Quasi-linear modeling of the interaction of resonant magnetic field perturbations with a tokamak plasma *

M. F. Heyn[1], I. B. Ivanov[1,2], <u>S. V. Kasilov</u>[1,3], W. Kernbichler[1], M. Mulec[1]

[1]*Association EURATOM-ÖAW, Institut für Theoretische Physik - Computational Physics, TU Graz, Austria*
[2]*Petersburg Nuclear Physics Institute, Russia*
[3]*Institute of Plasma Physics, National Science Center "Kharkov Institute of Physics and Technology", Ukraine*

Introduction

In this report the interaction of plasma with the external low frequency resonant magnetic field perturbations (RMPs) which occur in tokamaks due to the coil misalignment (error fields) or are set up deliberately for the creation of ergodic divertor configurations and for ELM mitigation is studied in kinetic approximation.

Basic equations

The form of the kinetic equation in action-angle variables $(\theta^\alpha, J_\alpha)$ convenient for account of collisions and turbulence effects is obtained when these variables are defined using the unperturbed canonical momentum [3],

$$\frac{\partial f}{\partial t} + \Omega^\alpha \frac{\partial f}{\partial \theta^\alpha} + e\left(\tilde{\mathbf{E}} + \frac{1}{c}\mathbf{v} \times \tilde{\mathbf{B}}\right) \cdot \left(\frac{\partial \mathbf{r}}{\partial \theta^\alpha}\frac{\partial f}{\partial J_\alpha} - \frac{\partial \mathbf{r}}{\partial J_\alpha}\frac{\partial f}{\partial \theta^\alpha}\right) = \frac{\partial}{\partial J_\alpha} D_A^{\alpha\beta} \frac{\partial f}{\partial J_\beta}. \qquad (1)$$

Here canonical angles and actions correspond to the gyrophase ϕ, poloidal angle ϑ, toroidal angle φ, perpendicular adiabatic invariant $J_\perp$ (magnetic moment), poloidal action J_ϑ, and toroidal moment p_φ, respectively, Ω^α are canonical frequencies, $\tilde{\mathbf{E}}$ and $\tilde{\mathbf{B}}$ is the RMP field, and the diffusion term in the r.h.s. describes the effect of turbulence in the weak turbulence approximation. In the linear approximation, expanding all quantities in Fourier series over canonical angles, $\tilde{f} = \sum_{\mathbf{m}} f_{\mathbf{m}} e^{im_\alpha \theta^\alpha}$, where $\mathbf{m} = (m_\phi, m_\vartheta, m_\varphi)$, and assuming harmonic dependence on time and radiation gauge for the perturbation field, one has

$$i(m_\alpha \Omega^\alpha - \omega) f_{\mathbf{m}} - \frac{\partial}{\partial J_\alpha} D_A^{\alpha\beta} \frac{\partial f_{\mathbf{m}}}{\partial J_\beta} = Q_{\mathbf{m}}, \qquad (2)$$

$$Q_{\mathbf{m}} \equiv -e\left[(\mathscr{E}_\alpha)_{\mathbf{m}} + \frac{\Omega^\beta}{\omega}\left(m_\alpha (\mathscr{E}_\beta)_{\mathbf{m}} - m_\beta (\mathscr{E}_\alpha)_{\mathbf{m}}\right)\right] \frac{\partial f_0}{\partial J_\alpha}, \qquad \mathscr{E}_\alpha = \mathbf{E} \cdot \frac{\partial \mathbf{r}}{\partial \theta^\alpha}. \qquad (3)$$

Without the diffusion term or with diffusion term replaced by a Krook collision term, $-\nu f_{\mathbf{m}}$, equation (2) was used in [1, 2] to derive the plasma response currents in the straight inhomogeneous cylinder tokamak model using also a specific finite Larmor radius (FLR) expansion. The results of the kinetic model [1] for the first order FLR expansion stay in agreement with results of MHD model [3] in the case of RMP's in DIII-D [2] despite the fact that the applicability of MHD approximation is strongly violated by FLR effects in those conditions.

Role of the anomalous transport, mode locking threshold

In the low frequency range, the plasma response currents are mainly described by the zero

cyclotron (Cherenkov) harmonics, $\mathbf{m}_0 = (0, m_\vartheta, m_\varphi)$ of the distribution function $f_\mathbf{m}$. For estimations, we use a slab model and retain only the spatial component of the diffusion term in Eq. (2),

$$i\left(k_\parallel v_\parallel - \omega\right) f_{\mathbf{m}_0} - D_\perp \frac{\partial^2 f_{\mathbf{m}_0}}{\partial r^2} = Q_{\mathbf{m}_0}, \qquad (4)$$

where $D_\perp = const$ is used for simplicity. Expanding parallel wavenumber $k_\parallel$ up to linear order around the resonant rational magnetic surface, $r = r_\mathrm{res}$, and introducing a new, dimensionless variable x such that

$$v_\mathrm{eff}\, x = k_\parallel' v_\parallel \left(r - r_\mathrm{res}\right) - \omega, \qquad v_\mathrm{eff} = \left(k_\parallel' v_\parallel\right)^{2/3} D_\perp^{1/3}, \qquad (5)$$

