High-precision dynamic orbit integration for spaceborne gravimetry

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Introduction

Dynamic orbits play an important role in the setup of the observation equations in low-low satellite-to-satellite gravity field determination. These orbits are determined through integration of the accelerations acting on a satellite, which can then be added to a known or estimated initial state.

We show investigations into the precision of an improved Encke approach[1] to the numerical integration of dynamic orbits.

Our approach allows for computation of dynamic orbits with repeatability at machine precision over a large swath of the spectral domain.

Methods

We compute 24h dynamic orbit arcs from real data by integrating all acting accelerations (as measured by the accelerometer and computed from gravitational background models) using a polynomial integration approach. An initial orbit is used as a Taylor point for the evaluation of force models.

The integrated orbit is then fitted to GPS observations. We use this fitted orbit as the Taylor point while repeating the integration. After some iterations, the orbit should converge to within machine accuracy.

By comparing the coordinates between two successive iterations of orbit integration, we can determine whether such convergence has occured. If the difference is large, the iteration should be continued. Once the magnitude of the differences does not change significantly between iterations, the integration method has reached its maximum convergence, and iteration can be aborted.

We can thus use this magnitude of the orbit difference between iterations after maximum convergence as an indicator of the integration method's quality.

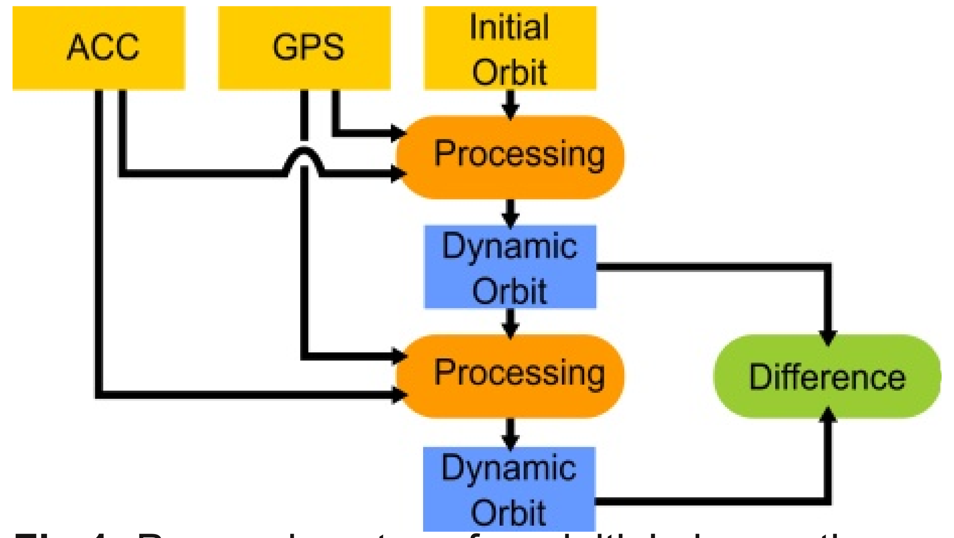

Fig 1: Processing steps from initial observations to comparison result

Encke approach to integration

The position of a satellite along its orbit can be interpreted as the sum of a well-described reference motion and the integral of all acting residual accelerations f not inluded in the reference motion.

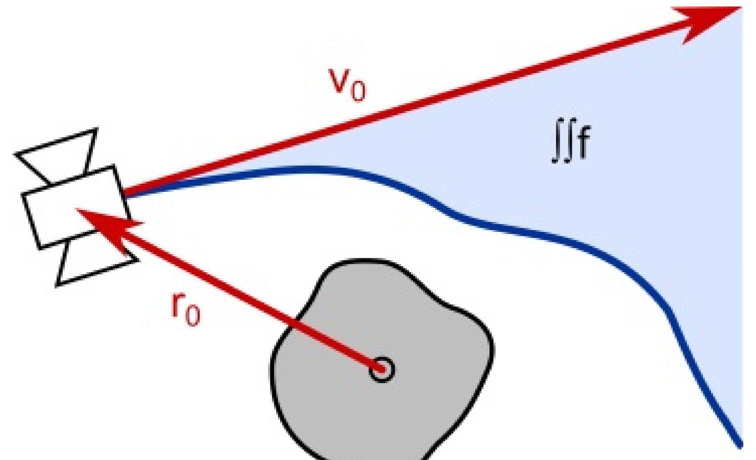

Fig 2: In the simplest case the reference motion is linear, as described by an initial position and velocity r_0, v_0. This may lead to the integrated accelerations f becoming large, and possibly numerically difficult.

