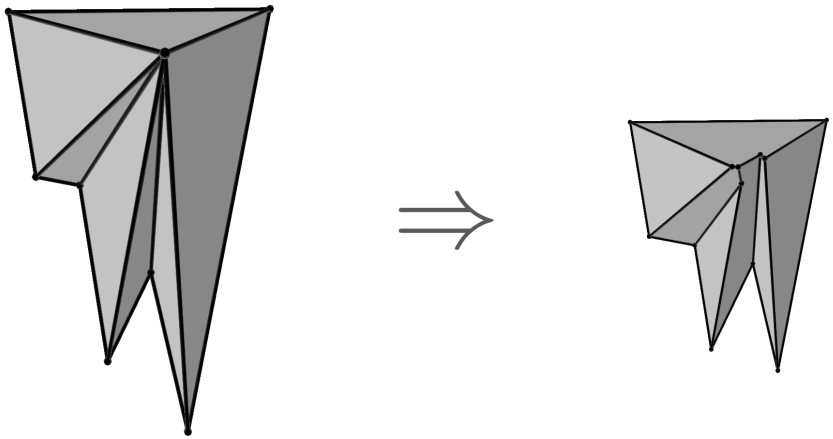
Examples of straight skeletons in $\mathbb{R}^3$



[G. Walzl. STRAIGHT SKELETONS From Plane to Space]

Mitered offsets of polytopes in $\mathbb{R}^3$

- Changes of various kinds: geometrical, combinatorial or topological
- Offset surface may be ambiguous $\Rightarrow$ different choices for the shrinking process and skeleton construction



Event detection in $\mathbb{R}^3$

- An event takes place where more than 3 planes intersect in a vertex
- Initially, various events might happen simultaneously, where vertices of higher degree are split (= initial events)
  - *Side note: In $\mathbb{R}^2$, we do not have initial events, since all vertices of a (valid) input polygon are of degree 2*
- Non-initial events occur later in the offsetting process, when vertices get a higher degree

Event handling in $\mathbb{R}^3$

- Type of event needs not be known in order to process the event correctly
  - *Side note: In $\mathbb{R}^2$, we are not confronted with the event handling problem, since the resolving is trivial*
- Vertex resolution is always possible via its offset arrangement (initial as well as non-initial events)
- Arrangement structure comprises all possible offset surfaces