Eq. (4) is reduced to the inhomogeneous Airy equation,

$$ixf_\mathbf{m} - \partial^2 f_\mathbf{m}/\partial x^2 = v_\mathrm{eff}^{-1}\, Q_\mathbf{m}, \qquad (6)$$

whose solutions have the smallest scale $x \sim 1$. The effective collision frequency v_eff here is the inverse decorrelation time τ_d, of the wave-particle phase by the combined effect of spatial diffusion and magnetic shear. Indeed, during this time the particle is randomly displaced over radius by $\delta r \sim \sqrt{D_\perp \tau_d}$. Such a displacement leads to the change of $k_\parallel$ and, consequently, to a random change of the wave-particle phase by $\tau_d k_\parallel' v_\parallel \delta r$. The condition that this change is of order one leads to (5). Such a decorrelation mechanism effectively reduces the resonant response current generated by RMP around the rational magnetic surface and, respectively, leads to stronger magnetic field penetration and a higher torque acting onto the plasma from the RMP. Estimating the Coulomb collision frequency v_e which enters the parallel conductivity $\sigma_\parallel = e^2 n_e/(m_e v_e)$ for the parameters typical for low density DIII-D discharges, $n_e = 3 \cdot 10^{13}$ cm^{-3}, $T_e = 3$ keV, one obtains $v_e \sim 5 \cdot 10^3$ s^{-1} while for v_eff using $k_\parallel' = sn_\varphi/(Rr_\mathrm{res})$ where $s = \frac{r}{q}\frac{dq}{dr}$ is the shear parameter and n_φ is the toroidal wavenumber, for $n_\varphi = 3$, $s \sim 1$, $D_\perp = 10^4$ cm^{2}s^{-1}, $R = 170$ cm and $r_\mathrm{res} \sim a = 80$ cm one obtains $v_\mathrm{eff} \sim 10^5$ s^{-1}, i.e., a one order of magnitude higher plasma resistivity. Such an increase of effective parallel resistivity by one order of magnitude is obtained also in the reactor case. Thus, the retainment of the spatial diffusion term only in the turbulent diffusion operator is justified: we suppose here that the turbulent diffusion in the velocity space is much weaker than the collisional diffusion.

Note that unlike the classical resistivity which depends only on temperature, the effective resistivity scales inversely with density so that resistive time $\tau_R = 4\pi r_\mathrm{res}^2 \sigma_\parallel/c^2$ scales with density linearly. Therefore, shielding of RMP's must increase with density too. According to [6] the threshold value of RMP amplitude $\tilde{B}_\mathrm{thr}$ needed to lock plasma rotation is determined by the ratio of the RMP torque and the viscous force which leads in the classical, resistive-inertial regime to the scaling $\tilde{B}_\mathrm{thr}^2 \sim \tau_H^{1/2} \tau_R^{3/4}$, where $\tau_H = R/(sn_\varphi v_A) \sim n_e^{1/2}$ is the Alfven time. Thus, instead of a very weak scaling with density, $\tilde{B}_\mathrm{thr} \sim n_e^{1/4}$, anomalous diffusion leads to the scaling $\tilde{B}_\mathrm{thr} \sim n_e^{1/2}$. This is still different from the experimentally observed linear scaling [4], however the above estimates do not take into account the poloidal mode coupling in the ideal plasma which has also a significant effect on the mode locking threshold [4].

Applicability of linear (quasilinear) approximation

In the low frequency range the main effect of the RMP on the plasma is caused by the radial perturbation magnetic field $\tilde{B}_r$ which leads to radial transport (the same as in ergodic magnetic fields, see [7]) and respective re-distribution of parallel plasma response current. Ignoring for

estimates the electrostatic field $\tilde{E}_\parallel$, the driving term $Q_{\mathbf{m}_0}$ in (4) has the form

$$Q_{\mathbf{m}_0} = -v_\parallel \left(\tilde{B}_r/B_0\right) \partial f_0/\partial r, \tag{7}$$

where B_0 is the main magnetic field. The nonlinear regime is achieved if the nonlinear wave-particle phase shift during the decorrelation time, $\tau_d k'_\parallel v_\parallel \delta r_{\mathrm{NL}}$, is greater than one. Here $\delta r_{\mathrm{NL}} = \tau_d v_\parallel \tilde{B}_r/B_0$ is radial resonant particle displacement due to the perturbation magnetic field. If this displacement is smaller than the random displacement by anomalous diffusion, δr (see above), nonlinear phase shift is negligible and the problem of RMP interaction with plasma can be treated within the linear (quasilinear) approximation. Note that the same result is obtained also from the direct estimation of the nonlinear terms in the kinetic equation (1). Expressing the ratio of displacements via the separatrix half-width, δr_{isl} of the islands created by the RMP,