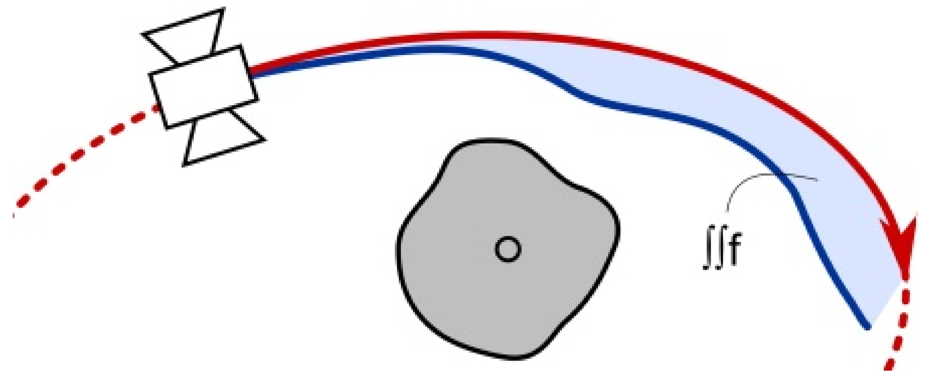

Fig 3: Encke suggests the use of the Kepler ellipse at the first epoch as a reference motion, leading to a smaller integral.

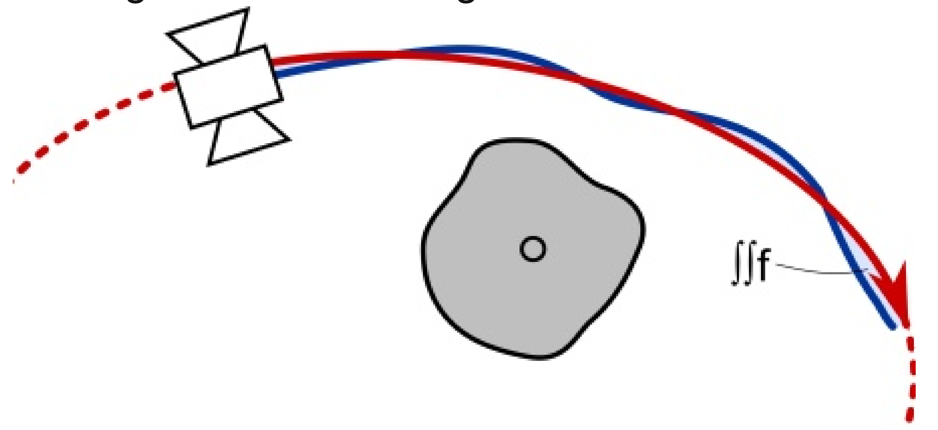

Fig 4: We refine this approach by determining a best-fit orbital ellipse, thus minimizing the energy of the integral of the accelerations.

