Mitered Offsets of Polyhedra

Daniel Lederer,
Institute of Theoretical Computer Science

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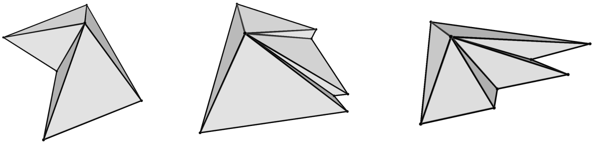
Problem definition

- Any (maybe non-convex) polytope in $\mathbb{R}^3$ is given

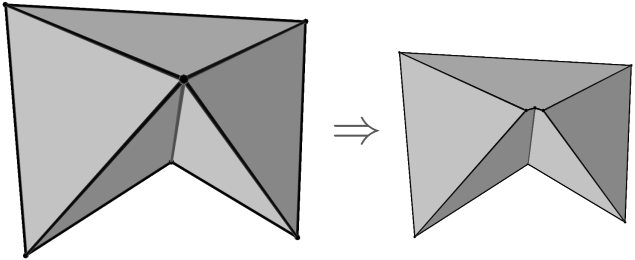


- We shrink the polyhedron: shifting facets inwards, in self-parallel way, with unit speed

$$= \textbf{Offsetting a polyhedron}$$

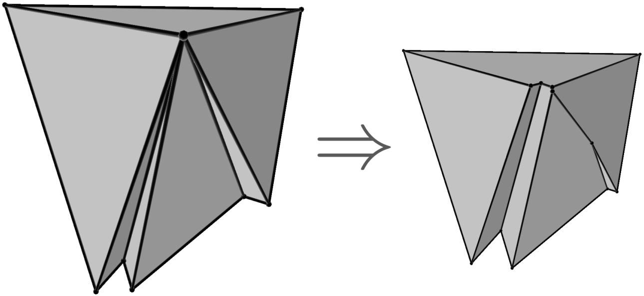
- Changes in geometric manner, structure or topology

Examples of offset surfaces



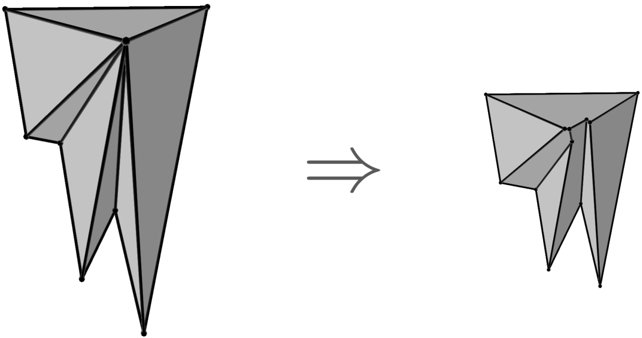
A vertex is split into 3 vertices and additional 2 edges are created

Examples of offset surfaces



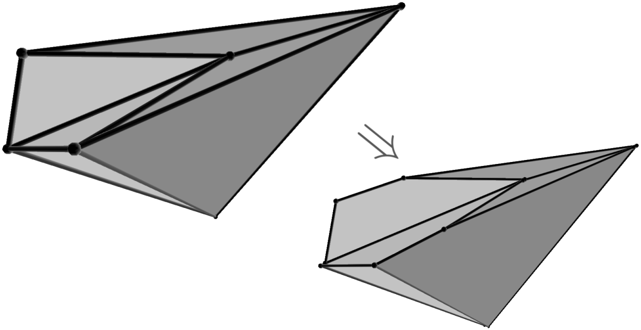
A vertex is split into 5 vertices and additional 4 edges are created

Examples of offset surfaces



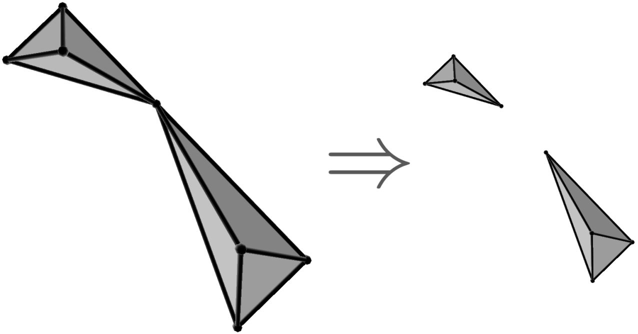
A vertex is split into 5 vertices and additional 4 edges are created

Examples of offset surfaces



Multiple vertices can be resolved at the same time

Examples of offset surfaces



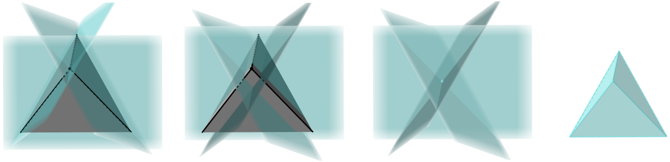
The polyhedron may split up into several parts

Our task

- Developing an algorithm that computes the offset polyhedron
- Algorithm should work for almost all different polyhedron structures
- Algorithm should be implementable and numerically robust
- Software solution using `C++`

Let’s take a closer look...

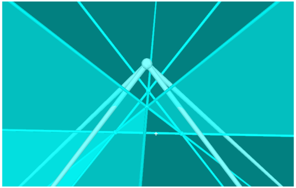
- Consider each vertex separately
- Each adjacent facet lies on a supporting plane
- Take the parallel offset of each plane and compute the intersections (= **arrangement)**



Small example of a 3-degree vertex. We recognize, the offset surface is topologically the same as the original one.

Let’s make it more interesting...

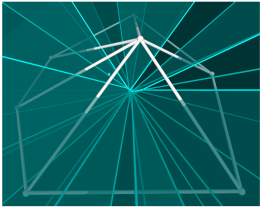
- Consider a vertex with degree 4
- Each plane intersects with every other plane
- Not all arrangement components are relevant for the offset surface



$\Rightarrow$

Arrangement of 4 planes results in $v = 4$, $e = 18$, $f = 28$ components

It doesn’t get any easier...

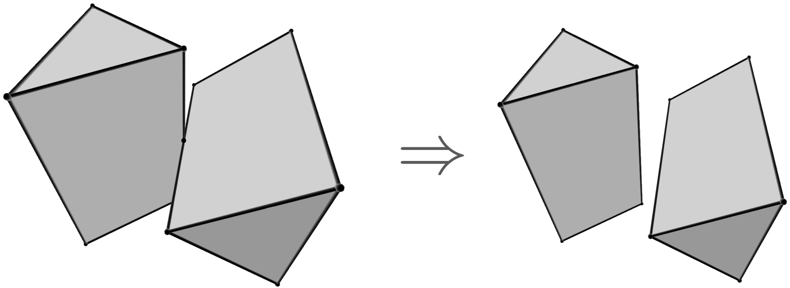


5 planes results in $v = 10$, $e = 40$, $f = 55$ components

6 planes results in $v = 20$, $e = 75$, $f = 96$ components

- As vertex degree increases, the task becomes more and more challenging

Another example



Vertices may also disappear and edges are merged

Solution

Consider each vertex *v* with degree $>$ 3 separately:

1. Shift the planes of facets inwards, adjacent to *v*
2. Compute arrangement of offset planes ($\Rightarrow$ dissection of space)
3. Find arrangement cells that contribute to the offset surface (visibility problem)
4. Merge these relevant cells

$\Rightarrow$ **Offset Surface**