$$\delta r_{\mathrm{NL}}/\delta r = \delta r_{\mathrm{isl}}^2/(16\delta r^2), \qquad \delta r_{\mathrm{isl}} = 4\left|\tilde{B}_r/(k'_\parallel B_0)\right|^{1/2}, \tag{8}$$

for the DIII-D parameters mentioned above one obtains that linear theory is applicable up to island sizes of the order of $\delta r_{\mathrm{isl}} \sim 1$ cm.

Quasilinear modelling

As shown in Ref [2] the RMP's interact primarily with the electron component of the plasma such that the torque is applied directly to this component. In order to estimate the consequences of this fact in the case of mode locking we use the collisionless quasilinear model. Omitting the turbulent diffusion term in (1), the quasilinear equation is obtained in the usual form,

$$\frac{\partial f_0}{\partial t} = \frac{\pi}{2} \sum_{\mathbf{m}} m_\alpha \frac{\partial}{\partial J_\alpha} |H_{\mathbf{m}}|^2 \delta\left(m_\beta \Omega^\beta - \omega\right) m_\gamma \frac{\partial f_0}{\partial J_\gamma}, \qquad H_{\mathbf{m}} = \frac{ie}{\omega} \Omega^\alpha (\mathscr{E}_\alpha)_{\mathbf{m}}. \tag{9}$$

Calculating the moments of this equation and adding the anomalous and neoclassical transport terms, the following set of balance equations for plasma density n_e, toroidal ion rotation frequency $V_{(i)}^\varphi$, electron and ion temperatures $T_{e,i}$ is obtained,

$$\frac{\partial n_e}{\partial t} + \frac{1}{S}\frac{\partial}{\partial r} S\left(\Gamma_{(e)}^{(EM)} + \Gamma_{(e)}^{(A)}\right) = S_{n_e}, \tag{10}$$

$$m_i n_i \langle g_{\varphi\varphi}\rangle \frac{\partial V_{(i)}^\varphi}{\partial t} - \frac{1}{S}\frac{\partial}{\partial r} S m_i n_i \langle g_{\varphi\varphi}\rangle \mu_\varphi^{(A)} \frac{\partial V_{(i)}^\varphi}{\partial r} + \frac{1}{c}\sqrt{g}B_0^\vartheta \sum_{e,i} e_{e,i}\left(\Gamma_{(e,i)}^{(EM)} + \Gamma_{(e,i)}^{(A)}\right) = S_{p_\varphi}^{(NBI)},$$

$$\frac{\partial}{\partial t}\frac{3}{2}n_{e,i}T_{e,i} + \frac{1}{S}\frac{\partial}{\partial r} S\left(Q_{(e,i)}^{(EM)} + Q_{(e,i)}^{(A)} + Q_{(e,i)}^{(NEO)}\right) = e_{e,i}\frac{\partial\Phi}{\partial r}\Gamma_{(e,i)}^{(EM)} + S_{nT(e,i)}^{(A)} + S_{nT(e,i)}^{(AUX)},$$

where r, S, g, $\langle g_{\varphi\varphi}\rangle$, B_0^ϑ, Φ, m_i and $\mu_\varphi^{(A)}$ are effective radius, flux surface area, metric determinant, flux surface averaged covariant component of the metric tensor, contra-variant poloidal component of the main magnetic field, equilibrium electrostatic potential, ion mass and anomalous toroidal viscosity coefficient, respectively. Here, the anomalous particle and heat fluxes densities $\Gamma_{(e,i)}^{(A)}$ and $Q_{(e,i)}^{(A)}$ are taken from the same simple Onsager-symmetric model as in [2], neoclassical heat flux $Q_{(i)}^{(NEO)}$ is taken into account only for ions, and sources denoted with $S^{(\cdots)}$ in the r.h.s of Eqs. (10) are chosen in order to sustain the initial profiles in absence of the RMP. The quasilinear particle and heat flux densities are given by

$$\Gamma^{(EM)}_{(e,i)} = -n_{e,i}\left(D_{11}A_1 + D_{12}A_2\right), \qquad Q^{(EM)}_{(e,i)} = -n_{e,i}T_{e,i}\left(D_{21}A_1 + D_{22}A_2\right),$$

$$A_1 = \frac{1}{n_{e,i}}\frac{\partial n_{e,i}}{\partial r} + \frac{e}{T_{e,i}}\frac{\partial \Phi}{\partial r} - \frac{3}{2T_{e,i}}\frac{\partial T_{e,i}}{\partial r}, \qquad A_2 = \frac{1}{T_{e,i}}\frac{\partial T_{e,i}}{\partial r}, \tag{11}$$