Separation of reference motion

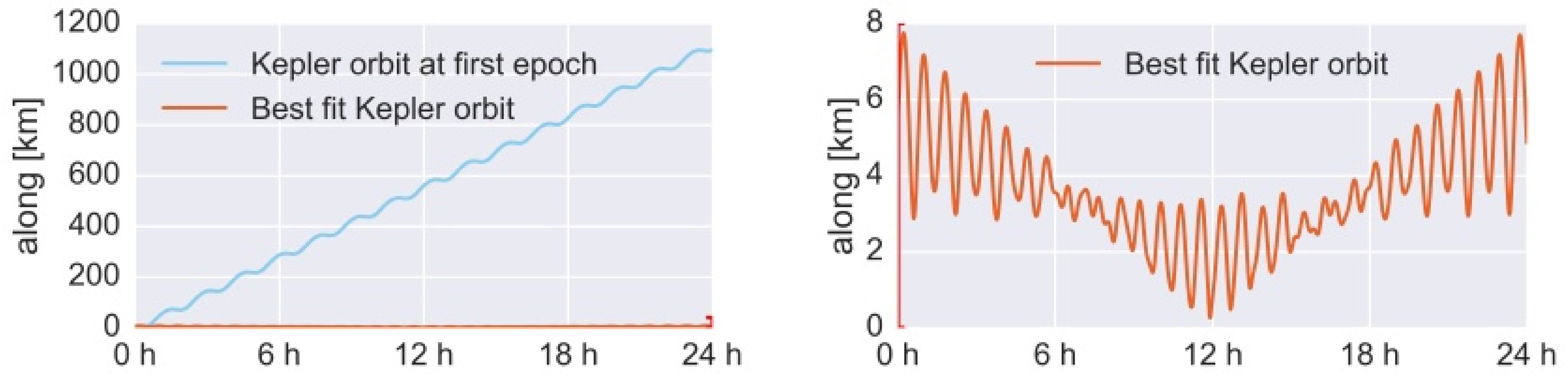

Fig 5: Separation between reference motion and integrated orbit over one day.

Equinoctial elements

The equinoctial elements[2] are non-singular for all eliptical orbits. Position and velocity can be derived from the equinoctial elements with high precision and efficiency, as no trigonometric functions are used. In terms of Kepler elements, the equinoctial elements are given by:

$a = a$ $\qquad h = e\sin(\omega+\Omega)$ $\qquad p = \tan(i/2)\sin\Omega$

$\lambda = M + \omega + \Omega$ $\qquad k = e\cos(\omega+\Omega)$ $\qquad q = \tan(i/2)\cos\Omega$

Precision of improved approach

We inspect the values for one coordinate at a random point along the orbit in two succesive iteration steps. This relates the numeric stability of the coordinates to the machine precision:

Linear motion:	**Best fit equinoctial:**
6436944.405557 93351 m	6436944.4056150 075 m
6436944.405557 85714 m	6436944.4056150 084 m

The improved Encke approach using a best fit Kepler ellipses is stable to 15 decimal digits. A double precision floating point number provides approximately 15 to 17 significant decimal digits.

Coordinate difference between iterations of dynamic orbit integration after maximum convergence

Spatial domain

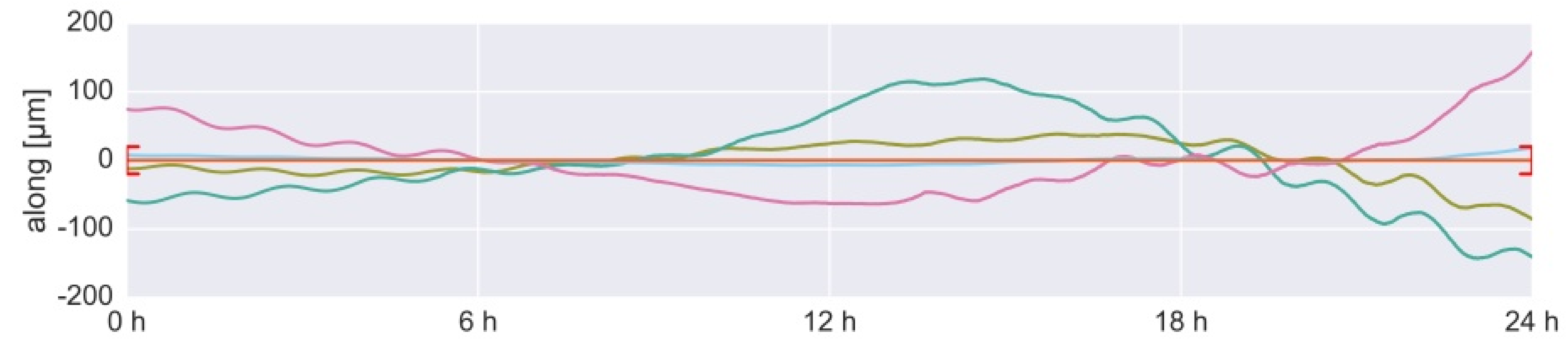

Fig 6a: Shows the difference between orbit iterations in the spatial domain. Integration error is found predominantly in the along-track axis, other axes are omitted.

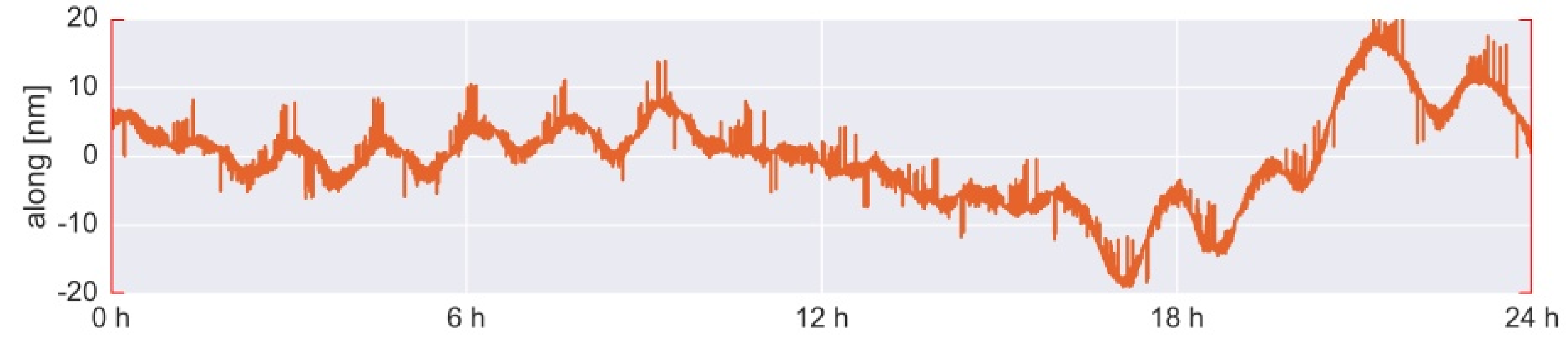

Fig 6b: Detail view of improved Encke method using equinoctial elements. Numerical artefacts due to machine precision visible.

Spectral domain

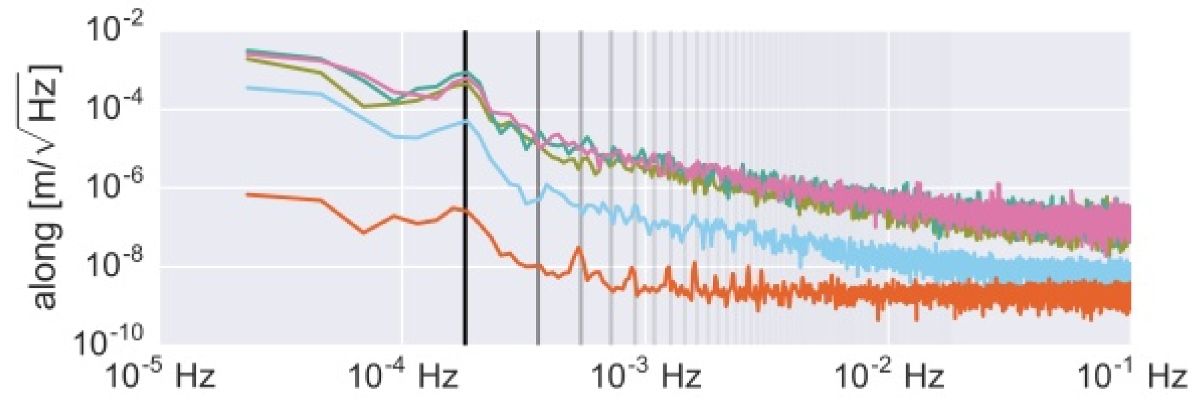

Fig 7: Shows the difference between orbit iterations in the spectral domain. All integration methods have the largest error at a frequency of once per revolution (dark black line). Flat parts of the spectrum indicate that the errors in this band are due to machine precision.