and diffusion coefficients D_{11}, $D_{12} = D_{21} = (1+Z^2)D_{11}$ and $D_{22} = \left(1+(1+Z^2)^2\right)D_{11}$ are

$$D_{11} = \sqrt{\pi}\left(v_T/B_0\right)^2 |Z/\omega_E|^3\, e^{-Z^2}\left|\omega_E\tilde{B}^r + ck_\perp\tilde{E}_\parallel\right|^2, \qquad Z = \omega_E/\left(\sqrt{2}k_\parallel v_T\right). \tag{12}$$

Here ω_E is the RMP frequency in the moving frame where radial electric field is absent, $k_\perp$ is the perpendicular component of the wavevector within the flux surface and v_T is thermal velocity. It can be checked that whenever quasilinear particle and heat fluxes become comparable with the anomalous fluxes, applicability of the linear (quasilinear) approach is violated. Thus, in the linear limit, the only equation (10) which needs to be solved is the equation for the toroidal velocity $V^\varphi_{(i)}$. Nevertheless, for estimation of the trend we solve the whole set for relatively large RMP amplitudes which are obtained when the rotation is locked. From the results for the steady state shown in Fig.1 it can be seen that the toroidal ion velocity is finite at the resonant surface.

Summary.

Within the kinetic theory it has been shown that the combined effect of the anomalous transport and of magnetic shear leads to an increased effective plasma resistivity which, in turn, causes higher RMP penetration and higher torque than previously expected. One of the consequences is stronger scaling of the mode locking threshold with density. The quasilinear modelling shows that for medium range RMP amplitudes mode locking does not lead to the complete braking of the ion component.

References

[1] M. F. Heyn, I. B. Ivanov, S. V. Kasilov, et al, Nucl. Fusion **46**, S159 (2006).
[2] M. F. Heyn, I. B. Ivanov, S. V. Kasilov, et al, Nucl. Fusion **48**, 024005 (2008).
[3] A. Cole and R. Fitzpatrick, Phys. Plasmas **13**, 032503 (2006).
[4] J. Park, M. J. Schaffer, J. E. Menard, et al, Phys. Rev. Lett. **99**, 195003 (2007).
[5] S. M. Mahajan and C. Y. Chen, Phys. Fluids **28**, 3538 (1985).
[6] R. Fitzpatrick, Phys. Plasmas **5**, 3325 (1998).
[7] I. B. Ivanov, S. V. Kasilov, W. Kernbichler, et al, JETP Letters **86**, 364 (2007).

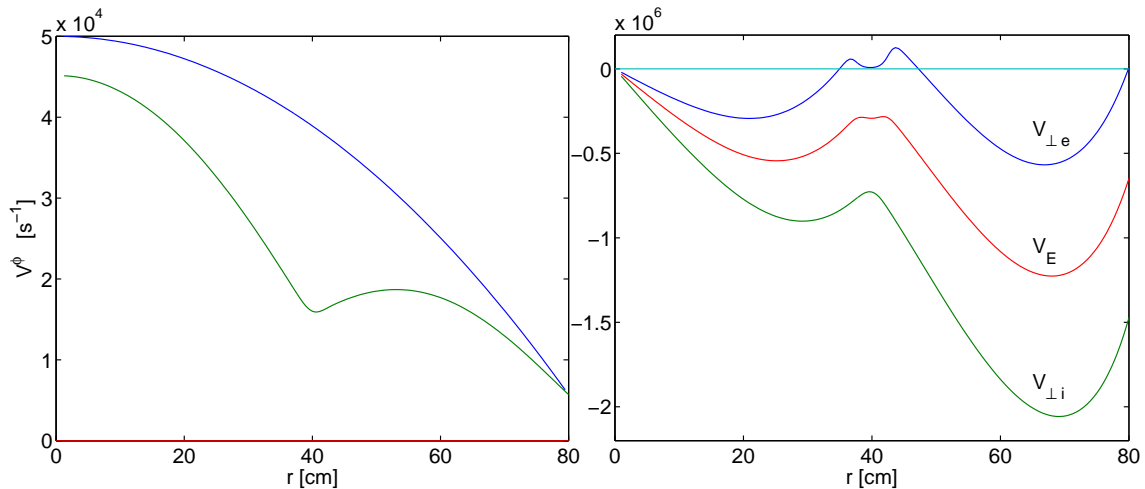Fig. 1. Toroidal rotation frequency $V^\varphi_{(i)}$ with (green) and without (blue) the perturbation field (left). Perpendicular rotation velocities, $V_\perp$ of electrons and ions and electric drift velocity V_E (right).