Type of reference motion and parametrization

- Initial position and velocity
- Kepler elements (first epoch)
- Kepler elements (best fit)
- Equinoctial elements (first epoch)
- Equinoctial elements (best fit)

Using a linear reference motion, we observe differences on the order of 100µm between successive integrations. This is magnitudes larger than for example the GRACE K-Band ranging accuracy. As in all other cases, the differences are largest in the along-track component.

Using a first epoch reference ellipse computed with Kepler elements, we can observe no improvement to the integration results over the linear reference motion (see figure 6a). The quality gain from computing a smaller integral is offset by the insufficient accuracy of the reference motion.

Minimizing the forces to be integrated by using a best-fit Kepler ellipse does not lead to better results. The reference motion computed from Kepler elements has insufficient accuracy when comuted in double precision arithmetic.

Going back to a a reference ellipses at the first epoch, use of equinoctial elements for the parametrization leads to significantly smaller deviations between iteration steps, on the order of 20µm (see figure 6a). The overall error in integration is improved by an order of magnitude (see figure 7).

By using a best-fit reference ellipse, we minimize the power of the computed integral. This leads to a deviation between iterations of only machine precision over a large part of the spectrum (see figure 7 and box Precision). Most of the remaining error is at very long wavelengths, starting at ~2/rev.

Propagation to range rate error

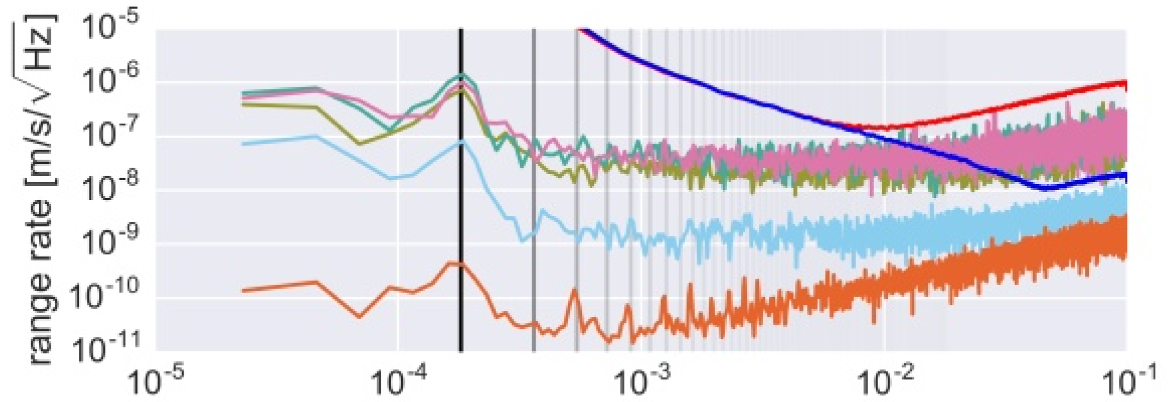

Fig 8: Figure 5: Propagation of orbit noise to range rate measurement. Fat red is a noise model for the GRACE KBR and ACC instrument noise, fat blue is a noise model for the GRACE-FO LRI instrument.

Results

We improved on Encke's method by using a best-fit Kepler ellipses as reference motion for dynamic orbit integration. This replaced an implementation based on a linear reference motion.

Using real GRACE data, we show that using equinoctial elements for the parametrization of this ellipse leads to a substantial increase in precision for the dynamic orbit integration.

Comparison to a noise model of the current GRACE KBR and accelerometer instruments indicates that a simple linear approach is sufficient for GRACE processing. Comparison with the expected LRI instrument noise of GRACE Follow-On demonstrates the applicability of this approach.

A need for even higher precision results would necessitate the consistent use of quadruple precision arithmetic, as the presented approach exhausts the precision available in double precision arithmetics in the high-frequency part of the spectrum.

References and Acknowledgements

[1] Encke, Johann Franz. “Über eine neue Methode der Berechnung der Planetenstörungen.” Astronomische Nachrichten 33
[2] Broucke, R. A., and P. J. Cefola. “On the Equinoctial Orbit Elements.” Celestial Mechanics 5, no. 3

The GRACE L1B data were obtained from the Physical Oceanography Distributed Active Archive Center (PO.DAAC) at the NASA Jet Propulsion Laboratory, Pasadena, CA.

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